2023

AP® Physics 2: Algebra-Based

Sample Student Responses and Scoring Commentary

Begin your response to **QUESTION 1** on this page.

PHYSICS 2

SECTION II

Time—1 hour and 30 minutes

4 Questions

Directions: Questions 1 and 4 are short free-response questions that require about 20 minutes each to answer and are worth 10 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

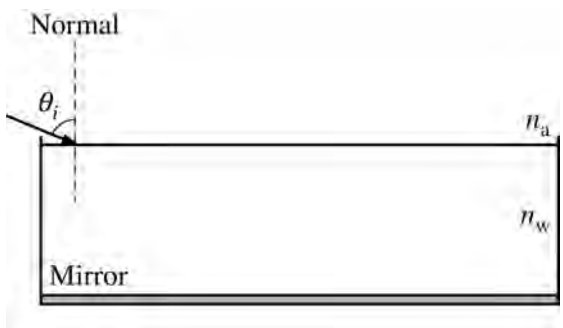
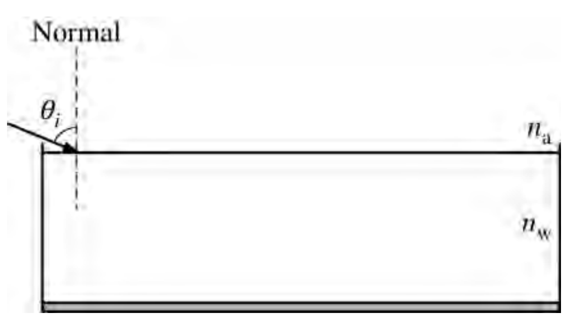


Figure 1

1. (10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.



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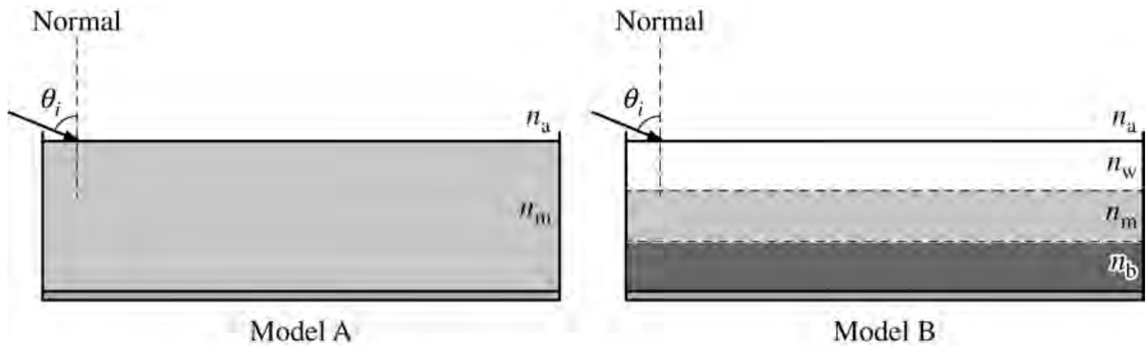

Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).

(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

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Continue your response to **QUESTION 1** on this page.

(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

Model A		Model B	
θ_i	Incident angle of the beam in air	θ_i	Incident angle of the beam in air
θ_1	Angle the beam makes with the normal in the mixture in Model A	θ_2	Angle the beam makes with the normal in the top layer in Model B
		θ_3	Angle the beam makes with the normal in the middle layer in Model B
		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

_____ θ_1 _____ θ_2 _____ θ_3 _____ θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

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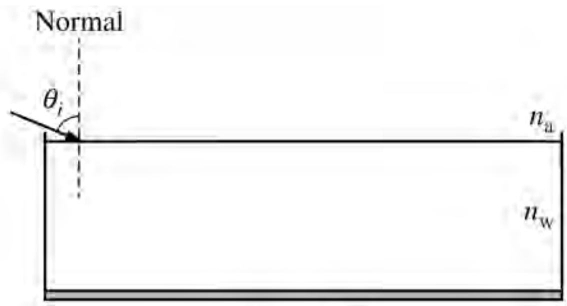
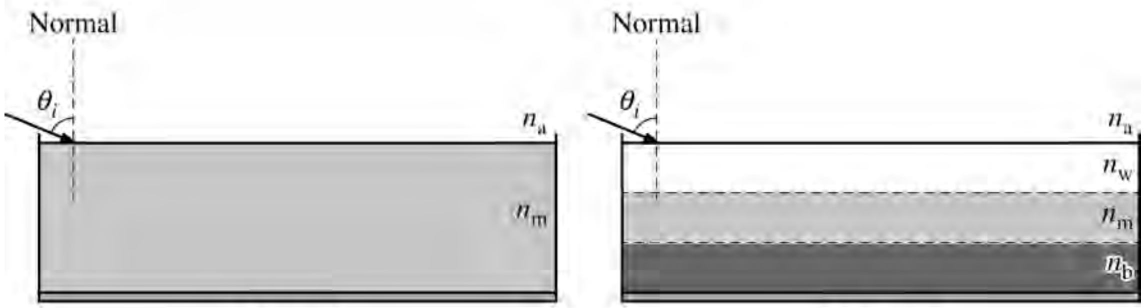


Original Tank



Model A

Model B

For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.

(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

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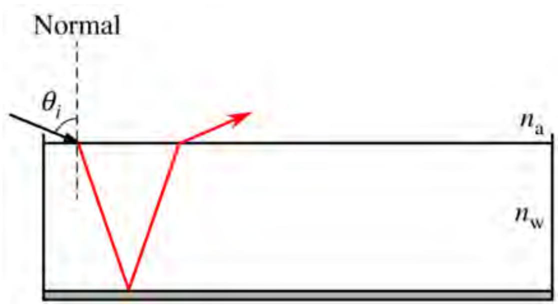
Question 1: Short Answer — 10 points

(a) For drawing a straight-line path from the entry point to the bottom of the tank with an angle from the normal that is less than θ_i — **1 point**

For drawing a continuous path that is symmetric about a vertical axis that intersects the mirror at the location where the beam of light is incident upon the mirror — **1 point**

Example Response



Total for part (a) — 2 points

(b) For indicating that the wavelength of light decreases without any incorrect statements — **1 point**

Example Response

As light travels from one medium to a medium that has a higher index of refraction, the speed of light decreases and the frequency of the light remains the same. Therefore, the wavelength of the light decreases, as described by the equation $\lambda = \dfrac{v}{f}$.

Total for part (b) — 1 point

(c)(i) For a correct application of Snell's law for two media boundaries — **1 point**

Scoring Note: If a test taker correctly applies Snell's law for air and the bottom layer, this point can be earned.

Example Response

$$\theta_4 = \sin^{-1}\left(\frac{n_a}{n_b}\sin\theta_i\right) \text{ OR } \sin\theta_4 = \frac{n_a}{n_b}\sin\theta_i$$

Example Solution

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
$$n_a \sin\theta_i = n_w \sin\theta_2 = n_m \sin\theta_3 = n_b \sin\theta_4$$

$$n_a \sin\theta_i = n_b \sin\theta_4$$
$$\sin\theta_4 = \frac{n_a}{n_b}\sin\theta_i$$
$$\theta_4 = \sin^{-1}\left(\frac{n_a}{n_b}\sin\theta_i\right)$$

(c)(ii)	For indicating that θ_4 alone is the smallest angle	**1 point**
	For indicating that θ_2 alone is the largest angle	**1 point**
	For indicating that $\theta_1 = \theta_3$	**1 point**
	For an explanation that correctly relates the index of refraction to an angle	**1 point**

Example Response

__2__ θ_1 __1__ θ_2 __2__ θ_3 __3__ θ_4

θ_2 has the greatest value because water has the lowest index of refraction. θ_1 and θ_3 are equal because each is in the same layer with the same index of refraction, but the angles are smaller than θ_2 because the index of refraction is larger in this layer. θ_4 has the smallest value because the bottom layer has the highest index of refraction.

Total for part (c) 5 points

(d)	For indicating that both d_A and d_B are less than d_w, with an attempt at a relevant explanation	**1 point**
	For correctly indicating that the horizontal distance traveled decreases with increasing refraction toward the normal	**1 point**

Example Response

Horizontal distances d_A and d_B are less than d_w. The light rays for all scenarios are entering from air. However, in models A and B, the light rays enter a medium with an index of refraction that is greater than that of water. Therefore, the light rays bend more toward the normal in models A and B than in the original tank. Bending more toward the normal results in a shorter horizontal distance traveled.

Total for part (d) 2 points

Total for question 1 10 points

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS 2

SECTION II

Time—1 hour and 30 minutes

4 Questions

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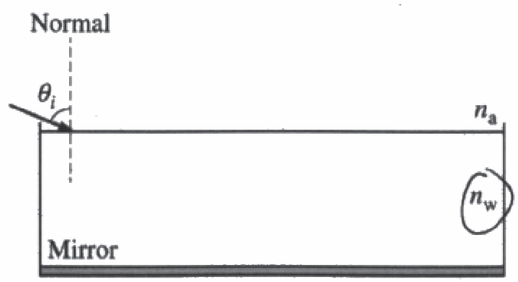


$n_a \sin\theta_i = n_w \sin\theta$

$n_w > n_a$

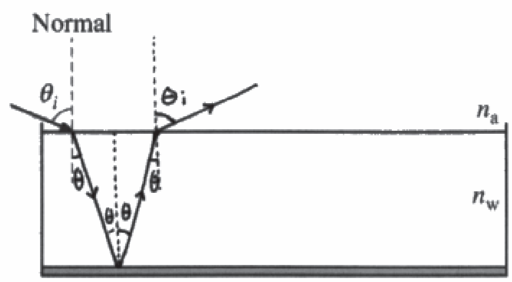
$\sin\theta < \sin\theta_i$

Figure 1

1. (10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.



Question 1

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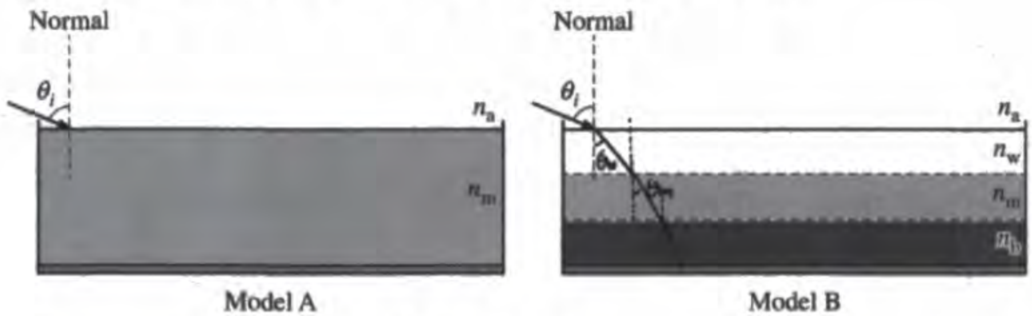

$n_m > n_w > n_a$ Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).

(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

As light enters the mixture, the observed wavelength decreases. Because n_m is larger than n_a, wavelength in the mixture is smaller than that in air as frequency remains constant.

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Q5198/3

Question 1

Continue your response to QUESTION 1 on this page.

(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

Model A		Model B	
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		θ_3	Angle the beam makes with the normal in the middle layer in Model B
		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

$n_a \sin\theta_i = n_w \sin\theta_2 = n_m \sin\theta_3 = n_b \sin\theta_4$

$\sin\theta_4 = \frac{n_a \sin\theta_i}{n_b}$ → $\therefore \theta_4 = \sin^{-1}\left(\frac{n_a \sin\theta_i}{n_b}\right)$

$\theta_4 = \sin^{-1}\left(\sin\theta_i \cdot \frac{n_a}{n_b}\right.$

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

2 θ_1 1 θ_2 2 θ_3 3 θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

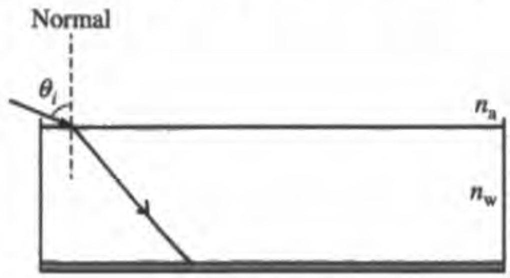
According to Snell's Law, in model B, $n_a \sin\theta_i = n_w \sin\theta_2 = n_m \sin\theta_3 = n_b \sin\theta_4$.

In model A, $n_a \sin\theta_i = n_m \sin\theta_1$. As $n_a < n_w < n_m < n_b$, θ_2 from $n_w \sin\theta_2$ has the greatest value. When comparing θ_1 and θ_3, ~~both~~ $n_m \sin\theta_1 = n_m \sin\theta_3 = n_a \sin\theta_i$, therefore $n_m \sin\theta_1 = n_m \sin\theta_3$. As $\sin\theta_1 = \sin\theta_3$, $\theta_1 = \theta_3$. As n_b is the largest, θ_4 from $n_b \sin\theta_4$ is smallest.

Question 1

Continue your response to **QUESTION 1** on this page.

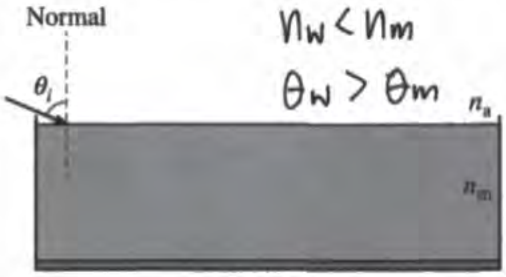
$n_a \sin\theta_i = n_w \sin\theta_w$



Original Tank

$n_w < n_m$

$\theta_w > \theta_m$



Model A

Model B

For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.

(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

Comparing d_A and d_w, d_A is less than d_w. Because $n_w < n_m$, angle of refraction in water is larger than that in model A. As the angle of refraction and reflection on mirror and therefore horizontal distance is larger, $d_w > d_A$.

d_B is also less than d_w. While both angles of refraction in water are the same, in model B, as the beam enters different liquids with higher indexes of refraction, the angles of refraction reduce. Therefore, the angle of reflection when hitting the mirror is less as well, resulting in a smaller d_B.

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Question 1

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PHYSICS 2

SECTION II

Time—1 hour and 30 minutes

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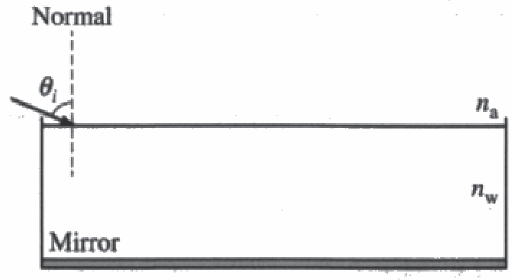

Figure 1

1. (10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

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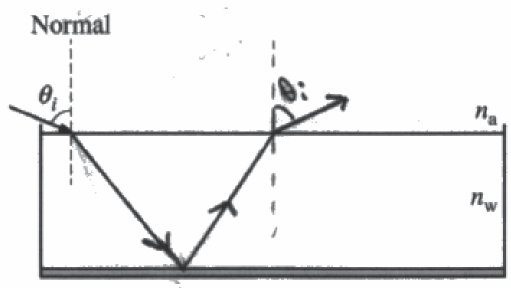

Question 1

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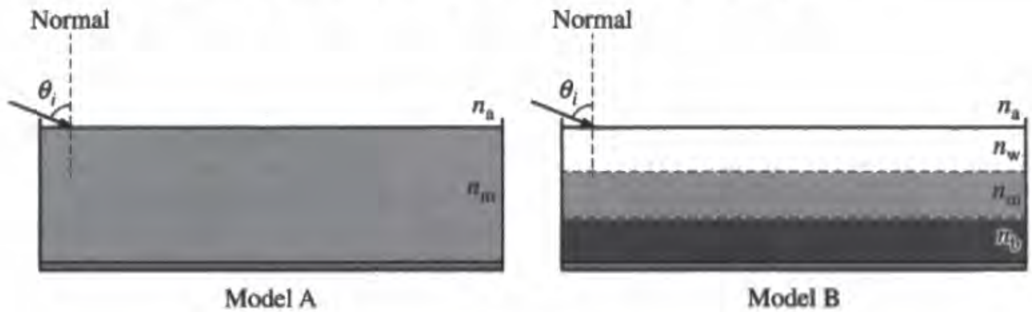

Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).

(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

The wavelength of light traveling into the mixture will be less than the wavelength of light traveling into the water. Because the index of refraction of the mixture is greater than that of the water, the speed of the light will be slower in the mixture. This means the light will be shorter in the mixture.

Question 1

Continue your response to QUESTION 1 on this page.

(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

Model A		Model B	
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		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

$n_a \sin\theta_i = n_b \sin\theta_4$

$\sin\theta_4 = \frac{n_a \sin\theta_i}{n_b}$

$\theta_4 = \sin^{-1}\left(\frac{n_a \sin(\theta_i)}{n_b}\right)$

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

2 θ_1 1 θ_2 2 θ_3 4 θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

The larger the index of refraction is, the larger the angle will be compared to the normal. Because the middle layer of model B has the same index of refraction as model A, they will have the same angle.

Question 1

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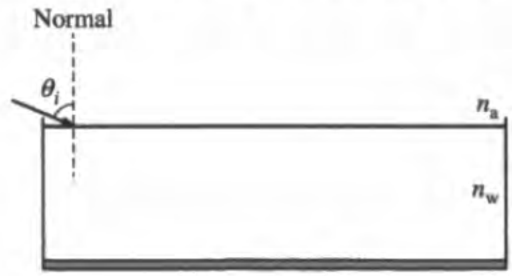
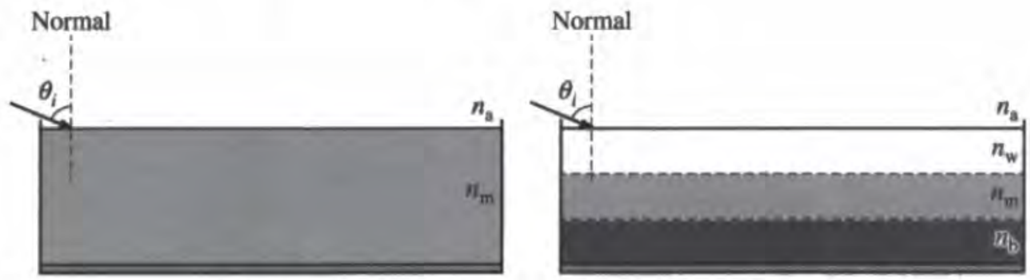


Original Tank



Model A   Model B

For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.

(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

dA is less than dw because the angle the light takes as it is in the tank in model A is larger than the angle in the water. This means the light in model A will hit the mirror closer in model A and will therefore be closer as it exits. The same is true for model B where dB is less than dw. Because the average index of refraction in model B is greater than in the original tank, the light will exist closer to where it entered.

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Question 1

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PHYSICS 2

SECTION II

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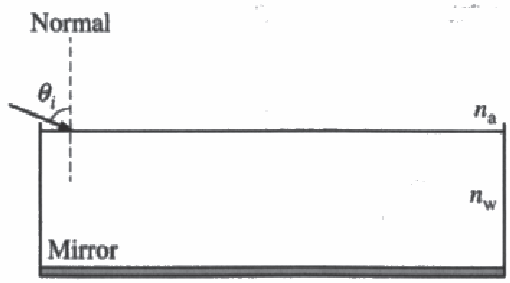

Figure 1

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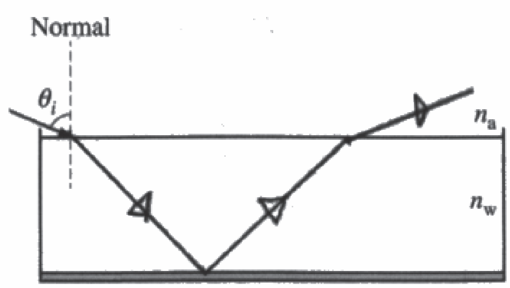

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Question 1

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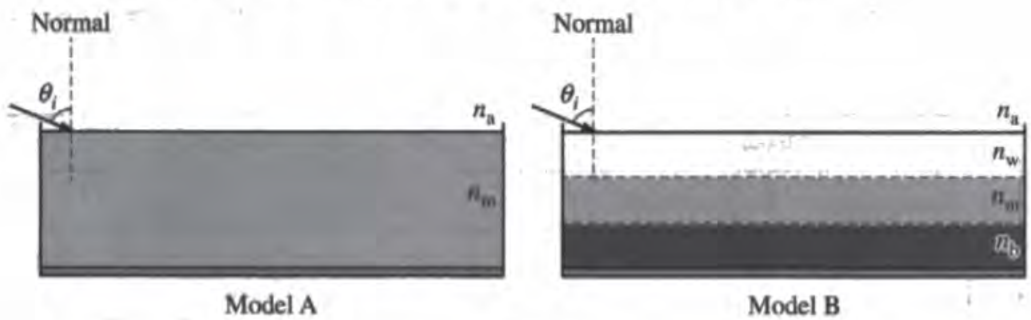

Figure 2

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(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

As the light goes into the mixture the wavelength decreases because $n_m > N_a$ which means we can use $n = \frac{c}{v}$ and make it $v = \frac{c}{N}$ to get the speed of light in the mixture and then use $\lambda = \frac{v}{f}$ and because v is lowering since $\frac{n_a}{n_m}$ is less than 1 and the frequency is constant λ must decrease.

Question 1

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i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

$n_a \sin\theta_i = n_w \sin\theta_2$ $\quad n_w \sin\theta_2 = n_m \sin\theta_3$

$n_m \sin\theta_3 = n_B \sin\theta_4$

$n_a \sin\theta_i = n_B \sin\theta_4$

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

1 θ_1 2 θ_2 3 θ_3 4 θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

Becase the lower into the substance the higher index of Refraction is which means the new angle is $\sin^{-1}\left(\frac{n_a \sin\theta}{n_w}\right)$ and this means the angles get progressivly smaller as in $n_a \sin\theta_i = n_B \sin\theta_4$ if nb is higher and $n_a \sin\theta_i$ is constant $\sin\theta_4$ must be lower

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Question 1

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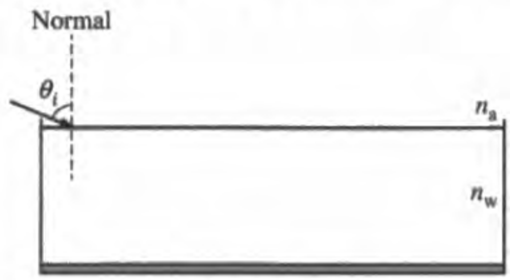
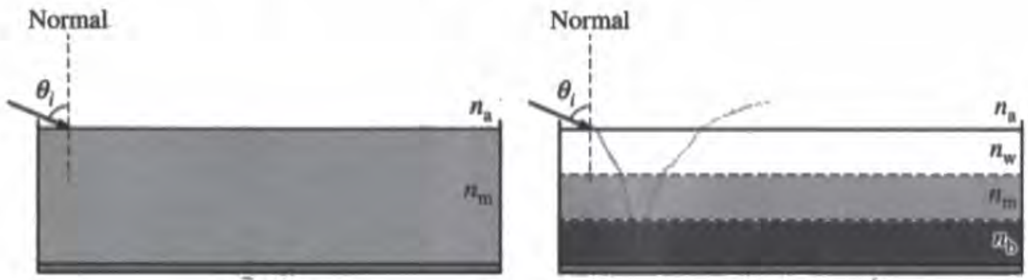


Original Tank



Model A Model B

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(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

$d_A < d_w$ because the water or mixture has a higher n or index of refraction than water it will hit the bottom at a steeper angle and return at the same angle which is smaller than the angle created by the original tank meaning it will travel less distance. The same applies for d_B except d_B is even smaller as the indexes of refraction progressively increase which means the angle will get progressively smaller and wont travel as far.

Question 1

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4 Questions

Directions: Questions 1 and 4 are short free-response questions that require about 20 minutes each to answer and are worth 10 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

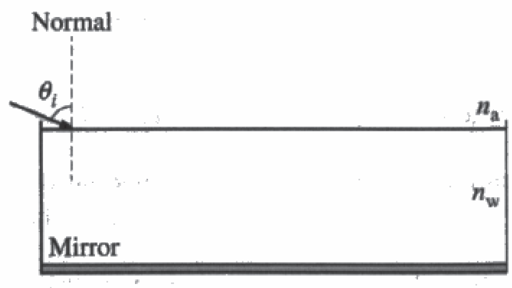
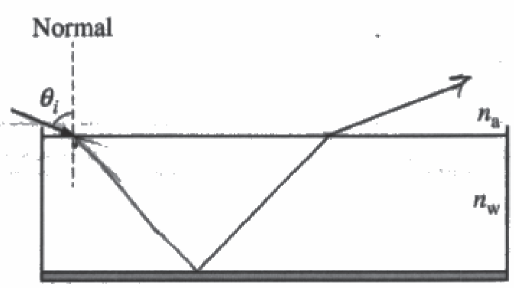


Figure 1

1. (10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.



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Question 1

Continue your response to QUESTION 1 on this page.

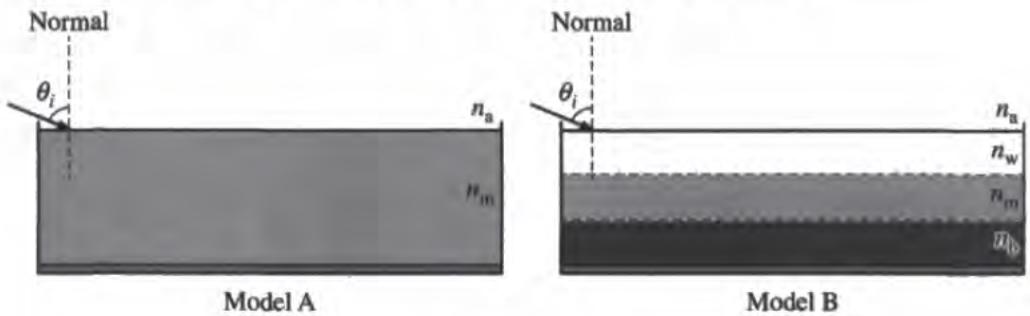

Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).

(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

As the beam travels from air into the mixture, the ~~[illegible]~~ magnitude of the wavelength ~~[illegible]~~ ~~or~~ stays the same but shifts directions.

Question 1

Continue your response to **QUESTION 1** on this page.

(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

Model A		Model B	
θ_i	Incident angle of the beam in air	θ_i	Incident angle of the beam in air
θ_1	Angle the beam makes with the normal in the mixture in Model A	θ_2	Angle the beam makes with the normal in the top layer in Model B
		θ_3	Angle the beam makes with the normal in the middle layer in Model B
		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

$$n_a \sin\theta_i = n_b \sin\theta_4$$

$$\sin\theta_4 = \frac{n_a \sin\theta_i}{n_b}$$

$$\theta_4 = \sin^{-1}\left(\frac{n_a \sin\theta_i}{n_b}\right)$$

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

1 θ_1 2 θ_2 3 θ_3 4 θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

~~[illegible]~~

According to $n_1 \sin\theta_1 = n_2 \sin\theta_2$, as the index of refraction increases, the angle has to decrease and the light is ~~[illegible]~~ entering mixtures with the greater index of refractions ~~[illegible]~~. So its angle must be decreasing.

Question 1

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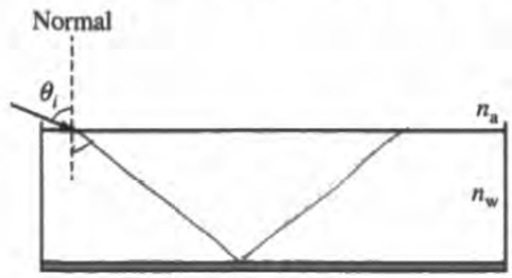
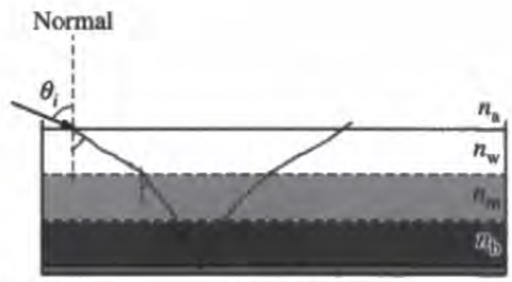


Original Tank

Model A



Model B

For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.

(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

Both d_A an d_B are less than d_w because Tanks A and B are filled with substances with greater index of refractions or a mixture of equal to and greater index of refractions, so the angle decreases, leading to an exit point closer to the entry point.

GO ON TO THE NEXT PAGE.

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS 2

SECTION II

Time—1 hour and 30 minutes

4 Questions

Directions: Questions 1 and 4 are short free-response questions that require about 20 minutes each to answer and are worth 10 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

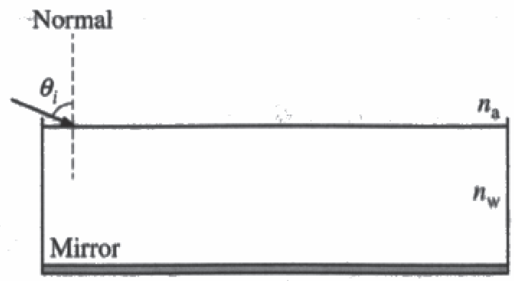
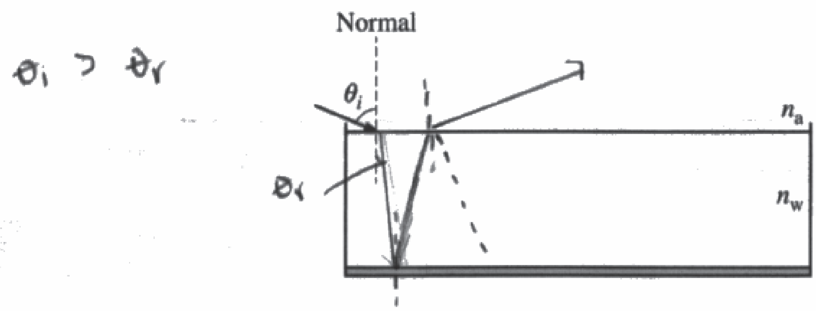


Figure 1

1. (10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.



Question 1

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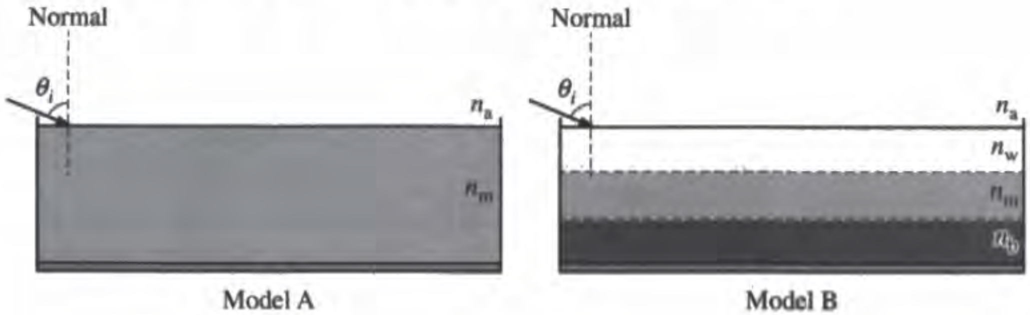

Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).

(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

Because $n_m > n_a$, the speed of the light is less in the mixture than in air. Therefore, the wavelength will decrease by $\lambda = \frac{v}{f}$ because the speed decreases due to the medium and frequency is determined by the source ∴ remains the same

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1

Continue your response to **QUESTION 1** on this page.

(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

Model A		Model B	
θ_i	Incident angle of the beam in air	θ_i	Incident angle of the beam in air
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		θ_3	Angle the beam makes with the normal in the middle layer in Model B
		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

Snell's Law

$$n_a \sin\theta_i = n_b \sin\theta_4$$

$$\sin\theta_4 = \frac{n_a \sin\theta_i}{n_b}$$

$$\theta_4 = \sin^{-1}\left(\frac{n_a \sin\theta_i}{n_b}\right)$$

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

1 θ_1 2 θ_2 3 θ_3 4 θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

1, 2, 3, 4 is the order of increasing indices of refraction. When a beam passes through a medium of less n to more n, then it bends more towards the normal.

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Question 1

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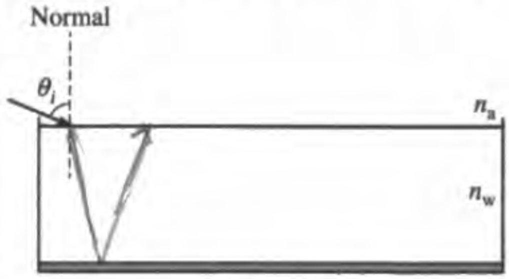
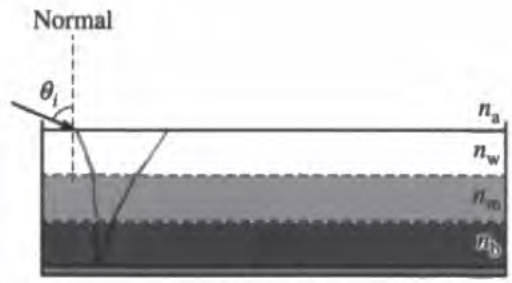


Original Tank

Model A



Model B

For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.

(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

In Model A, d_A would be less than d_w because we know $n_m > n_w$. so the angle of refraction will be less. ∴ the reflected ray would also have a smaller θ. In Model B, it would be the same $d_B = d_w$. This is because reflected ray passes through the same medium to get back to the air as it did going down, so it would have the same angle from θ_w.

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS 2

SECTION II

Time—1 hour and 30 minutes

4 Questions

Directions: Questions 1 and 4 are short free-response questions that require about 20 minutes each to answer and are worth 10 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

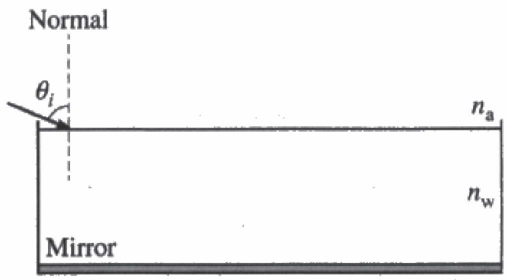
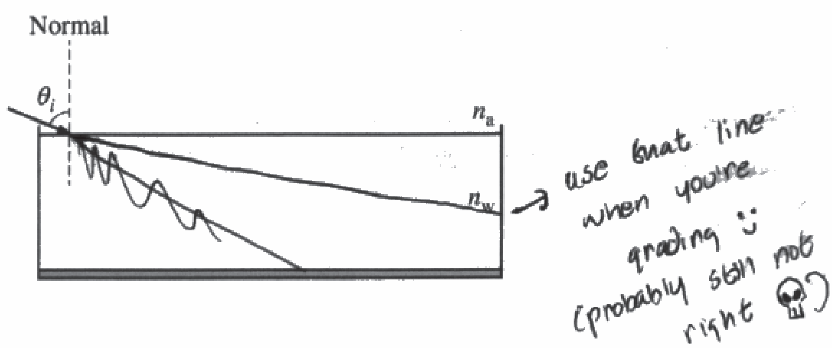


Figure 1

1. (10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.



Question 1

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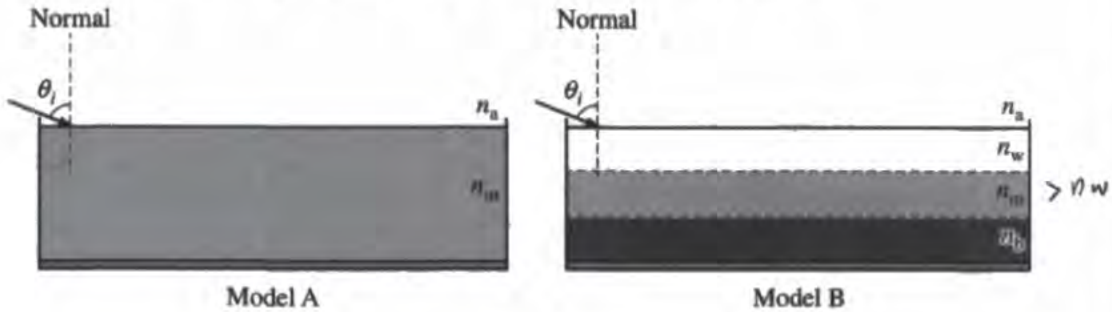

Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).

(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

The observed wavelength of light decreases because the index of refraction changes

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Question 1

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(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

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		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

$$n_a \sin\theta_i = n_b \sin\theta_4$$

$$\theta_4 = \arcsin\left(\frac{n_a \sin\theta_i}{n_b}\right)$$

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

1 θ_1 2 θ_2 3 θ_3 4 θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

A lower index of refraction means a larger angle created with the normal

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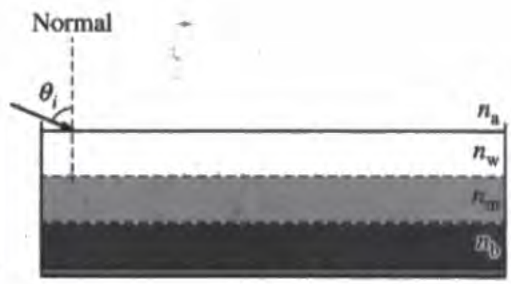
Question 1

Continue your response to QUESTION 1 on this page.

Original Tank

Model A



Model B

For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.

(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

Greater than d_w because they are exiting at a lower point in the tank

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS 2

SECTION II

Time—1 hour and 30 minutes

4 Questions

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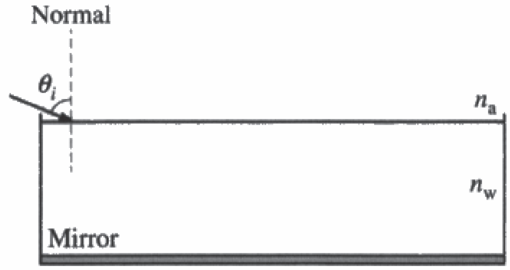
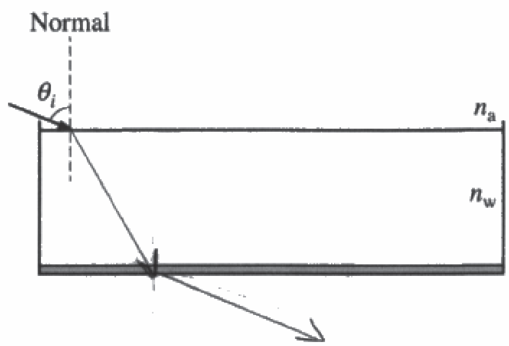


Figure 1

1. (10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.



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Question 1

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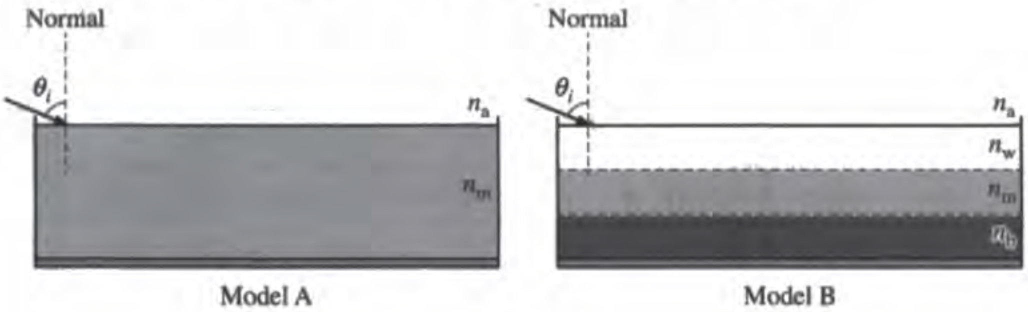


Figure 2

Sugar is then added to the water, resulting in a mixture that has a different index of refraction than water. A student considers two models, Model A and Model B, for how the sugar mixes with water. The models are shown in Figure 2.

Model A: The sugar is uniformly mixed throughout the water, resulting in a mixture with index of refraction n_m such that $n_m > n_w$.

Model B: Layers are formed of varying concentrations of sugar in the water. There are three distinct layers of equal volume. The top layer is only water (index of refraction n_w). The middle layer has the same concentration of sugar as the mixture in Model A (index of refraction n_m). The bottom layer has the highest concentration of sugar (index of refraction n_b).

(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

The wavelength will change when going from air to the mixture. The wavelength will increase, this is because the light will travel slower in the mixture creating a smaller frequency, increasing wavelength.

GO ON TO THE NEXT PAGE.

Question 1

Continue your response to **QUESTION 1** on this page.

(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

Model A		Model B	
θ_i	Incident angle of the beam in air	θ_i	Incident angle of the beam in air
θ_1	Angle the beam makes with the normal in the mixture in Model A	θ_2	Angle the beam makes with the normal in the top layer in Model B
		θ_3	Angle the beam makes with the normal in the middle layer in Model B
		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

$n_a \sin\theta_i = n_b \sin\theta_4$

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

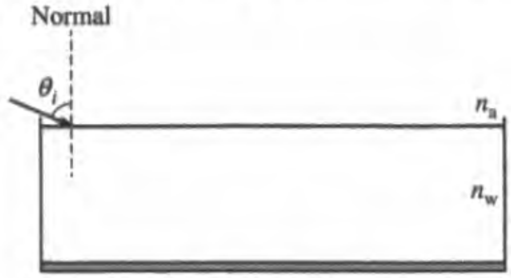
1 θ_1 2 θ_2 3 θ_3 4 θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

θ_1 has the largest angle because it travels the fastest, as the index of refraction changes, so does the speed, resulting in the rankings above. Making θ_1 the greatest, and θ_4 the smallest

Question 1

Continue your response to **QUESTION 1** on this page.



Original Tank

Model A

Model B

For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.

(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

d_A and d_B are equal to d_w. For the distance does not change but at the speed the light travels does.

Question 1

Begin your response to **QUESTION 1** on this page.

PHYSICS 2

SECTION II

Time—1 hour and 30 minutes

4 Questions

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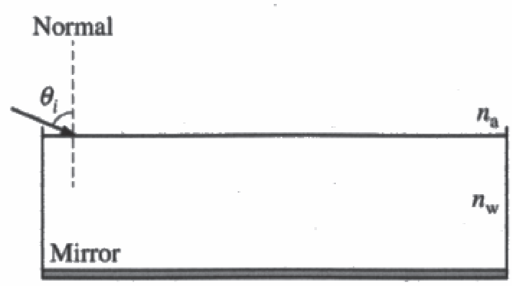
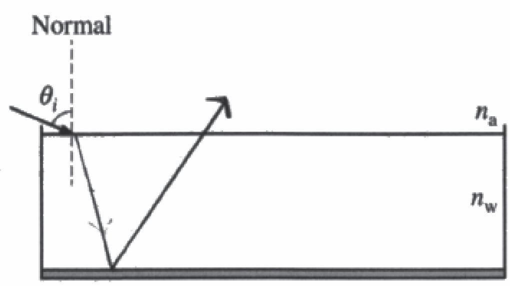


Figure 1

1. (10 points, suggested time 20 minutes)

A rectangular tank with a mirrored bottom is filled with water (index of refraction n_w). A beam of light passes from air (index of refraction n_a) into the water at angle θ_i from the normal, as shown in Figure 1. Index of refraction n_w is greater than index of refraction n_a.

(a) On the following diagram, sketch the entire path of the beam as the beam enters, travels through, and then exits the water.



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Question 1

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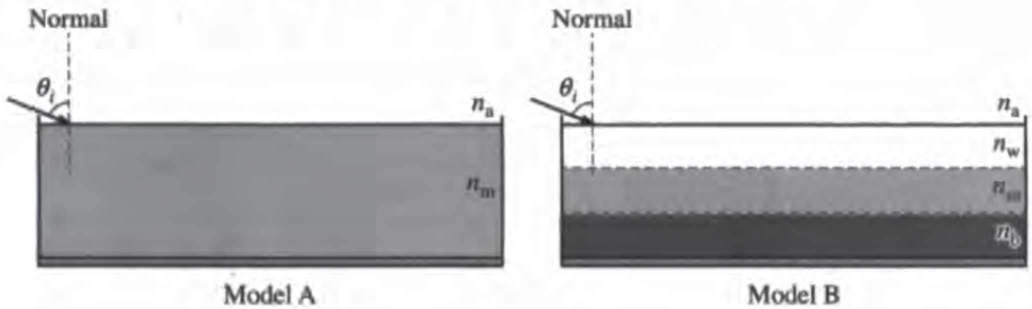

Figure 2

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(b) Consider Model A. Briefly describe how the observed wavelength of light changes, if at all, as the beam travels from air into the mixture.

The observed wavelength of light changes because the medeum that the light is going through changes and takes more force and energy to get through.

Question 1

Continue your response to **QUESTION 1** on this page.

(c) Relevant angles between the beam and the normal for the various layers present in models A and B are defined in the following table.

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		θ_4	Angle the beam makes with the normal in the bottom layer in Model B

i. Determine an expression for θ_4 in terms of θ_i, n_a, and n_b.

$n_a \sin\theta_4 = n_b \sin\theta_i$

ii. Rank the angles from greatest to least, with 1 being greatest. If two angles are the same value, give them the same ranking.

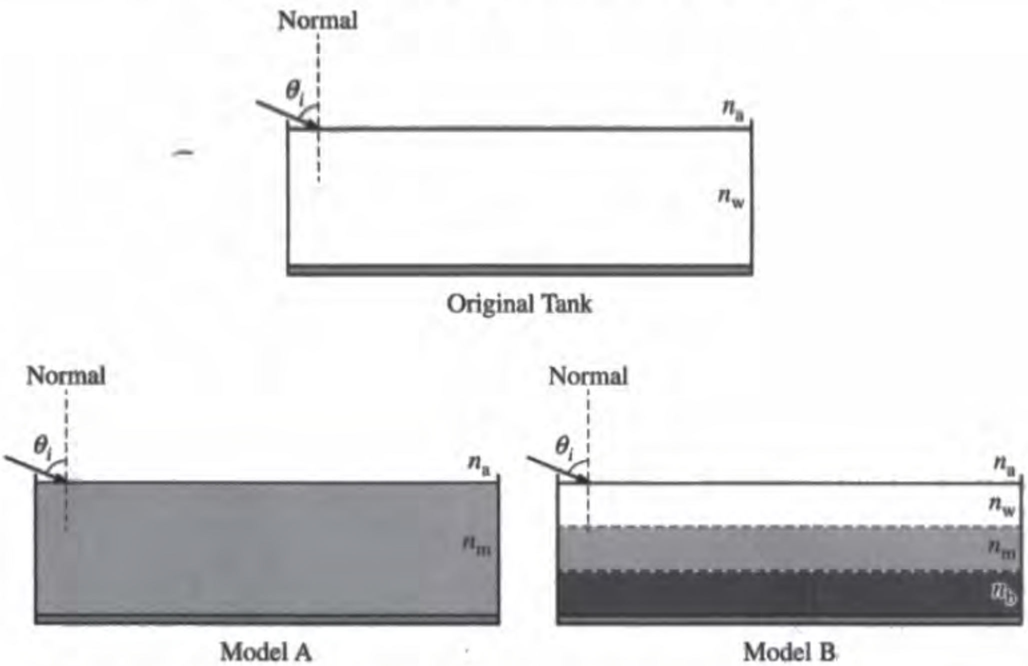
1 θ_1 4 θ_2 3 θ_3 2 θ_4

Briefly explain your reasoning using appropriate physics principles and/or mathematical models.

θ_1 is the farthest away from the normal and the θ_4 is then θ_3 then θ_2, θ_2 is the closest to the normal

Question 1

Continue your response to **QUESTION 1** on this page.



For the original tank filled with water, the beam is observed to exit the surface of the water a horizontal distance d_w from the entry point. For models A and B, the horizontal distances are d_A and d_B, respectively.

(d) Determine whether d_A and d_B are each greater than, less than, or equal to d_w. It is NOT necessary to compare d_A to d_B. Briefly justify your answer.

they are both greater then dw

Question 1

Sample Identifier: P2 Q1 Sample A
Score: 10

a.

- 1 point was earned. The response shows a straight-line path from the point of entry into the water toward the mirror with an angle of refraction that is less than θ_i.
- 1 point was earned. The response shows that the total path is symmetric about a vertical line through the reflection point.

b.

- 1 point was earned. The response states that the wavelength of the light in the sugar-water mixture decreases compared to the wavelength of the light in air, with no incorrect statements.

c.i.

- 1 point was earned. The response correctly applies Snell's law at multiple surfaces.

c.ii.

- 1 point was earned. The response correctly ranks θ_4 as the smallest angle.
- 1 point was earned. The response correctly ranks θ_2 as the largest angle.
- 1 point was earned. The response correctly indicates that θ_1 and θ_3 are equal.
- 1 point was earned. The response correctly relates a greater index of refraction to a smaller angle of refraction.

d.

- 1 point was earned. The response correctly states that both d_A and d_B will be less than d_W.
- 1 point was earned. The response correctly connects greater refraction toward the normal to shorter horizontal distances between the light beam's entry and exit points.

Sample Identifier: P2 Q1 Sample B
Score: 9

a.

- 1 point was earned. The response shows a straight-line path from the point of entry into the water toward the mirror with an angle of refraction that is less than θ_i.
- 1 point was earned. The response shows that the total path is symmetric about a vertical line through the reflection point.

b.

- 1 point was earned. The response states that the wavelength of the light in the sugar-water mixture decreases compared to the wavelength of the light in air, with no incorrect statements.

c.i.

- 1 point was earned. The response indicates a correct mathematical relationship between θ_4, θ_i, n_a, and n_b. Note that this relationship cannot be arrived at without correctly applying Snell's law at multiple surfaces.

c.ii.

- 1 point was earned. The response correctly ranks θ_4 as the smallest angle.
- 1 point was earned. The response correctly ranks θ_2 as the largest angle.
- 1 point was earned. The response correctly indicates that θ_1 and θ_3 are equal.
- 0 points were earned. The response incorrectly states that a larger index of refraction will result in a larger angle of refraction.

d.

- 1 point was earned. The response correctly states that both d_A and d_B will be less than d_W.
- 1 point was earned. The response correctly connects greater refraction toward the normal to a shorter horizontal distance between the light beam's entry and exit points. Although the reference to a "larger" angle is ambiguous, the connection to a higher index of refraction earns this point.

Sample Identifier: P2 Q1 Sample C
Score: 8

a.

- 1 point was earned. The response correctly shows a straight-line path from the point of entry into the water toward the mirror with an angle of refraction that is less than θ_i.
- 1 point was earned. The response shows that the total path is reasonably symmetric about a vertical line through the reflection point.

b.

- 1 point was earned. The response correctly states that the wavelength of the light in the sugar-water mixture decreases compared to the wavelength of the light in air, with no incorrect statements.

c.i.

- 1 point was earned. The response correctly applies Snell's law at multiple surfaces.

c.ii.

- 1 point was earned. The response correctly ranks θ_4 as the smallest angle.
- 0 points were earned. The response does not rank θ_2 as the largest angle.
- 0 points were earned. The response does not indicate that θ_1 and θ_3 are equal.
- 1 point was earned. The response correctly relates a greater index of refraction to a smaller angle of refraction.

d.

- 1 point was earned. The response correctly states that both d_A and d_B will be less than d_W.
- 1 point was earned. The response correctly connects greater amounts of refraction toward the normal to shorter horizontal distances between the light beam's entry and exit points.

Sample Identifier: P2 Q1 Sample D
Score: 7

a.

- 1 point was earned. The response correctly shows a straight-line path from the point of entry into the water to the mirror with an angle of refraction that is less than θ_i.
- 1 point was earned. The response shows that the total path is reasonably symmetric about a vertical line through the reflection point.

b.

- 0 points were earned. The response incorrectly states that the wavelength stays the same in the new substance.

c.i.

- 1 point was earned. The first line of the response correctly relates the indexes of refraction and angles of refraction for the top and bottom layers. This relationship cannot be arrived at without correctly applying Snell's law at multiple surfaces. Although it may be unclear in the first line whether the subscript on the first angle is an *i* or a 1, the rest of the work makes it clear that the subscript is correct.

c.ii.

- 1 point was earned. The response correctly ranks θ_4 as the smallest angle.
- 0 points were earned. The response does not rank θ_2 as the largest angle.
- 0 points were earned. The response does not indicate that θ_1 and θ_3 are equal.
- 1 point was earned. The response correctly relates a higher index of refraction to a smaller angle of refraction.

d.

- 1 point was earned. The response correctly states that both d_A and d_B will be less than d_W.
- 1 point was earned. The response correctly connects higher indexes of refraction to shorter horizontal distances between the light beam's entry and exit points.

Sample Identifier: P2 Q1 Sample E
Score: 6

a.

- 1 point was earned. The response correctly shows a straight-line path from the point of entry into the water toward the mirror with an angle of refraction that is less than θ_i.
- 1 point was earned. The response shows that the total path is reasonably symmetric about a vertical line through the reflection point.

b.

- 1 point was earned for stating that the wavelength of the light decreases in the mixture, with no incorrect statements.

c.i.

- 1 point was earned. The response correctly applies Snell's law at multiple surfaces and indicates a correct relationship between θ_4, θ_i, n_a, and n_b.

c.ii.

- 1 point was earned. The response correctly ranks θ_4 as the smallest angle.
- 0 points were earned. The response does not rank θ_2 as the largest angle.
- 0 points were earned. The response does not indicate that θ_1 and θ_3 are equal.
- 1 point was earned. The response correctly states that when a light beam passes into a medium with a higher index of refraction, the beam bends more toward the normal.

d.

- 0 points were earned. Although the response does correctly state that d_A is less than d_W, the response incorrectly states that d_B is equal to d_W.
- 0 points were earned. The response does not relate refraction toward the normal to the horizontal distance traveled by the beam.

Sample Identifier: P2 Q1 Sample F
Score: 4

a.

- 0 points were earned. The refracted path in the water is at an angle greater than θ_i.
- 0 points were earned. The path does not reflect off the mirror or emerge from the tank.

b.

- 1 point was earned. The response correctly states that the wavelength of the light in the sugar-water mixture decreases, with no incorrect statements.

c.i.

- 1 point was earned. The response indicates a correct mathematical relationship between θ_4, θ_i, n_a, and n_b. Note that this relationship cannot be arrived at without correctly applying Snell's law at multiple surfaces.

c.ii.

- 1 point was earned. The response correctly ranks θ_4 as the smallest angle.
- 0 points were earned. The response does not rank θ_2 as the largest angle.
- 0 points were earned. The response does not indicate that θ_1 and θ_3 are equal.
- 1 point was earned. The response correctly relates a larger angle of refraction to a lower index of refraction.

d.

- 0 points were earned. The response incorrectly states that both d_A and d_B are greater than d_W.
- 0 points were earned. The explanation does not involve refraction.

Sample Identifier: P2 Q1 Sample G
Score: 3

a.

- 1 point was earned. The response correctly shows a refracted path in the water with an angle of refraction less than θ_i.
- 0 points were earned. The path does not reflect off the mirror.

b.

- 0 points were earned. The response incorrectly states that the wavelength will increase in the sugar-water mixture.

c.i.

- 1 point was earned. The response indicates a correct relationship between θ_4, θ_i, n_a and n_b that could only be arrived at by applying Snell's law at multiple surfaces.

c.ii.

- 1 point was earned. The response correctly ranks θ_4 as the smallest angle.
- 0 points were earned. The response does not rank θ_2 as the largest angle.
- 0 points were earned. The response does not indicate that θ_1 and θ_3 are equal.
- 0 points were earned. Although the explanation does mention both indexes of refraction and both angles, the response does not link the value of the index of refraction to the value of the angle.

d.

- 0 points were earned. The response incorrectly states that both d_A and d_B are equal to d_W.
- 0 points were earned. The explanation does not involve refraction.

Sample Identifier: P2 Q1 Sample H
Score: 1

a.

- 1 point was earned. The response correctly shows a straight-line path from the point of entry into the water toward the mirror with an angle of refraction that is less than θ_i.
- 0 points were earned. The reflected path does not refract as the light emerges from the water into the air, and, therefore, is not vertically symmetric.

b.

- 0 points were earned. The response does not state how the wavelength of the light changes in the new medium.

c.i.

- 0 points were earned. The response does not correctly match subscripts on index of refraction quantities and angle quantities, and is, therefore, not a correct application of Snell's law.

c.ii.

- 0 points were earned. The response does not rank θ_4 as the smallest angle.
- 0 points were earned. The response does not rank θ_2 as the largest angle.
- 0 points were earned. The response does not indicate that θ_1 and θ_3 are equal.
- 0 points were earned. The response only restates the ranking without providing any explanation.

d.

- 0 points were earned. The response incorrectly states that both d_A and d_B are greater than d_W.
- 0 points were earned. No explanation is given.

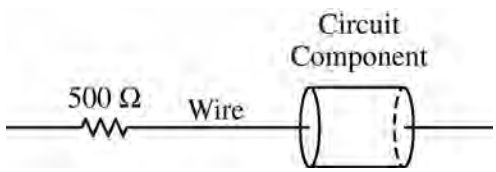
Begin your response to **QUESTION 2** on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.

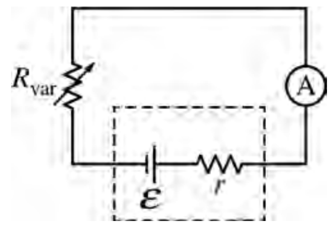


ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\ \Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

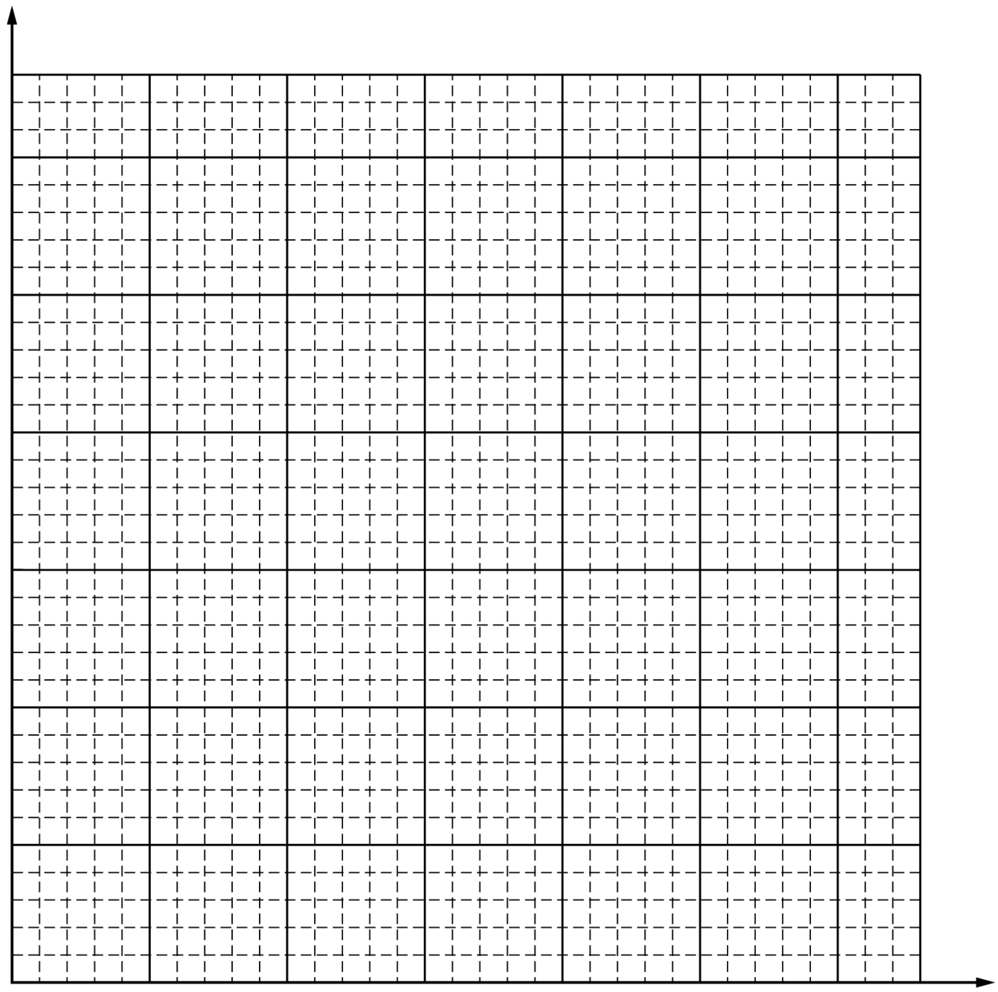
ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis:________________ Vertical Axis:________________

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

GO ON TO THE NEXT PAGE.

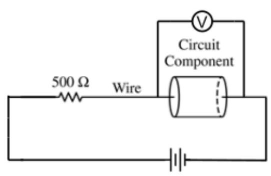
Question 2: Experimental Design 12 points

(a)(i) For a diagram including a source of potential difference (e.g., battery, power supply) that is in a complete circuit which results in a current in the unknown circuit component **1 point**

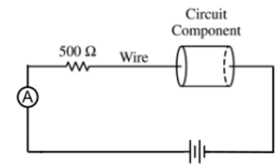
For a diagram including a measurement device that is appropriately connected in the circuit (e.g., voltmeter, ammeter) **1 point**

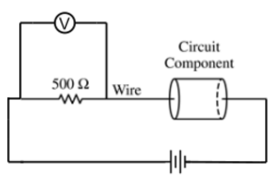
Scoring Note: A lightbulb that is connected in series with the circuit component is an acceptable alternative for an ammeter.

Example Responses



OR





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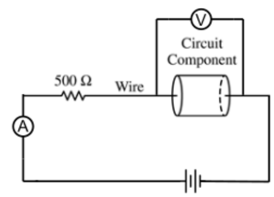
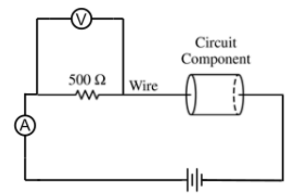


OR



(a)(ii) For describing a procedure that includes a measurement of **one** of the following: **1 point**

- the potential difference across the circuit component
- the current in the circuit
- the potential difference across the known resistor

For taking measurements at two different times after the circuit is closed or taking one measurement a long time after the circuit is closed, consistent with the procedure described **1 point**

Example Responses

Measure the current in the ammeter immediately after the circuit is closed and a long time after the circuit is closed.

OR

Measure the potential difference across the circuit component immediately after the circuit is closed and a long time after the circuit is closed.

OR

Measure the potential difference across the circuit component a long time after the circuit is closed.

OR

Measure the potential difference across the $500\,\Omega$ *resistor immediately after the circuit is closed and a long time after the circuit is closed.*

(a)(iii)	For describing a correct result of the experiment that indicates the current in the circuit decreases to zero over time or that the charge of the plates of the capacitor increases over time	**1 point**
	For describing a correct result of the experiment that indicates that the electric potential difference across the capacitor increases from zero over time	**1 point**

Example Response

Immediately after the circuit has been closed, a current should be measured. A long time after the circuit has been closed, a current of zero should be measured. This is because the initially uncharged capacitor becomes fully charged. This results in a potential difference across the capacitor that is equal to the potential difference across the battery a long time after the circuit has been closed. Therefore, according to Kirchhoff's loop rule, a potential difference will not be measured across any other circuit components.

Total for part (a) 6 points

(b)(i)	For an equation that correctly applies the loop rule	**1 point**

Example Response

$0 = +\varepsilon - Ir - IR_{\text{var}}$

(b)(ii)	For indicating appropriate quantities that, when graphed, result in a linear graph that allow students to determine emf ε	**1 point**

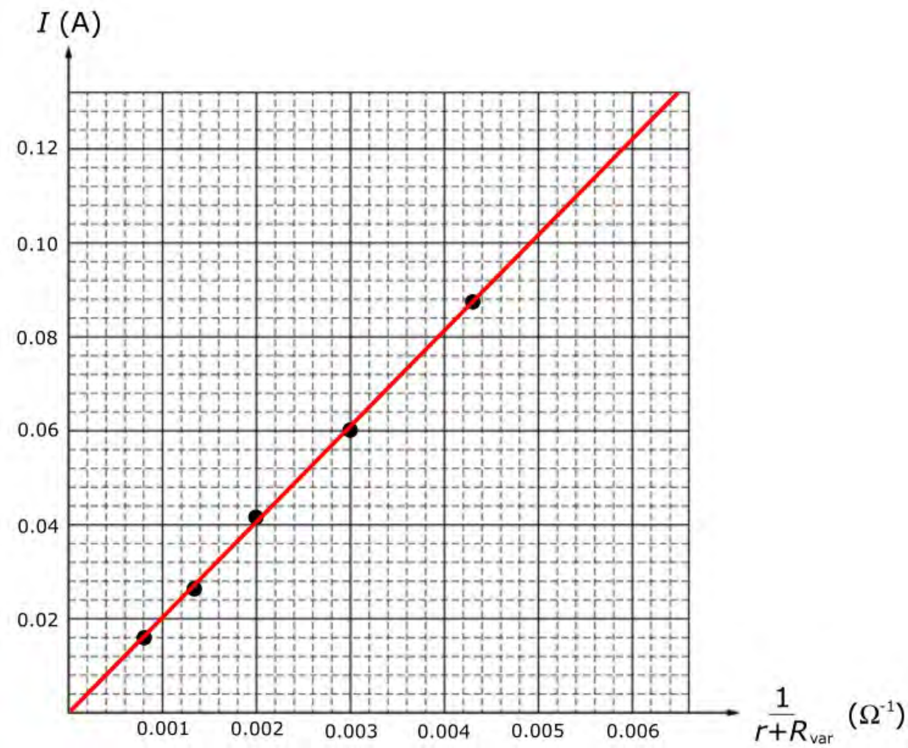
Example Responses

- $\dfrac{1}{r + R_{\text{var}}}$ *vs.* I
- I *vs.* IR_{var}
- R_{var} *vs.* $\dfrac{1}{I}$

(b)(iii)	For including numerical values on both axes with a linear scale and labeling the axes with appropriate labels and units	**1 point**
	For a graph in which data are plotted within at least half of the grid area	**1 point**

For drawing a best-fit line that approximates the trend of the data **1 point**

Example Responses



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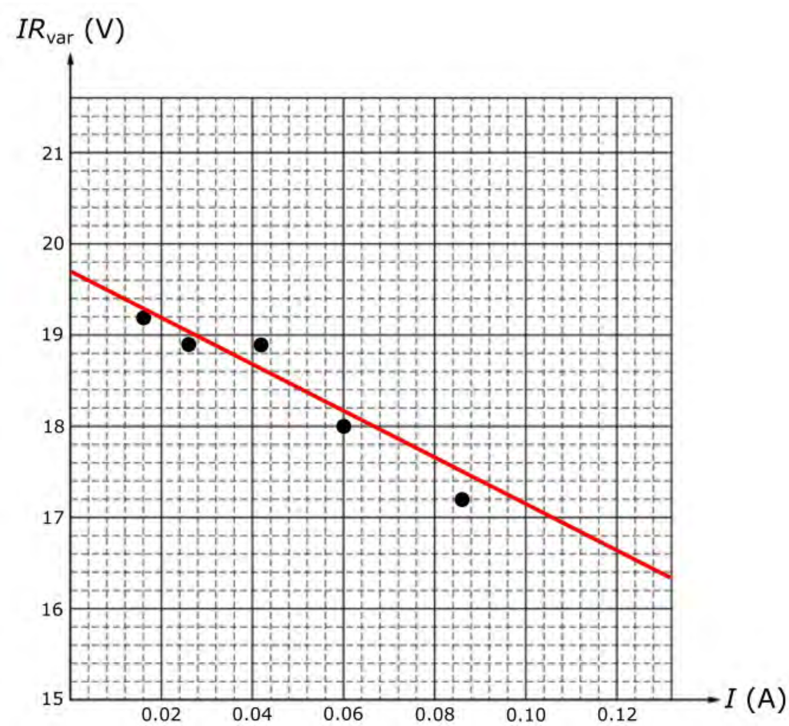


(b)(iv) For correctly using the graph to determine an experimental value for emf ε, including correct units, between 18.0 V and 22.0 V **1 point**

Example Solutions

I as a function of $\dfrac{1}{r + R_{\text{var}}}$

$$\varepsilon - Ir - IR_{\text{var}} = 0$$
$$\varepsilon - I\left(r + R_{\text{var}}\right) = 0$$
$$I\left(r + R_{\text{var}}\right) = \varepsilon$$
$$I = \varepsilon\left(\frac{1}{r + R_{\text{var}}}\right)$$
$$I = \varepsilon\left(\frac{1}{30\,\Omega + R_{\text{var}}}\right)$$
$$\text{Slope} = \varepsilon$$
$$\frac{\Delta y}{\Delta x} = \varepsilon$$
$$\frac{\left(0.08\text{ A} - 0.04\text{ A}\right)}{\left(0.004\,\Omega^{-1} - 0.002\,\Omega^{-1}\right)} \approx \varepsilon$$
$$\varepsilon \approx 20\text{ V}$$

OR

IR_{var} as a function of I

$$\varepsilon - Ir - IR_{\text{var}} = 0$$
$$\varepsilon - Ir = IR_{\text{var}}$$
$$IR_{\text{var}} = -Ir + \varepsilon$$
$$y\text{-intercept} = \varepsilon$$
$$\varepsilon \approx 20\text{ V}$$

Total for part (b) 6 points

Total for question 2 12 points

Question 2

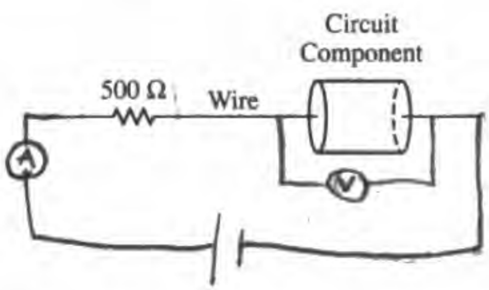
Begin your response to QUESTION 2 on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

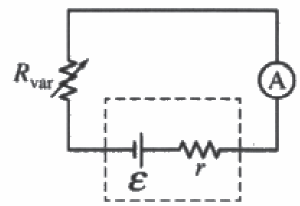
1) Place a battery with a known voltage in series with the 500Ω resistor and the circuit component
2) Place a voltmeter in parallel with the circuit component and ammeter in series with the resistor and circuit component
3) Measure the current and voltage at fixed intervals of 10 seconds.

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

The students would expect to see a decrease in the ammeter reading (the current) and an increase in the voltmeter reading (the potential difference) if the component is an uncharged capacitor. They would expect to see this because over time, the capacitor will store more charge and acquire a greater potential difference

Question 2

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\ \Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)	$\frac{1}{R_{var}+r}$ (Ω^{-1})	
0.087	200	0.0043	
0.060	300	0.0030	
0.042	450	0.0021	
0.027	700	0.0014	
0.016	1200	0.00081	

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$\mathcal{E} = I(r + R_{var})$

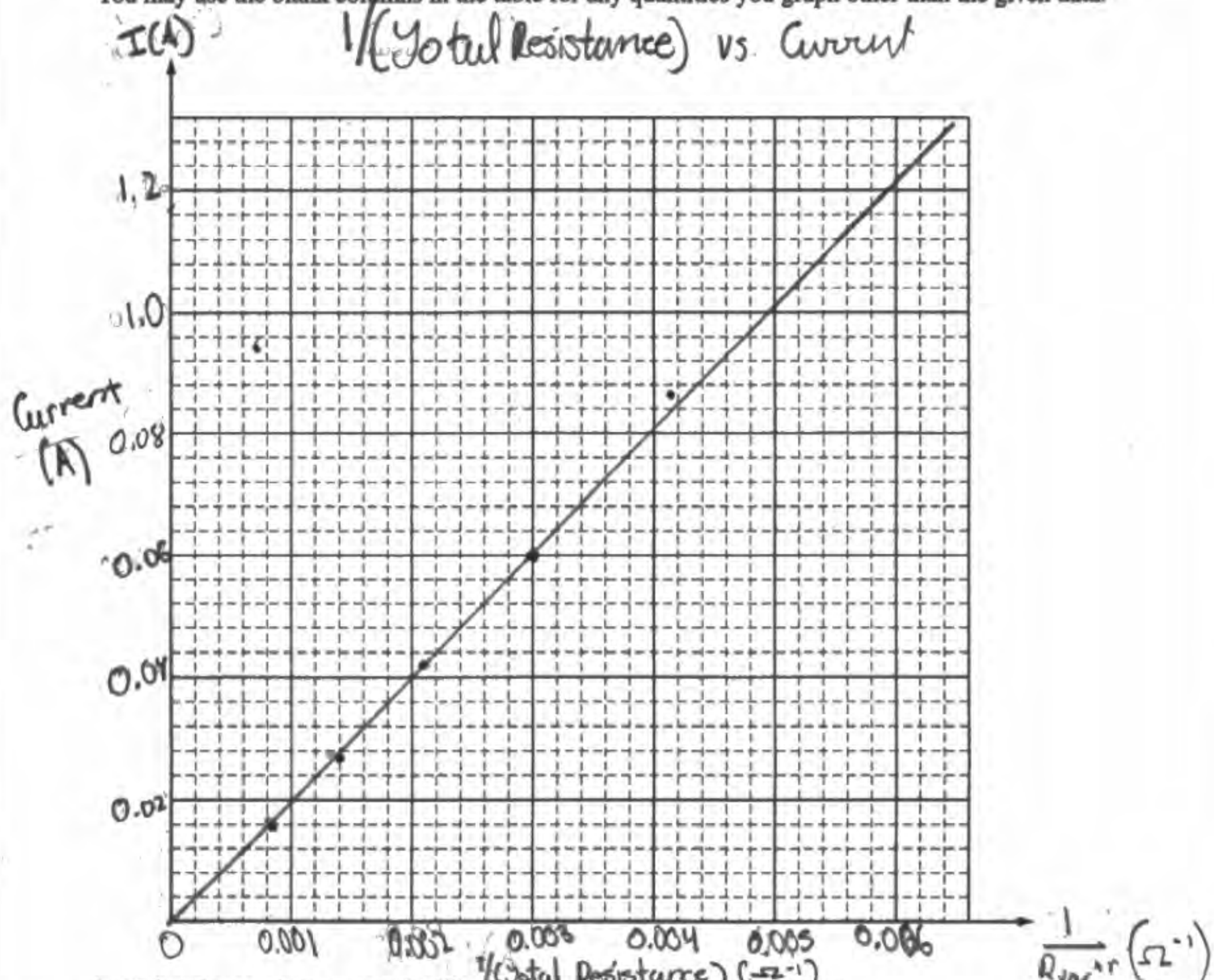
ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: 1/Total Resistance (Ω^{-1}) Vertical Axis: Current (A)

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

$$\mathcal{E} = \frac{I_2 - I_1}{\left(\frac{1}{R_{var_2}+r}\right) - \left(\frac{1}{R_{var_1}+r}\right)} = \frac{0.06 - 0.016}{0.003 - 0.0008} = 20\ V$$

$\mathcal{E} = 20\ V$

Question 2

Begin your response to QUESTION 2 on this page.

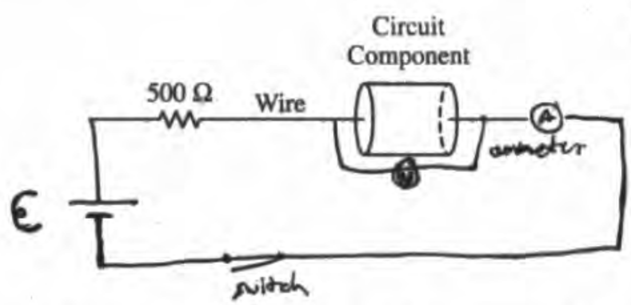
2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.

$I = \frac{V}{R}$



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

use an ammeter to measure the current of the circuit.

Close the circuit switch and observe the ammeter. If the current is constant, then the component is a resistor, if the current starts at $\frac{\varepsilon}{500\Omega}$ immediately after the switch closes and decreases to zero, then the component is a capacitor.

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

if the component is a capacitor, then a voltmeter connected in parallel with the component should see an increase in potential difference over time as the capacitor charges.

$V = \frac{Q}{C}$

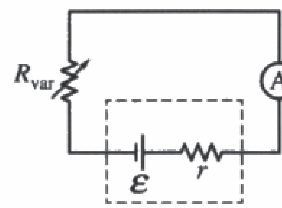
GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5186/7

Question 2

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)	$R_{var} + r = R_{eq}$	$1/R_{eq}$
0.087	200	230	0.0043
0.060	300	330	0.0030
0.042	450	480	0.0021
0.027	700	730	0.0014
0.016	1200	1230	0.00081

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$$I = \frac{\mathcal{E}}{R_{var} + r}$$

$$I = \frac{V}{R}$$

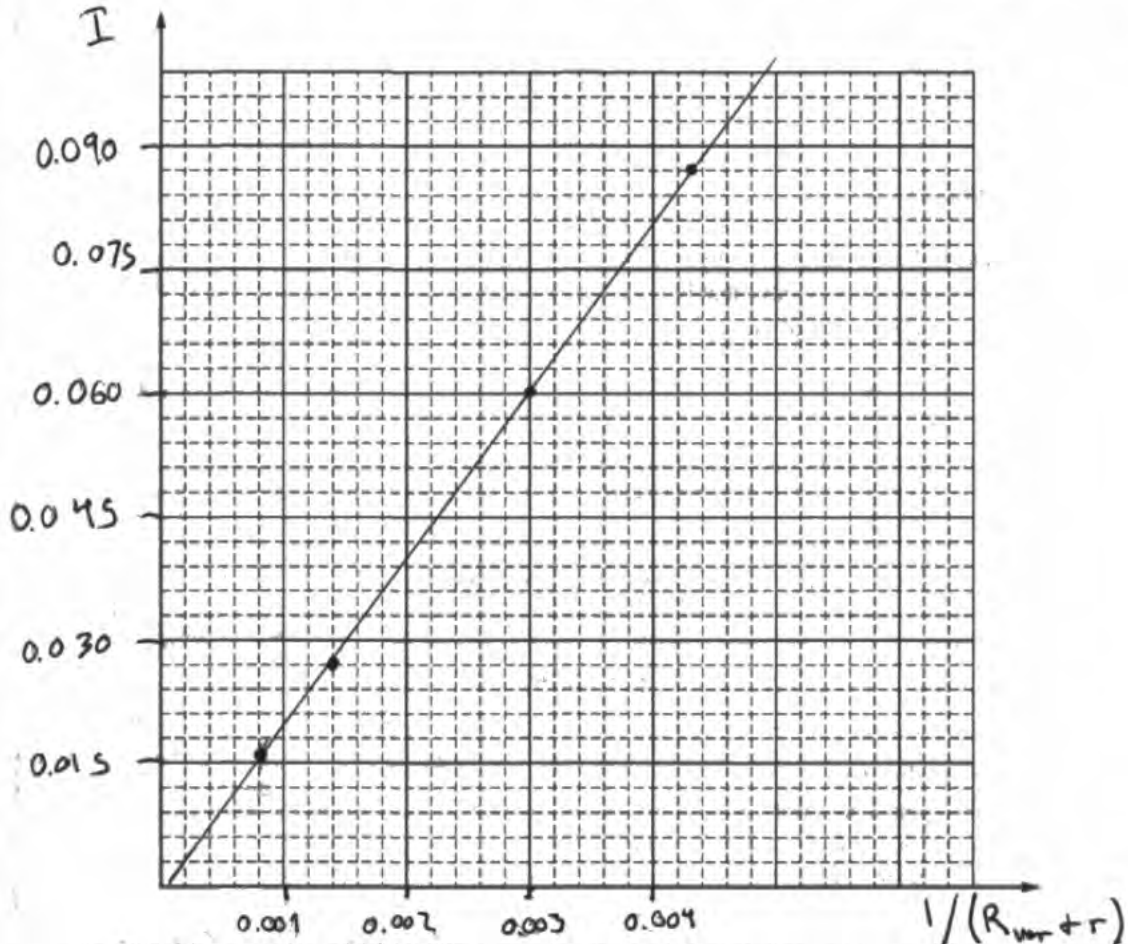
ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: $1/R_{eq} = 1/(R_{var} + r)$ Vertical Axis: I

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

$\frac{I}{1/R_{eq}}$ = slope = 20.3

1/Ω

~~V=20.3~~

$I = \frac{V}{R}$

$\mathcal{E} = 20\,V$

Question 2

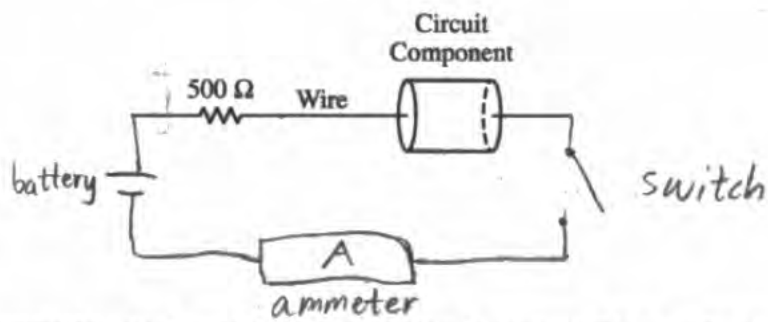
Begin your response to QUESTION 2 on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

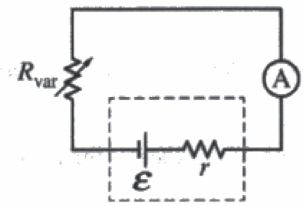
Measure the current in the ammeter immediately after the switch is closed and in several intervals as time passes

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

The student would expect the current in the ammeter to decrease as time passes after the switch is closed until there is eventually zero current. This effect is because a capacitor will increase in resistance as its charge increases.

Question 2

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)	$\frac{1}{I}$ ($\frac{1}{A}$)	$R_{var} + r$ (Ω)
0.087	200	11.49	230
0.060	300	16.67	330
0.042	450	23.81	480
0.027	700	37.04	730
0.016	1200	62.50	1230

$V = IR$

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$\mathcal{E} = I(R_{var} + r)$

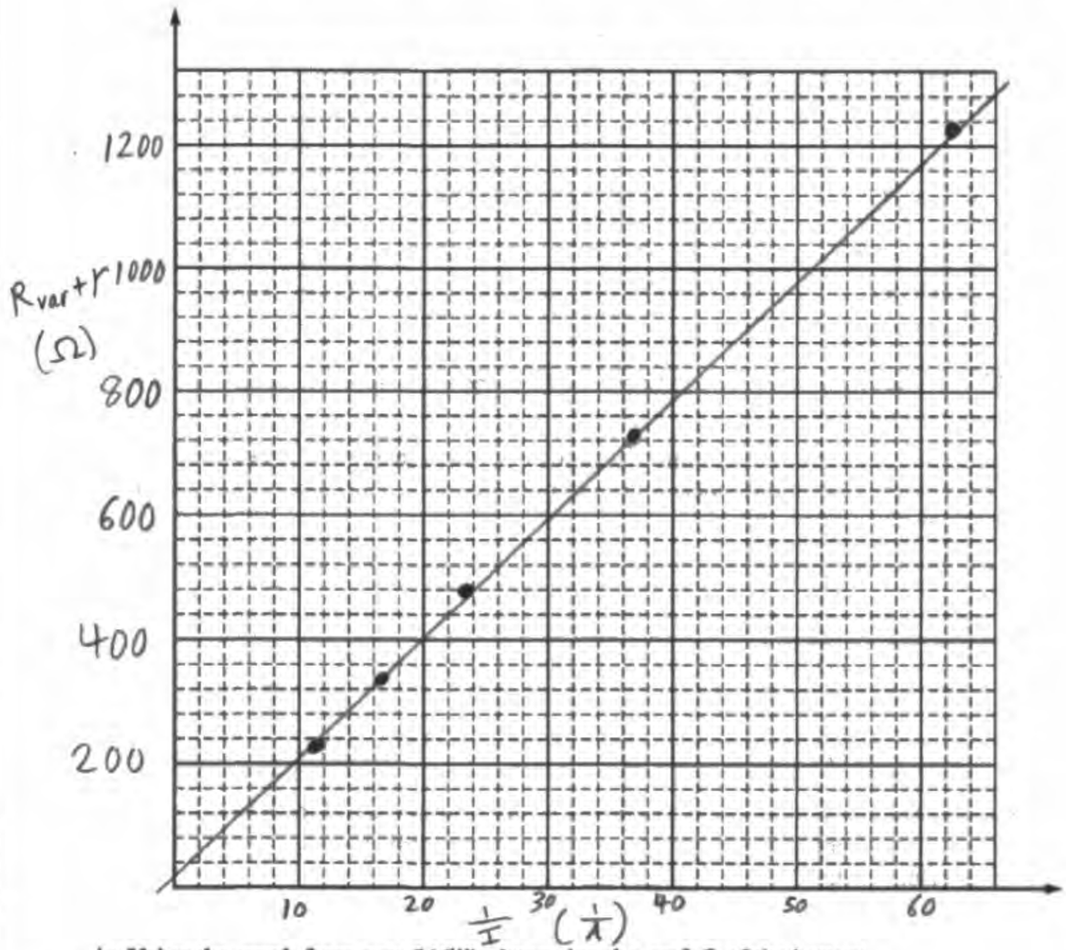
ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: $\frac{1}{I}$ Vertical Axis: $R_{var} + r$

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

$\mathcal{E} = 19.58V$

Question 2

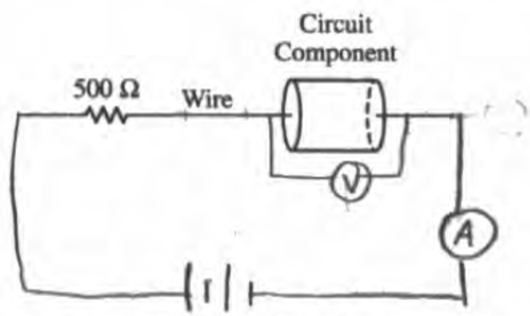
Begin your response to QUESTION 2 on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

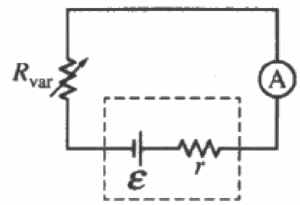
1. Connect the voltmeter in parallel to the circuit component and the ammeter to the circuit in series
2. Measure the voltage and current immediately after turning on the power supply or connecting the battery.
3. Wait 10 seconds before measuring the voltage and current again
4. Repeat step 3 5 times

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

If the component is an uncharged capacitor, then the potential difference and current should decrease nonlinearly and begin to approach 0. According to Ohm's Law ($\frac{V}{I}=R$), the current should decrease as the resistance increases, and after a long time in steady-state, there should be approximately ∞ resistance, which means that the current should be about 0 and therefore ΔV should be 0.

Question 2

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)	$\left(\frac{1}{k\Omega}\right)$ $\frac{1}{R_{eq}}$	
0.087	200	2	
0.060	300	1.667	
0.042	450	1.333	
0.027	700	1	
0.016	1200	0.667	

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$V = IR_{eq}$ $\quad$ $R_{eq} = R_{var} + r$

$V = \mathcal{E}$

$\therefore \mathcal{E} = I(R_{var} + r)$

ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: $\frac{1}{R_{eq}}$ $\left(\frac{1}{k\Omega}\right)$ $\quad$ Vertical Axis: I (A)

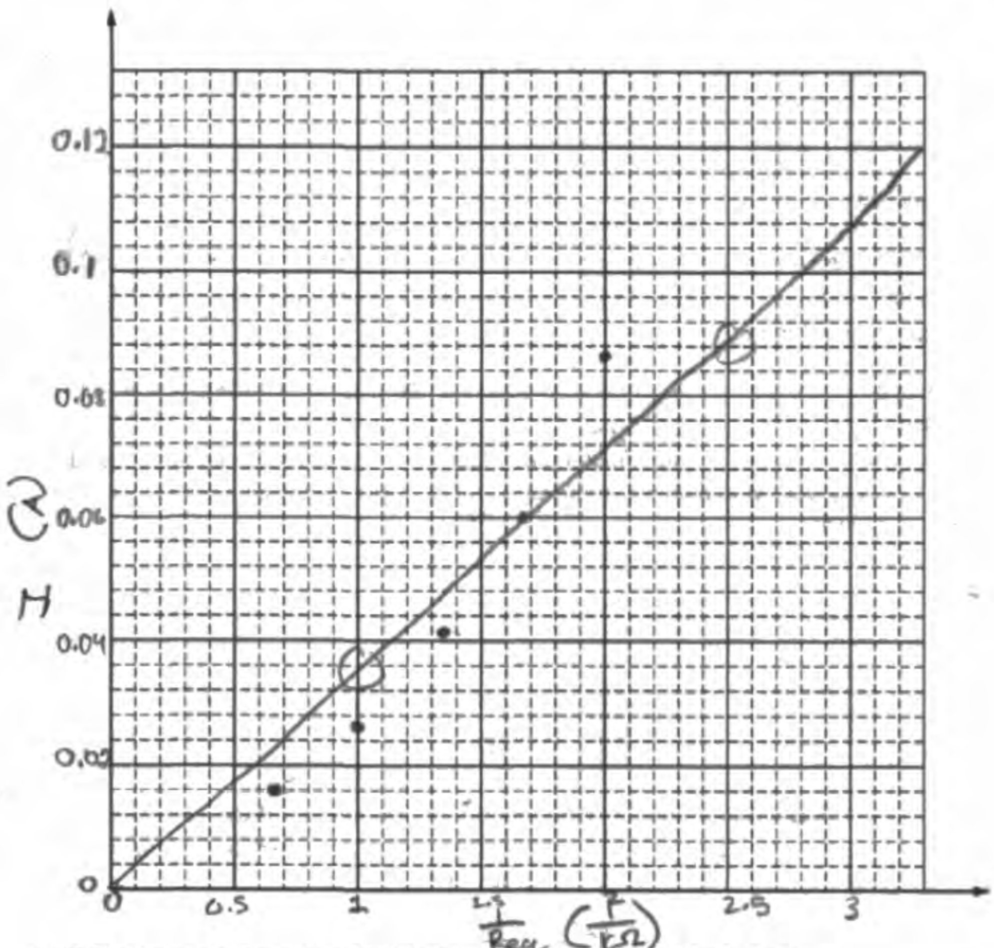
$$I = \frac{\mathcal{E}}{(R_{var} + r)}$$

GO ON TO THE NEXT PAGE.

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

Slope: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0.088 - 0.036}{2.5 - 1} = 0.0346$

$IR_{eq} = \mathcal{E}$

Question 2

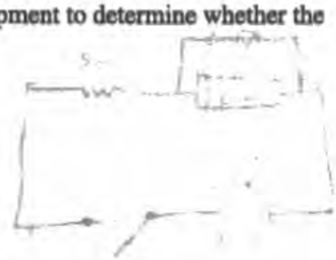
Begin your response to QUESTION 2 on this page.

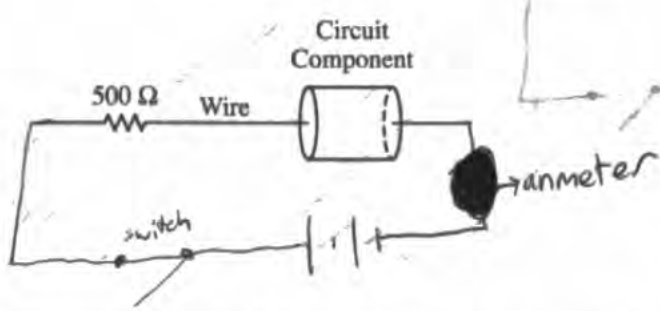
2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.





ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

~~Then, allow the system to persist with the closed switch for a long time. Measure the current again with the ammeter.~~

First, close the switch and record the current with the ammeter. Then, let the system persist for a long time. After, measure the current again with the ammeter.

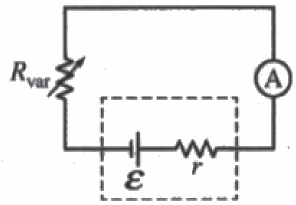
~~First, close the switch and record the current using the ammeter.~~

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

If the component was an uncharged capacitor, the potential difference across the component should change and eventually equal the emf.

Question 2

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$\mathcal{E} = I(R_{var} + r)$

ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

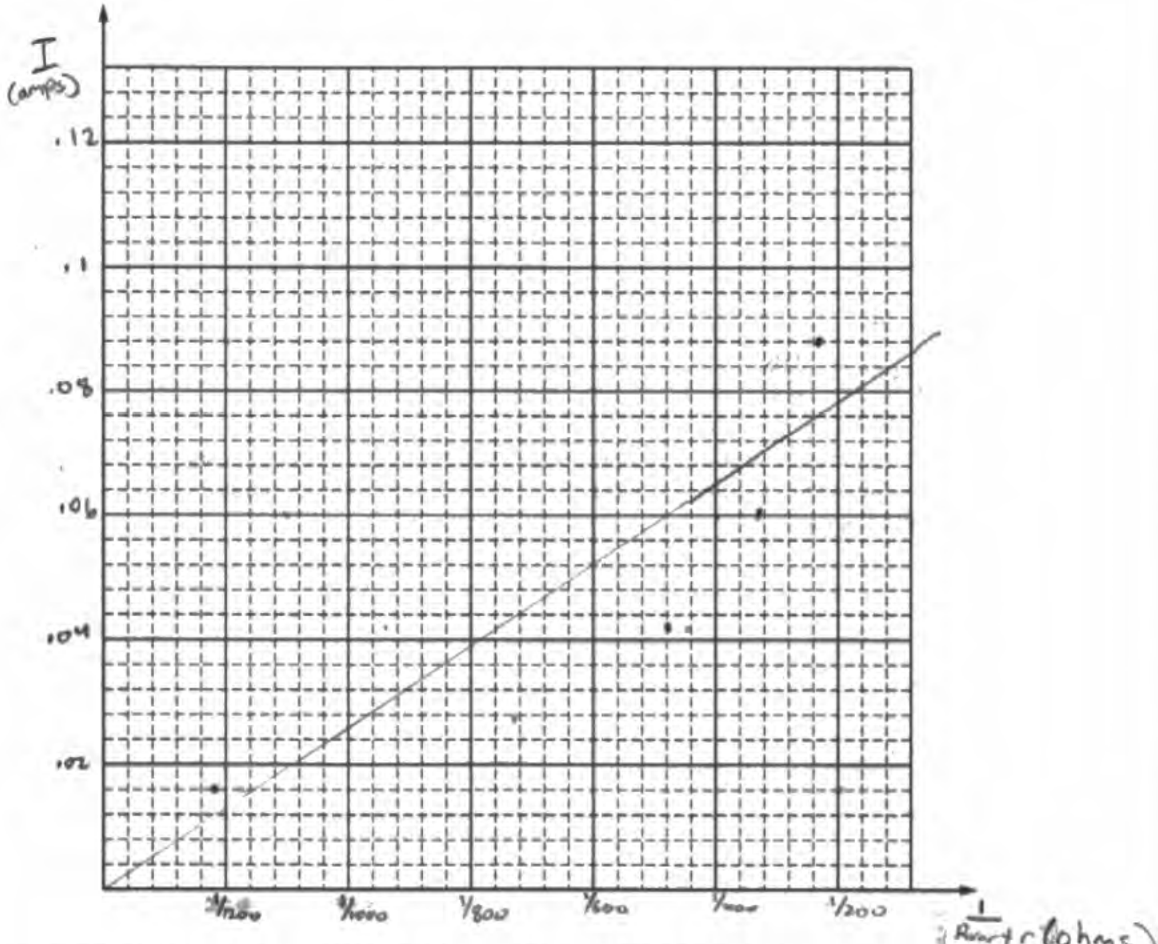
Horizontal Axis: $\frac{1}{(R_{var} + r)}$ Vertical Axis: I

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

$$\frac{.052-0}{1/600-0}$$

$\mathcal{E} = 31.2\ V$

Question 2

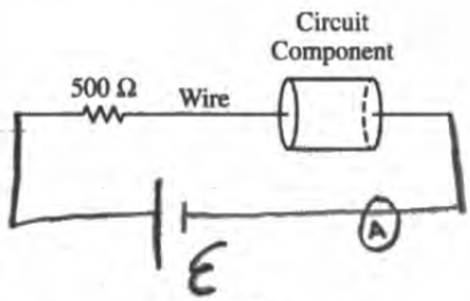
Begin your response to QUESTION 2 on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

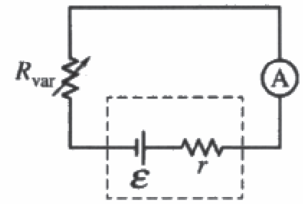
As shown in part (a)(i), the component's nature can be determined by adding a battery and an ammeter into the circuit. The students would closly monitor the ampmeter's readings and collect data accordingly.

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

If the component was a capacitor, after a time (T), the reading on the Ampmeter would be 0 amps. If the ampmeter's reading stays constant (relatively speaking), then the component must be a resistor.

Question 2

Continue your response to **QUESTION 2** on this page.



30 Ω

The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\ \Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)	~~$\mathcal{E}$ (V)~~	V (V)
0.087	200 230	~~17.4~~	17.4
0.060	300 330	~~19.8~~	18
0.042	450 480	~~20.16~~	18.9
0.027	700 730	~~19.71~~	18.9
0.016	1200 1230	~~19.68~~	19.2

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$V = IR \rightarrow \mathcal{E} = (I)(r + R_{var})$

~~$\mathcal{E} + \mathcal{E} = (I)(R_{var})$~~

ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: 1/resistance Vertical Axis: Voltage

$y = mx + b$

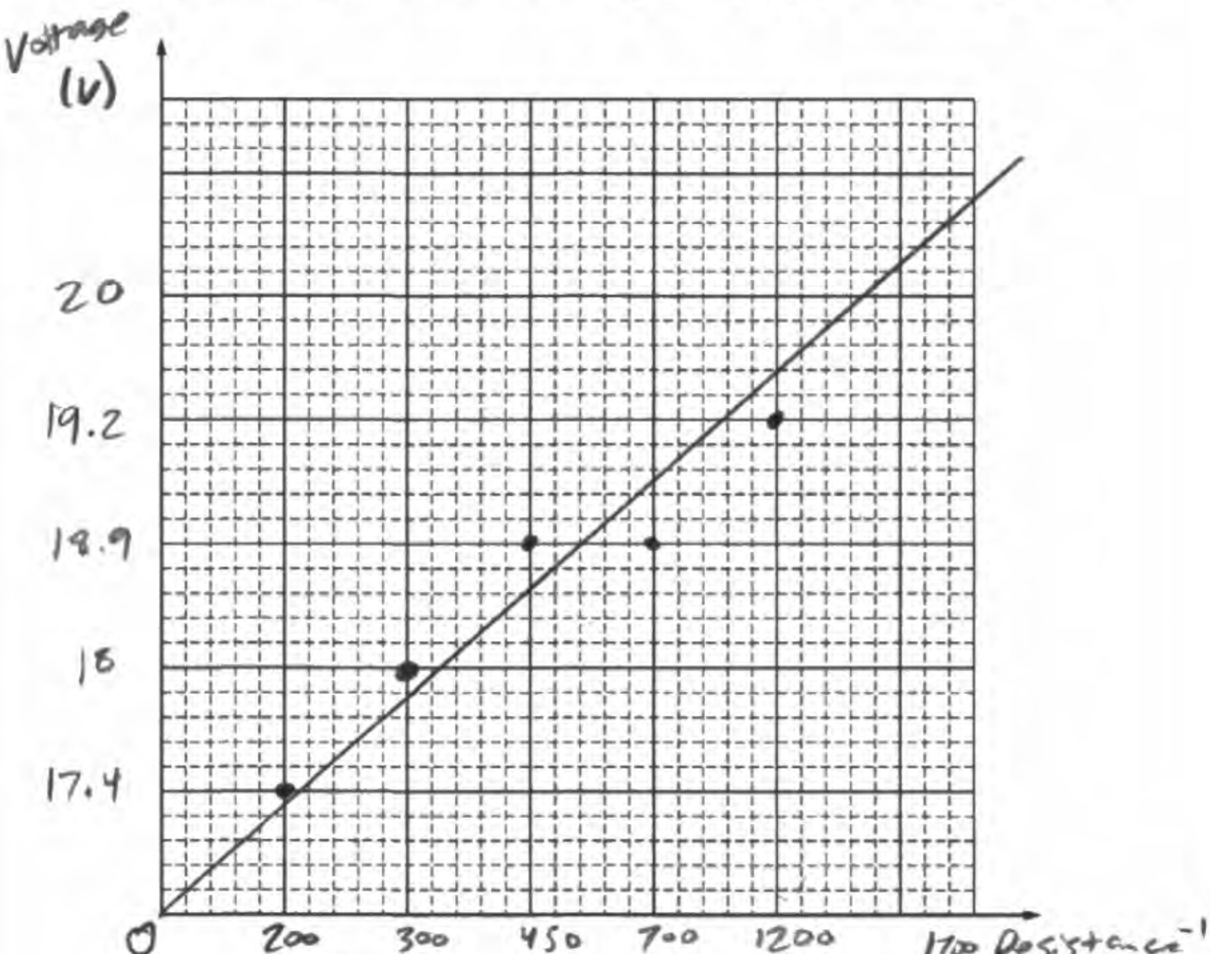
$V = IR$

$\mathcal{E} = \frac{V}{r}$ $\quad \mathcal{E}r = IR$

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

12 V

Question 2

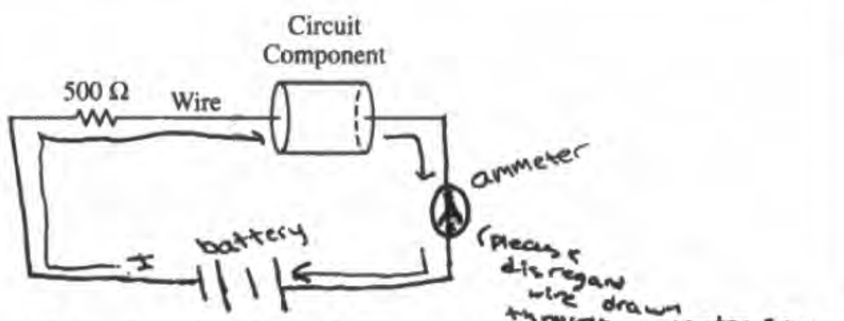
Begin your response to QUESTION 2 on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

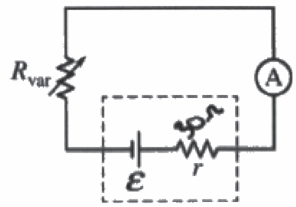
1.) Attach ammeter to turned off circuit.
2.) Turn on circuit. (let stay on for a while)
3.) Read and collect number on ammeter.
4.) Compare number collected to resistor mathmatically

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

The students would expect the number on the ammeter to be the same potential difference as the resistor because an uncharged capacitor in series shouldn't affect the potential difference or charge.

Question 2

Continue your response to QUESTION 2 on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\ \Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)	1/I (A)	
0.087	200	11.5	
0.060	300	16.7	
0.042	450	23.8	
0.027	700	37.0	
0.016	1200	62.5	

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$$\mathcal{E} = \frac{Ir}{R_{var}}$$

11.5
16.7
23.8
37.0
62.5

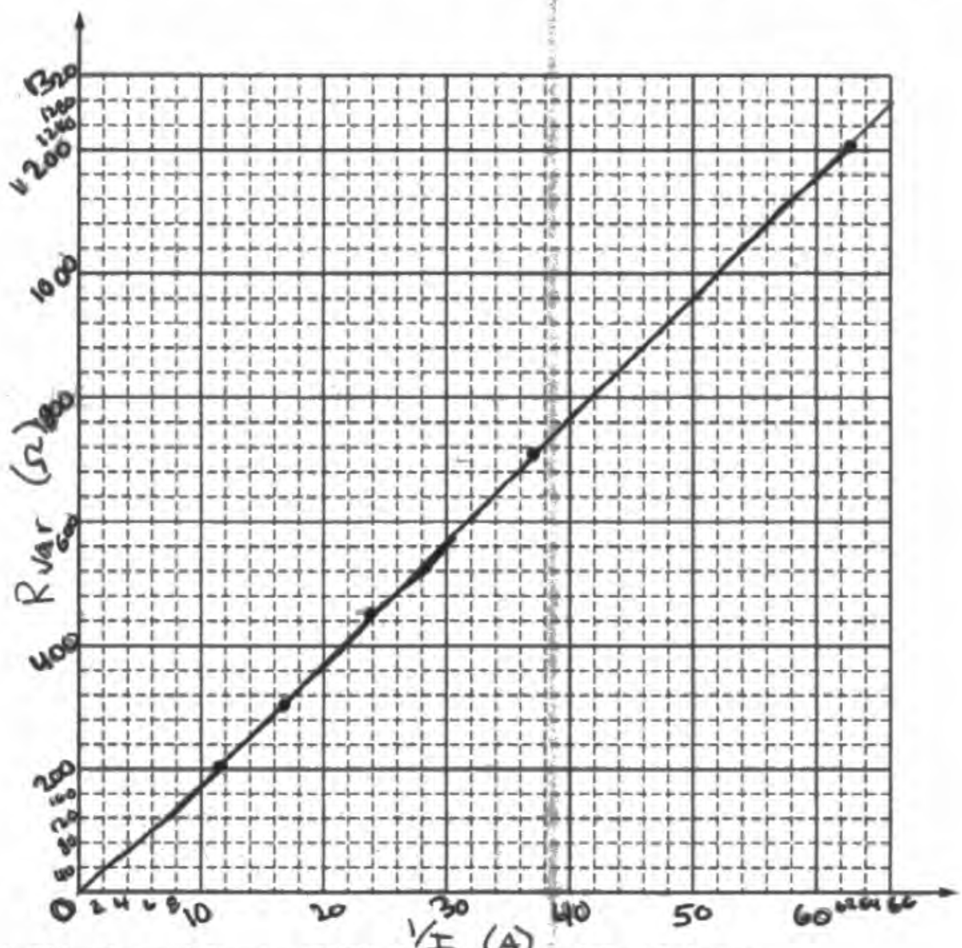
ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: 1/I (A) Vertical Axis: Rvar (Ω)

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

$\mathcal{E} = \frac{\Delta y}{\Delta x}$ $\mathcal{E} = \frac{y_1 - y_2}{x_1 - x_2}$ $\mathcal{E} = \frac{300-200}{16.7-11.5} = \frac{100}{5.2} = \boxed{19.23}$

Question 2

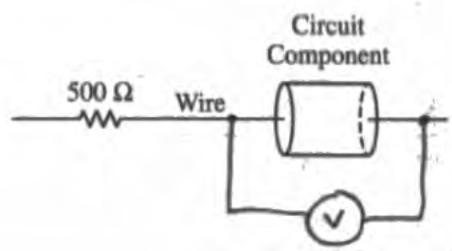
Begin your response to QUESTION 2 on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

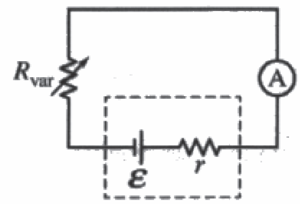
By placing a voltmeter, connected on either side, it will read either a non-zero potential difference or a potential difference of 0. A nonzero reading means it is a resistor, while if it reads 0, then it is an uncharged capacitor.

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

There would be 0 Volts of potential difference meaning the capacitor has no charge. $\Delta V = \frac{Q}{C}$, so if there is no potential difference, then there will be no charge.

Question 2

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)	$\frac{1}{I}$	$(r+R_{var})$
0.087	200	11.5	230
0.060	300	16.7	330
0.042	450	23.8	480
0.027	700	37.	730
0.016	1200	62.5	1230

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$\mathcal{E} = Ir + IR_{var}$ $\qquad$ $(r+R_{var}) = \frac{\mathcal{E}}{I}$

$\mathcal{E} = I(r+R_{var})$ $\qquad$ $R_{var} = -r + \frac{\mathcal{E}}{I}$

ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: $\frac{1}{I}$ $\qquad$ Vertical Axis: $(r+R_{var})$

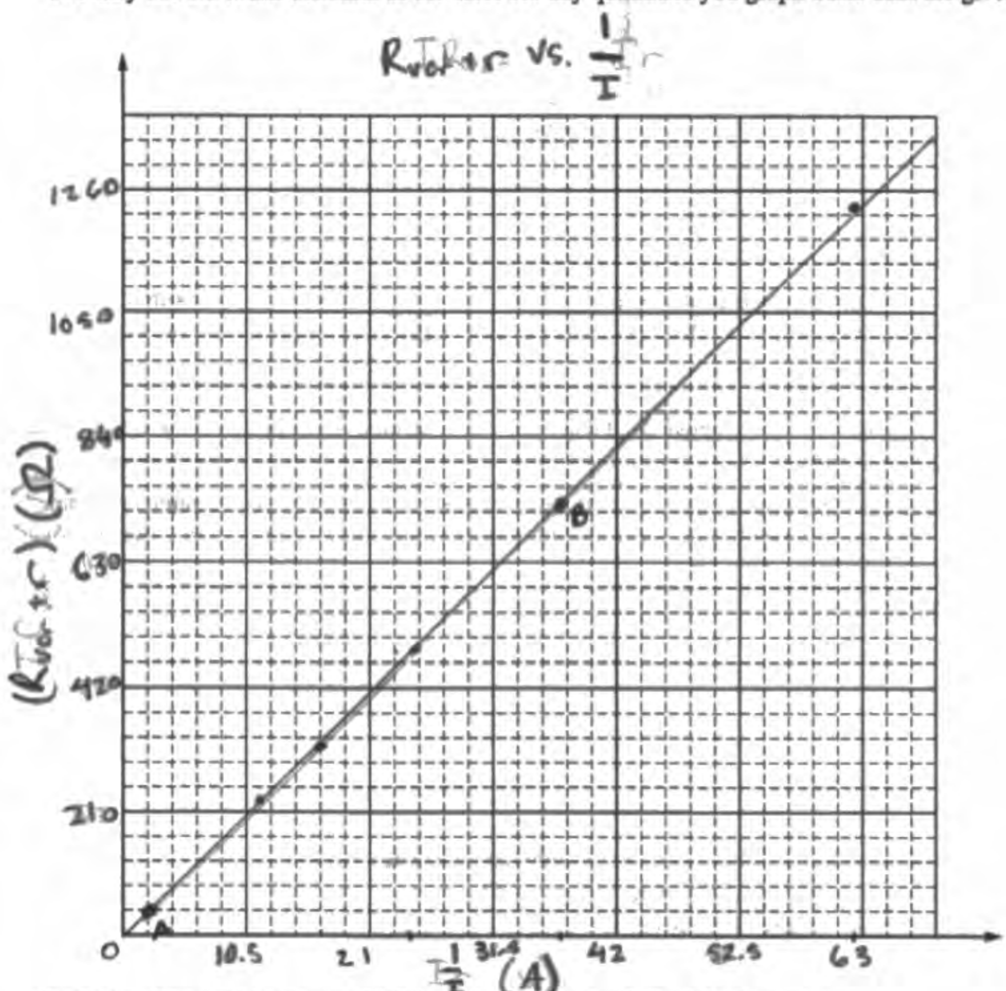
GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

$\frac{Y_B - Y_A}{X_B - X_A} \rightarrow \frac{730 - 42}{37 - 2.1} = 19.713$ * slope = $\mathcal{E}$

$\mathcal{E} \approx 19.713$

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2

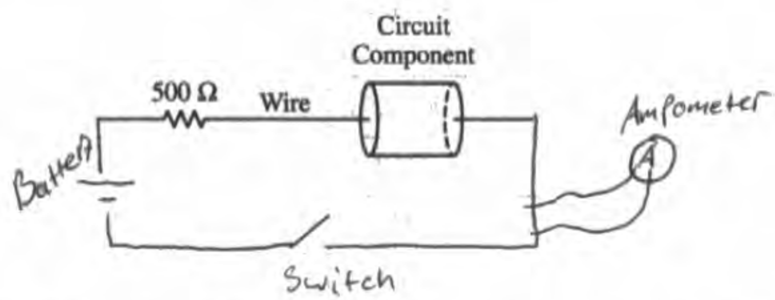
Begin your response to QUESTION 2 on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

- have students create the circuit found in (a)(i)
- have Students measure the current (I) at different times after closing the Switch
- record results and repeat to ~~correct~~ elliminate possible error

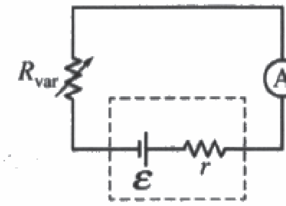
~~students will~~

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

When the switch is closed, the students would initially see no current if the ~~capacitor~~ unknown component was a capacitor but will see current after capacitor reaches full capacitence.

Question 2

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$ $\quad \mathcal{E} = \frac{I}{r} l v$

$\mathcal{E} = Blv$

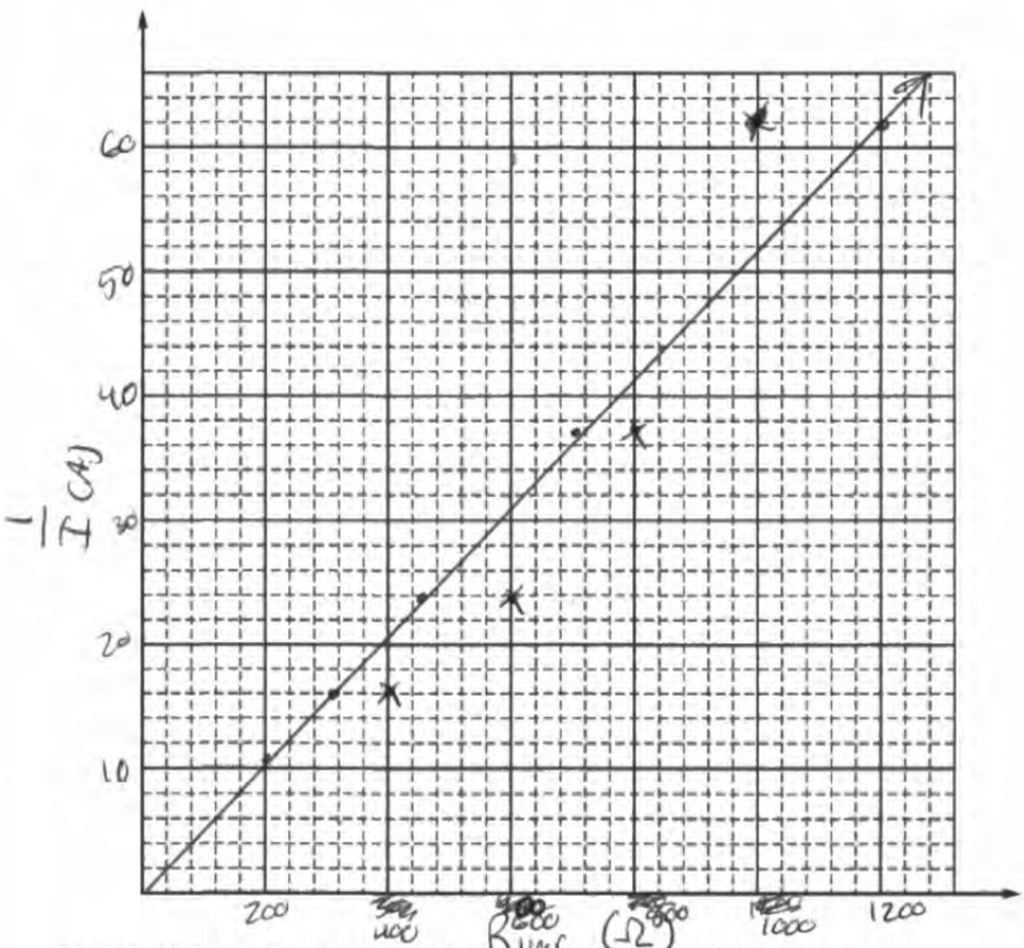
ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: R_{var} (Ω) Vertical Axis: $\frac{1}{I}$ (A)

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

$\mathcal{E} = \frac{I}{r}$ $\quad \mathcal{E} = \frac{0.087 - 0.060}{200 - 300}$

$\mathcal{E} = 2.7 \times 10^{-4}$

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5186/9

Question 2

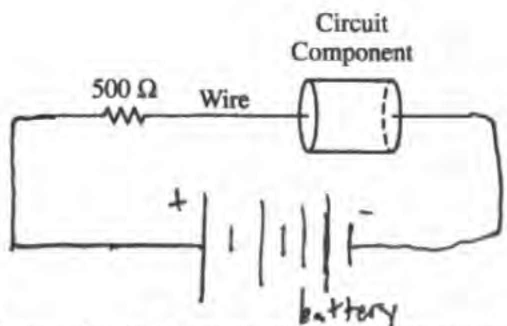
Begin your response to QUESTION 2 on this page.

2. (12 points, suggested time 25 minutes)

Students are given an unknown circuit component that is connected in series to a resistor with known resistance 500 Ω.

(a) The students are asked to experimentally determine whether the component is a resistor or an uncharged capacitor.

i. Complete the following diagram to show how to use standard circuit equipment to determine whether the component is a resistor or an uncharged capacitor.



ii. Describe an experimental procedure to determine whether the component is a resistor or an uncharged capacitor. Refer to the circuit equipment in the diagram drawn in part (a)(i).

You would use an ameter to find the current and if the system doesn't have the same current after traveling through. Than you can predict that it is a resistor due to resistors limiting current through systems.

iii. What results would the students expect if the component is an uncharged capacitor? Support your answer in terms of potential difference and charge.

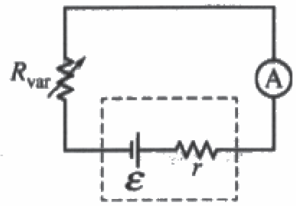
The potential difference would be around or equal to zero. The charge of the System would be neutral due to the Capacitor holding the charge.

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2

Continue your response to **QUESTION 2** on this page.



The students conduct a different experiment to determine the emf $\mathcal{E}$ of a battery that is not ideal and has internal resistance $r = 30\,\Omega$. The battery is connected to a variable resistor in a circuit, as shown. The students measure the current I through the circuit for different values of resistance R_{var} of the variable resistor that is connected to the battery. The following table contains the data collected.

I (A)	R_{var} (Ω)		
0.087	200		
0.060	300		
0.042	450		
0.027	700		
0.016	1200		

(b)

i. Write an equation describing the circuit in terms of $\mathcal{E}$, I, r, and R_{var}.

$$\mathcal{E} = \frac{Ir}{R}$$

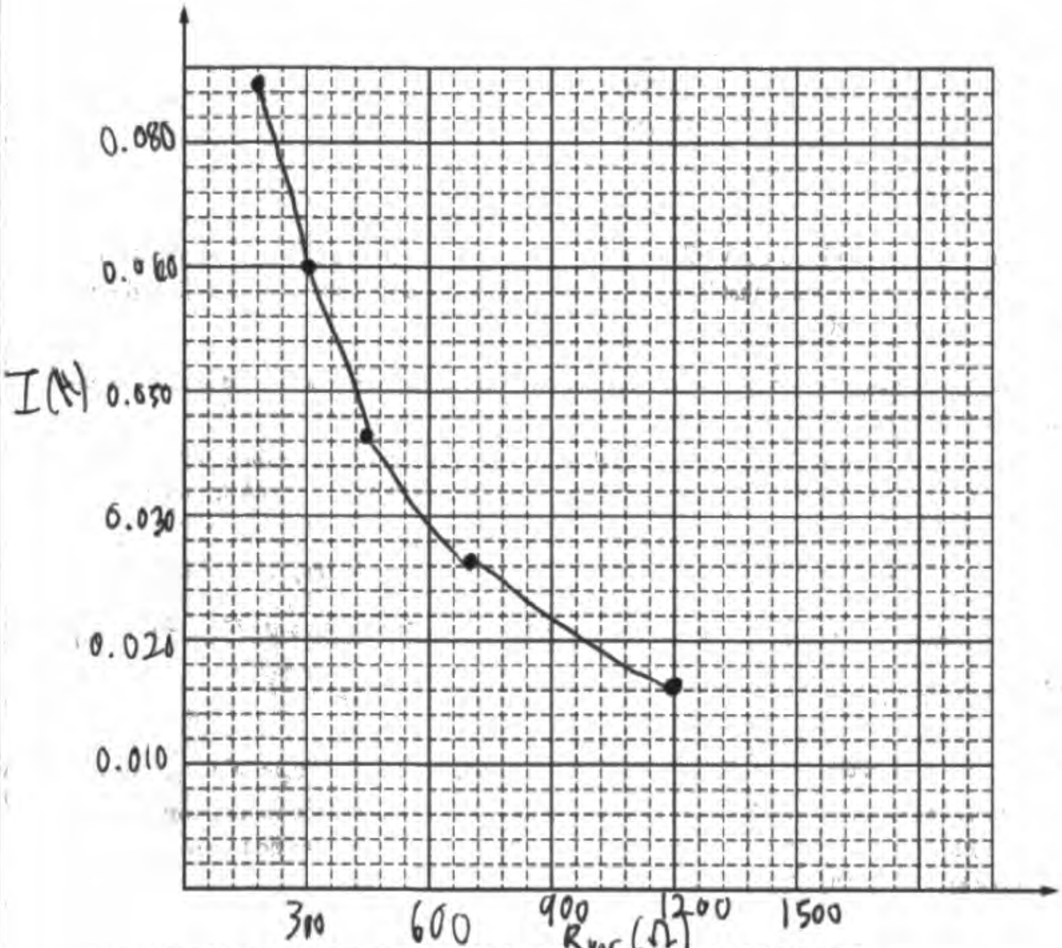
ii. Which quantities could be graphed to yield a straight line that could be used to calculate a numerical value for the emf $\mathcal{E}$ of the battery?

Horizontal Axis: I (A) Vertical Axis: R_{var} (Ω)

Question 2

Continue your response to **QUESTION 2** on this page.

iii. Plot the data points for the quantities indicated in part (b)(ii) on the following graph. Clearly scale and label all axes, including units. Draw a straight line that best represents the data.
You may use the blank columns in the table for any quantities you graph other than the given data.



iv. Using the graph from part (b)(iii), determine the emf $\mathcal{E}$ of the battery.

$$\mathcal{E} = \frac{(300)(20\Omega)}{(10\Omega)}$$

$$\mathcal{E} = 30\ \Omega$$

Question 2

Sample Identifier: P2 Q2 Sample A
Score: 12

a.i.

- 1 point was earned. The response correctly includes a source of potential difference in the diagram.
- 1 point was earned. The response correctly includes an appropriately connected measuring device in the diagram.

a.ii.

- 1 point was earned. The response correctly describes a procedure that includes a measurement of current and voltage.
- 1 point was earned. The response correctly describes a procedure that includes a measurement at different times.

a.iii.

- 1 point was earned. The response correctly indicates both that the current in the circuit would decrease over time and that the capacitor would store more charge.
- 1 point was earned. The response correctly indicates that the potential difference across the capacitor would increase over time.

b.i.

- 1 point was earned. The response correctly includes an equation that applies the loop rule.

b.ii.

- 1 point was earned. The response correctly indicates quantities that, when graphed, result in a linear graph that allows the determination of the emf of the battery.

b.iii.

- 1 point was earned. The response correctly labels both axes with a linear scale and with correct labels and units.
- 1 point was earned. The response correctly plots the data over more than half of the grid area.
- 1 point was earned. The response correctly draws a best-fit line that approximates the data.

b.iv.

- 1 point was earned. The response correctly uses the graph to determine an experimental value for the emf with correct units of volts.

Sample Identifier: P2 Q2 Sample B
Score: 11

a.i.

- 1 point was earned. The response correctly includes a source of potential difference in the diagram.
- 1 point was earned. The response correctly includes an appropriately connected measuring device in the diagram.

a.ii.

- 1 point was earned. The response correctly describes a procedure that includes a measurement of current.
- 1 point was earned. The response correctly describes a procedure that includes a measurement at different times.

a.iii.

- 1 point was earned. The response correctly indicates that the capacitor charge increases over time.
- 1 point was earned. The response correctly indicates that the potential difference across the capacitor would increase over time.

b.i.

- 1 point was earned. The response correctly includes an equation that applies the loop rule.

b.ii.

- 1 point was earned. The response correctly indicates quantities that, when graphed, result in a linear graph that allows the determination of the emf of the battery.

b.iii.

- 0 points were earned. The response correctly labels both axes with a linear scale and correct labels, but the axes do not include units.
- 1 point was earned. The response correctly plots the data over more than half of the grid area.
- 1 point was earned. The response correctly draws a best-fit line that approximates the data.

b.iv.

- 1 point was earned. The response correctly uses the graph to determine an experimental value for the emf with correct units of volts.

Sample Identifier: P2 Q2 Sample C
Score: 11

a.i.

- 1 point was earned. The response correctly includes a source of potential difference in the diagram.
- 1 point was earned. The response correctly includes an appropriately connected measuring device in the diagram.

a.ii.

- 1 point was earned. The response correctly describes a procedure that includes a measurement of current.
- 1 point was earned. The response correctly describes a procedure that includes a measurement at different times.

a.iii.

- 1 point was earned. The response correctly indicates both that the current in the circuit would decrease over time and that the capacitor would store more charge.
- 0 points were earned. The response does not correctly indicate that the potential difference across the capacitor would increase over time.

b.i.

- 1 point was earned. The response correctly includes an equation that applies the loop rule.

b.ii.

- 1 point was earned. The response correctly indicates quantities that, when graphed, result in a linear graph that allows the determination of the emf of the battery.

b.iii.

- 1 point was earned. The response correctly labels both axes with a linear scale and with correct labels and units.
- 1 point was earned. The response correctly plots the data over more than half of the grid area.
- 1 point was earned. The response correctly draws a best-fit line that approximates the data.

b.iv.

- 1 point was earned. The response correctly determines an experimental value for the emf with correct units of volts.

Sample Identifier: P2 Q2 Sample D
Score: 10

a.i.

- 1 point was earned. The response correctly includes a source of potential difference in the diagram.
- 1 point was earned. The response correctly includes an appropriately connected measuring device in the diagram.

a.ii.

- 1 point was earned. The response correctly describes a procedure that includes a measurement of current.
- 1 point was earned. The response correctly describes a procedure that includes a measurement at different times.

a.iii.

- 1 point was earned. The response correctly indicates both that the current in the circuit would decrease over time and that the capacitor would store more charge.
- 0 points were earned. The response does not correctly indicate that the potential difference across the capacitor would increase over time.

b.i.

- 1 point was earned. The response correctly includes an equation that applies the loop rule.

b.ii.

- 1 point was earned. The response correctly indicates quantities that, when graphed, result in a linear graph that allows the determination of the emf of the battery.

b.iii.

- 1 point was earned. The response correctly labels both axes with a linear scale, correct labels, and units.
- 1 point was earned. The response correctly plots the data over more than half of the grid area.
- 1 point was earned. The response correctly draws a best-fit line that approximates the data.

b.iv.

- 0 points were earned. The response does not correctly determine an experimental value for the emf. This is a result of incorrectly calculating the inverse of the total resistance.

Sample Identifier: P2 Q2 Sample E
Score: 8

a.i.

- 1 point was earned. The response correctly includes a source of potential difference in the diagram.
- 1 point was earned. The response correctly includes an appropriately connected measuring device in the diagram.

a.ii.

- 1 point was earned. The response correctly describes a procedure that includes a measurement of current.
- 1 point was earned. The response correctly describes a procedure that includes a measurement at different times.

a.iii.

- 0 points were earned. The response does not correctly indicate either that the current decreases or that the capacitor charges.
- 1 point was earned. The response correctly indicates that the potential difference across the capacitor would increase over time and be equal to the emf of the battery.

b.i.

- 1 point was earned. The response correctly includes an equation that applies the loop rule.

b.ii.

- 1 point was earned. The response correctly indicates quantities that, when graphed, result in a linear graph that allows the determination of the emf of the battery.

b.iii.

- 0 points were earned. The response does correctly label the axes, but the response does not correctly label both axes with a linear scale or include the correct units for the x-axis.
- 1 point was earned. The response correctly plots the data over more than half of the grid area.
- 0 points were earned. The response does not correctly draw a best-fit line that approximates the data.

b.iv.

- 0 points were earned. The response does not correctly determine the emf of the battery.

Sample Identifier: P2 Q2 Sample F
Score: 8

a.i.

- 1 point was earned. The response correctly includes a source of potential difference in the diagram.
- 1 point was earned. The response correctly includes an appropriately connected measuring device in the diagram.

a.ii.

- 1 point was earned. The response correctly describes a procedure that includes a measurement of the current in the circuit.
- 1 point was earned. The response correctly describes a procedure that includes a measurement at different times by closely monitoring the ammeter and looking at the ammeter reading after a time, as described in part (a)(iii).

a.iii.

- 1 point was earned. The response correctly indicates that the current decreases over time.
- 0 points were earned. The response does not correctly indicate anything about the potential difference across the capacitor.

b.i.

- 1 point was earned. The response correctly includes an equation that applies the loop rule.

b.ii.

- 0 points were earned. The response does not correctly indicate quantities that, when graphed, result in a linear graph that allows the determination of the emf of the battery.

b.iii.

- 0 points were earned. The response does not correctly label the horizontal axis. The variable resistance in ohms is plotted instead of the reciprocal of the variable resistance. Furthermore, the scale is not linear.
- 1 point was earned. The response correctly plots the data over more than half of the grid area.
- 1 point was earned. The response correctly draws a best-fit line that approximates the data.

b.iv.

- 0 points were earned. The response does not correctly determine the emf of the battery.

Sample Identifier: P2 Q2 Sample G
Score: 7

a.i.

- 1 point was earned. The response correctly includes a battery in a complete circuit.
- 1 point was earned. The response correctly includes a measurement device in the circuit.

a.ii.

- 1 point was earned. The response describes the use of a measurement device correctly by measuring the current in the circuit.
- 1 point was earned. The response correctly describes taking one measurement after a long time.

a.iii.

- 0 points were earned. The response does not describe a correct result of the experiment that indicates the current in the circuit decreases to zero over time. The response only mentions measuring the current in different locations in the circuit.
- 0 points were earned. The response does not describe a correct result of the experiment that indicates that the electric potential difference across the capacitor increases from zero over time.

b.i.

- 0 points were earned. The response does not include an equation that correctly applies the loop rule.

b.ii.

- 1 point was earned. The response correctly indicates appropriate quantities that, when graphed, result in a linear graph that would allow for a determination of the emf.

b.iii.

- 0 points were earned. The response correctly includes a linear scale and labeled quantities, but the horizontal axis has incorrect units.
- 1 point was earned. The response correctly displays the data plotted within at least half of the grid area.
- 1 point was earned. The response correctly displays a best-fit line that approximates the trend of the data.

b.iv.

- 0 points were earned. The response does not have correct units.

Sample Identifier: P2 Q2 Sample H
Score: 6

a.i.

- 0 points were earned. The response does not include a source of potential difference in a complete circuit.
- 1 point was earned. The response includes a measurement device that is appropriately connected in the circuit.

a.ii.

- 1 point was earned. The response correctly describes the use of a measurement device to measure the potential difference.
- 0 points were earned. The response does not describe taking measurements at different times or after a long time.

a.iii.

- 0 points were earned. The response does not describe a correct result of the experiment that indicates the current in the circuit decreases to zero over time.
- 0 points were earned. The response does not describe a correct result of the experiment that indicates that the electric potential difference across the capacitor increases from zero over time. Furthermore, the potential difference across a resistor in an open circuit would also have a potential difference of zero.

b.i.

- 1 point was earned. The response correctly applies the loop rule.

b.ii.

- 1 point was earned. The response correctly indicates appropriate quantities that, when graphed, result in a linear graph that would allow for a determination of the emf.

b.iii.

- 0 points were earned. The response correctly includes linear scales on both axes, and the correct label and units are present on the vertical axis. However, the label is correct on the horizontal axis with incorrect units. The units should be $\frac{1}{\text{A}}$.
- 1 point was earned. The response correctly displays the data plotted within at least half of the grid area.
- 1 point was earned. The response correctly displays a best-fit line that approximates the trend of the data.

b.iv.

- 0 points were earned. The response does correctly determine the value of the emf of the battery, but the response does not include the correct units.

Sample Identifier: P2 Q2 Sample I
Score: 6

a.i.

- 1 point was earned. The response correctly includes a source of potential difference in the diagram.
- 0 points were earned. The response does not correctly include an appropriately connected ammeter in the diagram.

a.ii.

- 1 point was earned. The response correctly describes a procedure that includes a measurement of the current in the circuit.
- 1 point was earned. The response correctly describes a procedure that includes a measurement at different times.

a.iii.

- 0 points were earned. The response incorrectly indicates that the current increases over time.
- 0 points were earned. The response does not indicate anything about the potential difference across the capacitor.

b.i.

- 0 points were earned. The response does not correctly include an equation that applies the loop rule.

b.ii.

- 1 point was earned. The response correctly indicates quantities that, when graphed, result in a linear graph that allows the determination of the emf of the battery.

b.iii.

- 0 points were earned. The response does correctly label the horizontal axis and includes a linear scale for both axes, but the response does not correctly include the correct units for the vertical axis.
- 1 point was earned. The response correctly plots the data over more than half of the grid area.
- 1 point was earned. The response correctly draws a best-fit line that approximates the data.

b.iv.

- 0 points were earned. The response does not correctly determine the emf of the battery.

Sample Identifier: P2 Q2 Sample J
Score: 3

a.i.

- 1 point was earned. The response correctly includes a battery in a complete circuit.
- 0 points were earned. The response does not correctly include a measurement device that is appropriately connected in the circuit.

a.ii.

- 1 point was earned. The response describes the appropriate use of a measurement device by measuring the current in the circuit.
- 0 points were earned. The response does not describe taking measurements at different times or after a long time.

a.iii.

- 0 points were earned. The response does not describe a correct result of the experiment that indicates the current in the circuit decreases to zero over time.
- 0 points were earned. The response does not describe a correct result of the experiment that indicates that the electric potential difference across the capacitor increases from zero over time.

b.i.

- 0 points were earned. The response does not include an equation that correctly applies the loop rule.

b.ii.

- 0 points were earned. The response does not indicate appropriate quantities that, when graphed, result in a linear graph that would allow for a determination of the emf.

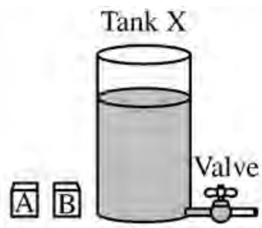
b.iii.

- 0 points were earned. The response does correctly include numerical values on both axes with a linear scale, but the scale on the y-axis is not linear.
- 1 point was earned. The response correctly displays the data plotted within at least half of the grid area.
- 0 points were earned. The response does not correctly display a smoothly drawn best-fit line that approximates the trend of the data.

b.iv.

- 0 points were earned. The response does not correctly determine the emf of the battery.

Begin your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

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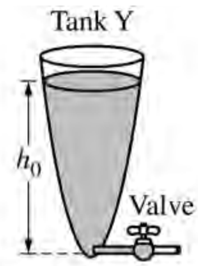
Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

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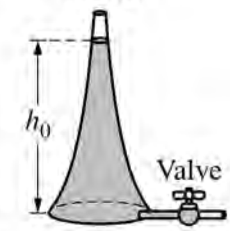
Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

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Question 3: Quantitative/Qualitative Translation 12 points

(a)(i) For a statement that collisions from the water particles exert upward forces on the block and collisions from the air particles exert downward forces on the block **1 point**

For a statement indicating that the force from the water is greater than the force from the air **1 point**

Example Response

The air particles collide with the top of the block and exert downward forces on the block. The water particles collide with the bottom of the block and exert upward forces on the block. The force exerted by the water particles is greater than the force exerted by the air particles. Therefore, the result of these forces is an upward buoyant force from the particles.

(a)(ii) For indicating that Block A has a greater density than Block B because Block A displaces a larger volume of water, thus the buoyant force on Block A is greater than the buoyant force on Block B **1 point**

Example Response

Because Block A *displaces a greater volume of fluid, the buoyant force on Block* A *is greater than the buoyant force on Block* B*. Because the buoyant force and gravitational force are balanced for both blocks, Block* A *must weigh more than Block* B*. Because the blocks have the same volume, Block* A *is more dense than Block* B*.*

Total for part (a) 3 points

(b)(i) For using Bernoulli's equation to derive the relationship between v_p and h **1 point**

For indicating that $P_2 = P_1$ **1 point**

For correct substitutions of the heights and speeds **1 point**

Example Solution

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho v_s^2 + \rho g h = \rho g(0) + \frac{1}{2}\rho v_p^2$$

$$v_p^2 = v_s^2 + 2gh$$

$$v_p = \sqrt{v_s^2 + 2gh}$$

(b)(ii) For using the continuity equation to derive the relationship between v_p and R **1 point**

For correct substitutions for the expressions of areas and speeds **1 point**

Example Solution

$A_1 v_1 = A_2 v_2$

$\pi R^2 v_s = \pi r^2 v_p$

$v_p = \frac{R^2}{r^2} v_s$

(b)(iii)	For using conservation principles to justify that when $R \gg r$, then $v_s \ll v_p$	**1 point**
	For indicating a very small value of v_s will have a negligible effect on v_p	**1 point**

Example Response

When the cross sectional area of the tank is very large compared to the cross sectional area of the pipe, the speed v_s of the surface of the water is much less than the speed of the water v_p exiting the pipe due to the constant volume flow rate. As a result, the speed of the surface of the water can be approximated as zero, so the speed of the water exiting the pipe can be approximated as $v_p = \sqrt{2gh}$.

Total for part (b) 7 points

(c)	For correctly relating the decrease in v_p to the decrease in the height of the surface of the water h	**1 point**
	For correctly relating the decrease in v_s to the increase in radius R	**1 point**

Example Response

According to the equation in part (b)(i), $v_p = \sqrt{v_s^2 + 2gh}$. As h decreases, v_p decreases.

When solving the equation in part (b)(ii) for v_s, it can be shown that $v_s = \frac{r^2}{R^2} v_p$. Therefore, an increase in R results in a decrease in v_s. Because v_p decreases with decreasing h, by using the same expression from part (b)(ii) in the case in which v_p decreases and R increases, it can be shown that the speed v_s of the water at the surface decreases.

OR

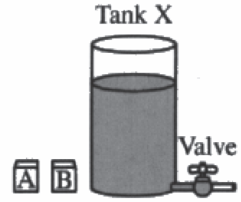
If the two equations from parts (b)(i) and (b)(ii) are solved simultaneously for v_s as a function of h and R, it can be shown that $v_s = \sqrt{\frac{2gh}{\frac{R^4}{r^4} - 1}}$. Therefore, as h decreases and R increases, v_s decreases.

Total for part (c) 2 points

Total for question 3 12 points

Question 3

Begin your response to QUESTION 3 on this page.



Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

The air particles exert a downwards force on the blocks, but the water particles under it exert upwards forces. The stronger force of the water particles below (at a greater pressure) lead to an upwards buoyant force.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

A has a greater density.

$\rho = \frac{m}{V}$

$F_g - F_B = 0$

$mg - \rho V g = 0$

→ $mg = \rho_w V g$

$m = \rho_w V$

$V_A > V_B$ because more has to be submerged to touch bottom first.

→ If $V_A > V_B$, $m_A > m_B$

$\rho = \frac{m}{V}$

If $m_A > m_b$, $\rho_A > \rho_b$

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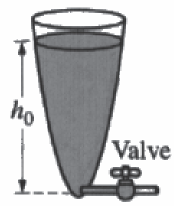
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 3

Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$h = h_0 - v_s t$

$P_1 + \rho g h_1 + \frac{1}{2}\rho v_s^2 = P_1 + \rho g h_2 + \frac{1}{2}\rho v_p^2$

$\rho g h + \frac{1}{2}\rho v_s^2 = \frac{1}{2}\rho v_p^2$

$2gh + v_s^2 = v_p^2$

$v_p = \sqrt{v_s^2 + 2gh}$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

$A_1 v_1 = A_2 v_2$

$\pi R^2 v_s = \pi r^2 v_p$

$\frac{R^2}{r^2} v_s = v_p$

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

when R is much greater than r, $\frac{r^2}{R^2}$ is very small, $v_p\left(\frac{r^2}{R^2}\right) = v_s$, so v_s is very small (close to 0). $v_p = \sqrt{v_s^2 + 2gh}$, but becaus v_s is very small, it is negligible and $v_p = \sqrt{2gh}$

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Question 3

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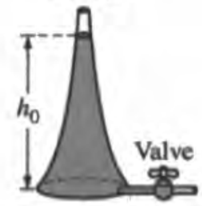
Tank Z



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

~~As the water goes down, R gets much larger than r. If $\frac{R^2}{r^2}V_s = V_P$, $V_s = V_P\left(\frac{r^2}{R^2}\right)$, and V_s will get smaller.~~

~~$V_P = \sqrt{V_s^2 + 2gh}$ shows that V_P lessens when h decreases, so as the water level goes down, V_P will decrease.~~

$\frac{R^2}{r^2}V_s = V_P$

$V_P = \sqrt{V_s^2 + 2gh}$

$\frac{R^2}{r^2}V_s = \sqrt{V_s^2 + 2gh}$

$\frac{R^4}{r^4}V_s^2 = V_s^2 + 2gh$

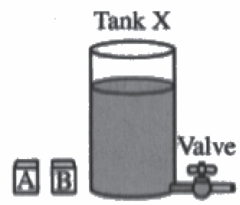
$\left(\frac{R^4}{r^4} - 1\right)V_s^2 = 2gh$

$V_s = \sqrt{\frac{2gh}{\left(\frac{R^4}{r^4} - 1\right)}}$

Because h↓ and R↑, V_s will decrease over time

Question 3

Begin your response to QUESTION 3 on this page.



Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

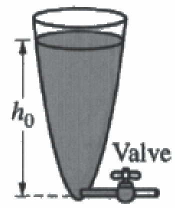
When the blocks are placed in water, the air particles above them are pushing down on them with atmospheric pressure, while the weight of the water displaced is pushing up on them in an attempt to bring the water back to its original spot. There is an upward buoyant force exerted on the block because the pressure from the water particles is more than the pressure from air particles on an equal area sized the block.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block A has greater density because both blocks have the same dimensions and, using the relative densities of the block compared to water, since $\frac{\rho_A}{\rho_w}$ represents the fraction of the volume of an object under water, and Block A and B have equal dimensions, if Block A touched the bottom first, more of it's volume was submerged in water, therefore $\frac{\rho_A}{\rho_w} > \frac{\rho_B}{\rho_w}$ meaning $\rho_A > \rho_B$ where ρ is density.

Question 3

Continue your response to **QUESTION 3** on this page.



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

Bernoulli's formula: $P_1 + \rho_1 g h_1 + \rho_1 v_s^2/2 = P_2 + \rho_2 g h_2 + \rho_2 v_p^2/2$. Since $P_1 = P_2 = 1atm$, and $\rho_1 = \rho_2$, the equation becomes $\rho g h_1 + \rho v_s^2/2 = \rho g h_2 + \rho v_p^2/2$. comparing relative heights, & dividing by ρ we have $gh + v_s^2/2 = v_p^2/2$. Mult. by 2, $2gh + v_s^2 = v_p^2$. Square root. $v_p = \sqrt{2gh + v_s^2}$.

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

$A_1 v_s = A_2 v_p$ so $v_p = \frac{A_1 v_s}{A_2}$. $A_1 = \pi R^2$ and $A_2 = \pi r^2$ so the equation is $v_p = \frac{\pi R^2 \cdot v_s}{\pi r^2} = \frac{R^2}{r^2} v_s$. So, $v_p = \frac{R^2}{r^2} v_s$

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

$A_1 v_s = A_2 v_p$ so $v_s = \frac{A_2 v_p}{A_1} = \frac{\pi r^2 v_p}{\pi R^2} = \frac{r^2 v_p}{R^2}$. If R is sufficiently greater than r, $v_s = \frac{r^2 v_p}{R^2}$ approaches 0, so $v_p = \sqrt{v_s^2 + 2gh}$ has the v_s^2 approach 0, making it negligible and $v_p = \sqrt{0 + 2gh} = \sqrt{2gh}$

Question 3

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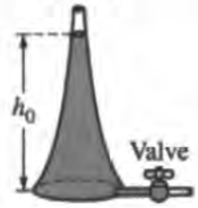

Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

The speed v_s decreases because, $v_p = \frac{R^2}{r^2} v_s$, and because the flowrate stays constant, we can rewrite this equation as $v_p r^2 = R^2 v_s$ where $v_p r^2$ stays constant since ~~doesn't change~~ the valve's cross-sectional area doesn't change. Therefore, $R^2 v_s$ stays constant and as water exits the other end of the pipe, R^2 increases, so to ~~stay constant~~ maintain constant flow rate, v_s decreases.

Question 3

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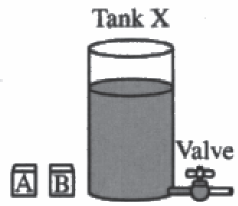

Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

There is an upwards buoyant force exerted on each block because the density of particles in the water is greater than that of air. This means that, on average, there are more water particles colliding with the blocks from below than there are particles colliding with the blocks from the above air. This difference in number of collisions is responsible for the upwards buoyant force.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block A has a greater density because by touching the bottom of the tank first, we know it was more submerged than block B. Because block A was deeper submerged, it displaced more water than block B, and because the buoyant force is proportional to the volume of water displaced by an object, there was a greater buoyant force on block A, this mean that to be at rest initially, block A must have had a greater downwards force of gravity, meaning it had more mass, and because both blocks had the same dimensions, block A was more dense.

F

Question 3

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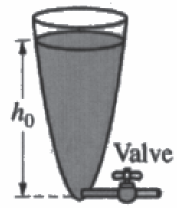

Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$P_1 + \rho g y_1 + \frac{1}{2}\rho v^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v^2$

$P_0 + \rho g h + \frac{1}{2}\rho v_s^2 = P_0 + \frac{1}{2}\rho v_p^2$ ← the vertical displacement is relative to the bottom of the tank and there's atmospheric pressure at both ends

$2\rho g h + \rho v_s^2 = \rho v_p^2$

$2gh + v_s^2 = v_p^2$ $\quad$ $v_p = \sqrt{v_s^2 + 2gh}$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

$A_1 v_s = A_2 v_p$

$\pi R^2 v_s = \pi r^2 v_p$

$v_p = \frac{R^2}{r^2} v_s$

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim. Given $v_p = \frac{R^2}{r^2} v_s$, we can find that v_s is proportional to $\frac{r^2}{R^2}$. This means that if R is significantly greater than r, v_s is about 0, meaning that v_s^2 is very close to 0. This means you can remove the v_s^2 from $v_p = \sqrt{v_s^2 + 2gh}$, to estimate using $v_p = \sqrt{2gh}$.

Question 3

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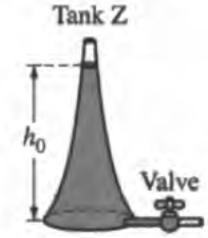

Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

The speed at which the surface of the water moves downwards decreases. This is because, using the equation $v_p = \frac{R^2}{r^2} v_s$, we find that $v_s = \frac{r^2}{R^2} v_p$. This means that v_s is inversely proportional to the radius at the top of the water. Since that radius increases as the water level lowers, v_s decreases as the level lowers. Similarly, using the equation $v_p = \sqrt{v_s^2 + 2gh}$, we find that v_s is directly proportional to h, so as the level decreases, so does v_s.

Question 3

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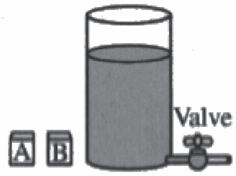

Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

The water particles exerts a force upward as the water particles at the bottom of the block resist the weight of the block more so than the air particles pushing down on the top of the block because the water molecules are denser than air particles.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

As water depth decreases, there is less pressure to exert a greater force up on the block. Thus, Block A had the greatest density as its greater density means greater mass and greater force of gravity as Block A and B had identical dimensions. With this greater Fg, Block A could reach the bottom first against a decreasing upward force.

Question 3

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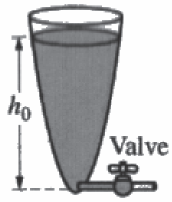

Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$P_1 + \rho g h_0 + \frac{1}{2}\rho v_s^2 = P_2 + \rho g h_f + \frac{1}{2}\rho v_p^2$

$\rho g h + \frac{1}{2}\rho v_s^2 = \frac{1}{2}\rho v_p^2$

$2gh + v_s^2 = v_p^2$

$v_p = \sqrt{2gh + v_s^2}$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

$A_1 V_1 = A_2 V_2$

$\pi (R^2) v_s = \pi (r^2) v_p$

$v_s R^2 = v_p r^2$

$v_p = \frac{R^2}{r^2} v_s$

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim. If R is much greater than r, than the ratio between vp and vs is so high that vs makes minimal difference in calculating vp through $\sqrt{2gh + v_s^2}$. A greater R implies a greater h; therefore, $\sqrt{2gh}$ is still revalent in vp calculations.

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Question 3

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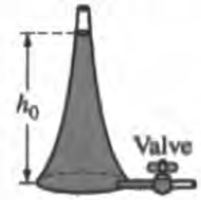
Tank Z



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

Decrease.

- As water exits, the height of the water decreases. By $v_p = \sqrt{2gh + v_s^2}$, a decrease in h leads to a decrease in speed.
- In this shape, R^2/r^2 is small in the equation $v_p = \frac{R^2}{r^2} v_s$. The radius at the top is smaller the the radius at the bottom, leading to a small final speed than initial speed as R^2/r^2 is less than 1, showing a smaller vp than vs in the equation $v_p = \frac{R^2}{r^2} v_s$.

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Question 3

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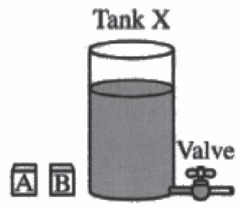

Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

There is an upwards bouyant force because water has a higher density than air, meaning it has more particles in a given space. This means that more water particles collide with the blocks than air particles, so the force is upwards because the water is beneath the block.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block B. This is because $F_B = \rho V g$ and we know that V and g are the same for both blocks so it can be said $F_B = \rho$. Because block B is still Floating, its F_B must be greater than that of block A as block A's F_B is no longer countering F_g. Therefore, because $F_B = \rho$ and block B's F_B is greater, $\rho_B > \rho_A$

Question 3

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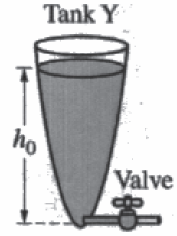

Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$P_s + \rho g y_s + \frac{1}{2}\rho v_s^2 = P_p + \rho g y_p + \frac{1}{2}\rho v_p^2$

$P_s = P_p = P_0$ $y_p = 0$ → $\rho g y_s + \frac{1}{2}\rho v_s^2 = \frac{1}{2}\rho v_p^2$ $2gy_s + v_s^2 = v_p^2$

$v_p = \sqrt{2gy_s + v_s^2}$ $y_s = h$ $\boxed{v_p = \sqrt{v_s^2 + 2gh}}$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2}v_s$.

$A_s = \pi R^2$ $A_p = \pi r^2$

$A_s v_s = A_p v_p$

$\pi R^2 v_s = \pi r^2 v_p$ $v_p = \frac{\pi R^2 v_s}{\pi r^2} = \boxed{\frac{R^2 v_s}{r^2}}$

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

This is because when R is much greater than r, v_s becomes negligible, as can be seen through $v_p = \frac{R^2 v_s}{r^2}$. Because v_s is negligible, it can be ignored in $v_p = \sqrt{v_s^2 + 2gh}$

Question 3

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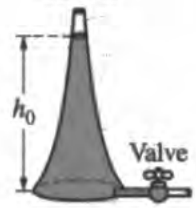

Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

V_s increases. This is because rearanging the equation from part (b)(ii) gives $V_s = \frac{r^2 V_p}{R^2}$ because r is constant and R increases as time passes, V_s is shown to increase. Additionally, rearanging part (b)(i) gives $V_s = \sqrt{V_p^2 + 2gh}$. Because V_p can be expected to increase and is squared, despite h decreasing, the right side of the equation can be expected to increase with time, meaning V_s must increase as well.

Question 3

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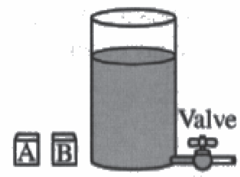

Note: Figure not drawn to scale.

Figure 1

3. **(12 points, suggested time 25 minutes)**

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

The air and water molecules collide with each other and momentum is conserved. Since momentum is conserved, the air molecules are pushing the denser water molecules up.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block A is denser than Block B. The blocks have the buoyant force and F_g exerted on them. Since they have the same volume, F_B will be the same for both blocks since they occupy the same volume of water. F_g will be greater for the denser material since $m = \rho V$ and mass is directly proportional to density when V is constant. Therefore, the denser block will feel a greater downward net force since $\Sigma F = F_B - F_g$, and Block A is denser than Block B since it accelerated to the bottom of the tank faster.

Question 3

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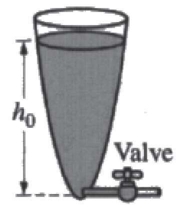
Tank Y



Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$\cancel{P_1} + \cancel{\rho} g y_1 + \frac{1}{2}\cancel{\rho} v_1^2 = \cancel{P_2} + \cancel{\rho g y_2} + \frac{1}{2}\cancel{\rho} v_2^2$

$P_1 = P_2$
$\rho_1 = \rho_2$
$y_2 = 0$

$g y_1 + \frac{1}{2} v_1^2 = g y_2 + \frac{1}{2} v_2^2$

$\frac{1}{2} v_1^2 + g y_1 = \frac{1}{2} v_2^2$

$v_1^2 + 2 g y_1 = v_2^2$

$v_s^2 + 2gh = v_p^2$

$\boxed{v_p = \sqrt{v_s^2 + 2gh}}$

$v_2 = v_p$
$v_1 = v_s$
$y_1 = h$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

$A_1 = R^2 \pi$
$A_2 = r^2 \pi$
$v_1 = v_s$
$v_2 = v_p$

$A_1 v_1 = A_2 v_2$

$R^2 \cancel{\pi} \cdot v_s = r^2 \cancel{\pi} \cdot v_p$

$R^2 v_s = r^2 v_p$

$\boxed{v_p = \frac{R^2}{r^2} \cdot v_s}$

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

When R is suffuently greater than r, the speed v_s can be considered to be 0 m/s since it moves so slowly relative to the speed v_p. Therefore, because $A_{top} > A_{bottom}$ by such a large factor; $v_s = 0$ compared to the enormous speed of v_p that is much greater than v_s. from the equation in part (ii), $\frac{R^2}{r^2}$ is so great that v_s is miniscule to v_p.

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Question 3

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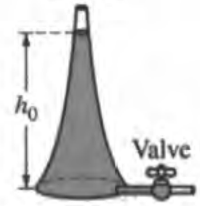
Tank Z



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

$V_p = \sqrt{V_s^2 + 2gh}$ $V_s = \sqrt{V_p^2 - 2gh}$

At time $t = 0$, the valve of Tank Z is opened.

$V_p = \frac{R^2}{r^2} V_s$ $V_s = \frac{r^2}{R^2} V_p$

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

V_s increases over time.

When rearranged to be a function V_s, the equations from part b i and b ii are $V_s = \sqrt{V_p^2 - 2gh_0}$ and $V_s = \frac{r^2}{R^2} V_p$

As time increases, h_0 approaches 0, so V_s must increase since it is directly proportional to V_p. Additionally, the top surface area of the water increases over time as water exits the tank, so the ratio of $\frac{r^2}{R^2}$ becomes closer to 1, increasing the value of V_s when V_p is constant or increasing.

Therefore, since the height of the water decreases and area of the water surface increases, the speed V_s increases based on $V_s = \sqrt{V_p^2 - 2gh_0}$ and $V_s = \frac{r^2}{R^2} V_p$

$t\uparrow$ $h_0\downarrow$ $V_s = \sqrt{V_p^2 - (0)} \Rightarrow V_s = V_p$ and $V_s \uparrow$

$t\uparrow$ $R\uparrow$

Question 3

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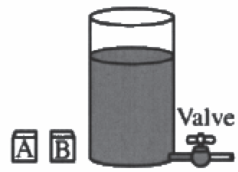

Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

Because the densities of the blocks are less than that of the water, they cannot displace the water and overcome the air pressure of the top of the tank.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block A, because it was partially submerged deeper into the water meaning it had a greater amount of water displaced meaning greater density.

Question 3

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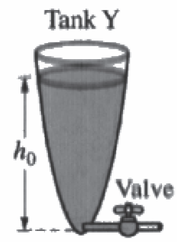

Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\rho g h_0 + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$$

$$2gh_0 + v_1^2 = v_2^2$$

$$\boxed{\sqrt{2gh_0 + v_1^2} = v_2}$$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

$$A_1 v_p = A_2 v_s$$

$$\pi r^2 v_p = \pi R^2 v_s$$

$$\boxed{v_p = \frac{R^2}{r^2} v_s}$$

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

because v_p is equal to $\sqrt{v_s^2 + 2gh}$, when R is sufficiently greater than r, Vs can also be ignored.

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Question 3

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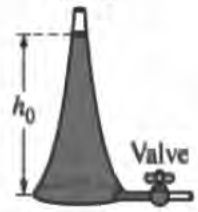

Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

The water's velocity idecreases, because as time goes on, there will be less water on top of the point of the valve and its height would decrease and according to the equation $v_p = \sqrt{v_s^2 + 2gh_0}$, this means that the velocity will lower at the valve

Question 3

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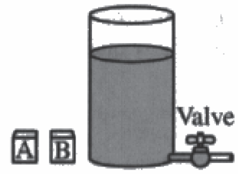


Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

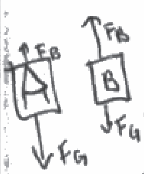
Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water. ρg✓

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

Water particles are denser than the blocks but the blocks are denser than the air. Gravity pushes down on the blocks but that is not enough to fully submerge the blocks as the water particles push upward, as they are allowed to move freely since they are liquid.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.



Block A has greater density since $\rho = \frac{m}{V}$ and volume is not accountable since both blocks have the same dimensions since Block A is denser, and heavier, the bouyant force does not = the gravity force and the block A goes downward. More downward force

Question 3

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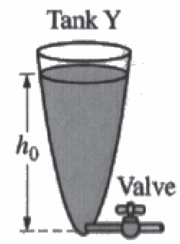

Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

~~P_1~~ + ~~$\rho g h_0$~~ $+ \frac{1}{2}\rho v_1^2 =$ ~~P_2~~ $+$ ~~ρ~~ $g h_2 + \frac{1}{2}\rho v_2^2$ $\sqrt{v_1^2 = 2gh^2 + v_2^2}$

$2 \cdot \frac{\frac{1}{2}\rho v_1^2}{\rho} = \left(\rho g h_2 + \frac{1}{2}\rho v_2^2\right)2$ $v_1 = \sqrt{v_2^2 + 2gh}$

density same $v_p = \sqrt{v_s^2 + 2gh}$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$.

$\frac{v_p}{v_s} = \frac{R^2}{r^2}$] top] bottom   ] everything else is negligible so only this ratio matters ∴

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

When the radius R is sufficiently greater v_s^2 is almost negligible since its decreasing at such a slow rate.

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Question 3

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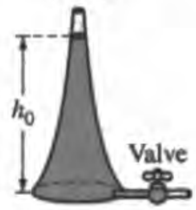
Tank Z



Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

The speed vs ~~de~~increases as the water moves downward since there is ~~less~~ more force on top. $v_p = \frac{R^2}{r^2} v_s$ (R^2 smaller, r^2 bigger, overall goes down)

so $\frac{R^2}{r^2} v_s$ - velocity increases. ~~then~~

~~[crossed out]~~

~~[crossed out]~~

$v_p = \sqrt{v_s^2 + 2gh}$ depends on the ~~[crossed out]~~ height to go down to make the v_s^2 greater so the v_p can remain the same.

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Question 3

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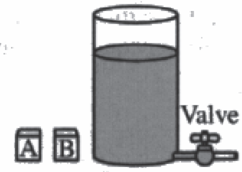
Tank X



Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

$F_b = \rho V g$. There is an upward buoyant force because the denser water particle's are pushing up with a greater force than the combination of air particles and block and gravity pushing down.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block A has greater density. $F_b = \rho V g$. Gravity and volume are the same for both A and B so density must be higher in the block that hits the bottom first.

Question 3

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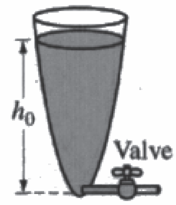

Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

$V_p < \Delta h$

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \sqrt{\frac{R^2}{r^2}}v_s$.

$V_p > R \longrightarrow R = \frac{\sqrt{V_p r^2}}{v^2}$

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim.

When R is a lot greater than r, $\sqrt{V_s}^2$ in the equation $v_p = \sqrt{vs^2 + 2gh}$ is negligible because the radius R is so large.

Question 3

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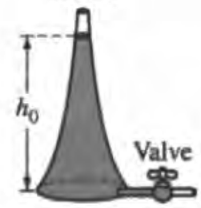

Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

As the water comes out, its speed will decrease. At First, $v_p = \sqrt{2gh}$ can be used, but as the radius increases, $v_p = \sqrt{v_s^2 + 2gh}$ must be used and this equation results in a larger amount.

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Question 3

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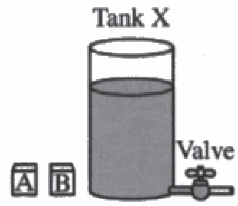

Note: Figure not drawn to scale.

Figure 1

3. (12 points, suggested time 25 minutes)

Tank X is a large cylindrical tank that is partially filled with water, as shown in Figure 1. The bottom of Tank X is connected to a short horizontal pipe. A valve that is initially closed can be opened to allow water to flow through the pipe and exit through the other end of the pipe.

(a) Two blocks, A and B, have identical dimensions and are placed in the tank. Both blocks float at rest and are partially submerged in the water.

i. The water and air can be modeled as consisting of individual particles that are in continuous random motion. In terms of interactions with both water and air particles, explain why there is an upward buoyant force exerted on each block.

The reason the blocks float is because the density of the block is less than that of the water. ~~The~~ buoyant force of the water is stronger and that's why there is an upward buoyant force.

ii. The valve is then opened, and water flows out through the pipe. The surface of the water moves downward. When Block A touches the bottom of Tank X, Block B is still above the bottom of Tank X. Which block has a greater density? Briefly explain your reasoning.

Block A has a greater density than Block B because although both blocks have less density than water, Block A has the greater density which is shown by the ~~fact~~ that Block A touched the bottom first when greater amounts of gravitational force was added as more and more water left the pipe.

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Question 3

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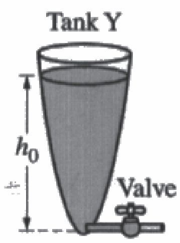

Note: Figure not drawn to scale.

Figure 2

Tank Y is a large tank with the top open to the air, as shown in Figure 2. The bottom of Tank Y is connected to a short horizontal pipe of radius r with a closed valve. Tank Y is filled with water to height h_0 above the horizontal pipe. Tank Y is specially designed so that when the valve is opened, the surface of the water moves downward at constant speed v_s.

(b) At time $t = 0$, the valve is opened.

i. Derive the relationship between the speed v_p at which water exits the pipe and the changing height h of the surface of the water above the pipe to show that $v_p = \sqrt{v_s^2 + 2gh}$.

The speed at which water exits the pipe has to be constant as a lesser area, like the bottom of Tank Y, will have a greater pressure to make up for the missing gravitational pressure that comes from ho decreasing.

ii. Derive the relationship between v_p and the changing radius R of the top surface of the water to show that $v_p = \frac{R^2}{r^2} v_s$. The speed at which the water exits the pipe has to be matching with the decrease in area. The decrease in area will require more pressure and that's what $\frac{R^2}{r^2}$ is required and derived in the equation.

iii. When the radius R of the tank is sufficiently greater than r, the speed v_p can be approximated as $v_p = \sqrt{2gh}$. Justify this claim. The reason $v_p = \sqrt{2gh}$ and not just $v_p = gh$ is because of R, which is a much greater radius than r and must therefore have a lesser speed because of more area and that is why the square root and 2 are required.

Question 3

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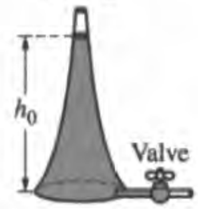


Note: Figure not drawn to scale.

Figure 3

Tank Z is a large tank whose top is open to the air and is shaped as shown in Figure 3. The bottom of Tank Z is connected to a short horizontal pipe with a closed valve. Tank Z is filled with water to a height h_0 above the horizontal pipe.

At time $t = 0$, the valve of Tank Z is opened.

(c) Does the speed v_s at which the surface of the water moves downward increase, decrease, or remain the same over time as water exits the other end of the pipe? Justify your answer by using or referencing equations from both part (b)(i) and part (b)(ii).

To prove that the speed decreases, we can use the equation in (b)(i) $v_p = \sqrt{v_s^2 + 2gh}$ which shows that there is less of a height decrease because there is a lesser velocity at the bottom of the tank because of a larger radius increase which also proves the equation in (b)(ii) that the flow of the valve will be much slower

Question 3

Sample Identifier: P2 Q3 Sample A
Score: 12

a.i.

- 1 point was earned. The response correctly indicates the air particles exert downward forces on the block and the water particles exert upward forces on the block.
- 1 point was earned. The response indicates that the force exerted from the water is greater than the force exerted from the air.

a.ii.

- 1 point was earned. The response claims that Block A has a greater density than Block B and uses a correct analysis of the forces exerted on the block to show the greater volume of water displaced by Block A means that Block A has a greater mass, and, therefore, greater density.

b.i.

- 1 point was earned. The response uses Bernoulli's equation to attempt to derive the relationship between v_p and h.
- 1 point was earned. The response shows that the pressures at the two locations (surface and pipe exit) are the same by eliminating the terms.
- 1 point was earned. The response correctly substitutes the speeds and heights into the expression.

b.ii.

- 1 point was earned. The response uses the continuity equation to derive the relationship between v_p and R.
- 1 point was earned. The response correctly substitutes values for areas and speeds.

b.iii.

- 1 point was earned. The response references conservation principles (the equation in part (b)(ii)) to show that surface speed is very small.
- 1 point was earned. The response indicates the term for v_s is not needed in the expression because v_s is negligible.

c.

- 1 point was earned. The response derives a mathematical expression for v_s as a function of R and h based on the expressions from parts (b)(i) and (b)(ii) that includes the correct functional dependence between v_p and h.
- 1 point was earned. The response derives a mathematical expression for v_s as a function of R and h based on the expressions from parts (b)(i) and (b)(ii) that includes the correct functional dependence between v_s and R.

Sample Identifier: P2 Q3 Sample B
Score: 11

a.i.

- 1 point was earned. The response correctly indicates the air particles push down on the block and the water particles push up on the block.
- 1 point was earned. The response indicates that the force exerted from the water is greater than the force exerted from the air because the pressure is greater beneath the block.

a.ii.

- 1 point was earned. The response uses a correct application of specific gravity that is derived from a correct analysis of the buoyant and gravitational forces exerted on the block, to conclude that the density of Block A is greater because a greater volume of Block A is submerged than Block B.

b.i.

- 1 point was earned. The response uses Bernoulli's equation to attempt to derive the relationship between v_p and h.
- 1 point was earned. The response shows that the pressures at the two locations (surface and pipe exit) are the same.
- 1 point was earned. The response correctly substitutes the speeds and heights into the expression.

b.ii.

- 1 point was earned. The response uses the continuity equation to derive the relationship between v_p and R.
- 1 point was earned. The response correctly substitutes values for areas and speeds.

b.iii.

- 1 point was earned. The response references conservation principles (the equation in part (b)(ii)) to show that surface speed approaches zero as R increases.
- 1 point was earned. The response states that v_s is negligible and can be eliminated from the expression.

c.

- 0 points were earned. The response does not attempt to connect the decreasing height to a decreasing value of v_p as the water exits the pipe.
- 1 point was earned. The response uses the equation from (b)(ii) to relate an increasing R to a decreasing v_s.

Sample Identifier: P2 Q3 Sample C
Score: 10

a.i.

- 0 points were earned. Although the response refers to collisions above and below the block, the response does not specify that the air particles push down on the block and the water particles push up on the block.
- 1 point was earned. The response indicates that the force exerted from the water is greater than the force exerted from the air.

a.ii.

- 1 point was earned. The response claims Block A has a greater density than Block B and uses a correct analysis of the forces exerted on the block to state that the greater volume of water displaced by Block A means that Block A has a greater buoyant force and, therefore, greater mass and density.

b.i.

- 1 point was earned. The response uses Bernoulli's equation to attempt to derive the relationship between v_p and h.
- 1 point was earned. The response shows that the pressures at the two locations (surface and pipe exit) are the same by indicating they are both atmospheric pressure and eliminating the terms.
- 1 point was earned. The response correctly substitutes the speeds and heights into the expression.

b.ii.

- 1 point was earned. The response uses the continuity equation to derive the relationship between v_p and R.
- 1 point was earned. The response correctly substitutes values for areas and speeds.

b.iii.

- 1 point was earned. The response references conservation principles (the equation in part (b)(ii)) to show that surface speed is very close to zero when R is much greater than r.
- 1 point was earned. The response indicates that v_s can be removed from the expression because v_s is very close to zero.

c.

- 0 points were earned. Although the response uses the equation from part (b)(i) to relate the decreasing height to speed, the response incorrectly relates the height to v_s. The response does not attempt to connect the decreasing height to the decreasing speed as the water exits the pipe.
- 1 point was earned. The response uses the equation from part (b)(ii) to relate an increasing R to a decreasing v_s.

Sample Identifier: P2 Q3 Sample D
Score: 9

a.i.

- 1 point was earned. The response correctly indicates that the water particles exert upward forces on the block and the air particles exert downward forces on the block.
- 0 points were earned. Although the response does indicate that the water exerts "more" force, it unclear whether this is in reference to more than the gravitational force or more than the force exerted from the air.

a.ii.

- 0 points were earned. Although the response correctly identifies that Block A has the greater density, the response does not provide the correct reasoning. The response does not reference the buoyant force or volume of fluid displaced and instead claims that Block A weighs more without providing support.

b.i.

- 1 point was earned. The response uses Bernoulli's equation to attempt to derive the relationship between v_p and h.
- 1 point was earned. The response shows that the pressures at the two locations are the same by eliminating the terms from Bernoulli's equation.
- 1 point was earned. The response correctly substitutes the height and the speed at each location.

b.ii.

- 1 point was earned. The response uses the continuity equation to derive the relationship.
- 1 point was earned. The response correctly substitutes the areas and speeds at the surface and pipe. Although an extraneous $\frac{1}{2}$ term was included on each side of the equation, and because the response eliminated the $\frac{1}{2}$ term, the addition of the $\frac{1}{2}$ term was ignored. The substitutions for the radii were correct.

b.iii.

- 1 point was earned. The response references the conservation principle in part (b)(ii) to relate the ratio of the speeds to the ratio of the areas and explains that the surface speed is very small when R is very large.
- 1 point was earned. The response indicates that the small value of the surface speed results in a value that has minimal effect on the expression from part (b)(i).

c.

- 1 point was earned. The response uses the expression from part (b)(i) to correctly indicate that the speed the water exits the pipe decreases as height decreases.
- 0 points were earned. Although the response does attempt to relate surface radius to speed, the response does not correctly describe the functional dependence of surface speed on radius.

Sample Identifier: P2 Q3 Sample E
Score: 8

a.i.

- 0 points were earned. The response does not indicate that the air particles push down on the block and the water particles push up on the block.
- 1 point was earned. The response indicates that the water exerts a greater upward force on the block as a result of more collisions between the water particles and the block than air particles and the block.

a.ii.

- 0 points were earned. The response does not indicate that Block A is denser than Block B.

b.i.

- 1 point was earned. The response uses Bernoulli's equation to attempt to derive the relationship between v_p and h.
- 1 point was earned. The response shows that the pressures at the two locations (surface and pipe exit) are the same.
- 1 point was earned. The response correctly substitutes the height and the speed at each location.

b.ii.

- 1 point was earned. The response uses the continuity equation to derive the relationship between the speed at the pipe exit and the changing radius.
- 1 point was earned. The response correctly substitutes the areas and speeds at the surface and at the exit of the pipe.

b.iii.

- 1 point was earned. The response references the conservation equation in part (b)(ii) to show that the surface speed is very small when R is very large.
- 1 point was earned. The response indicates that the surface speed is small enough to be neglected in the expression.

c.

- 0 points were earned. The response uses an incorrect expression and does not show the correct functional dependence between the height and the speed of the water exiting the pipe.
- 0 points were earned. Although the response uses the correct expression from part (b)(ii), the response concludes that the surface speed will increase, rather than decrease, as a result of the increasing R.

Sample Identifier: P2 Q3 Sample F
Score: 7

a.i.

- 0 points were earned. The response does not indicate that the water pushes up on the block or that the air pushes down.
- 0 points were earned. The response does not indicate that the force exerted from the water is greater than the force exerted from the air.

a.ii.

- 0 points were earned. Although the response does claim Block A has a greater density than Block B, the response incorrectly claims that the buoyant force on each block is the same.

b.i.

- 1 point was earned. The response uses Bernoulli's equation to attempt to derive the relationship between v_p and h.
- 1 point was earned. The response shows that the pressures at the two locations (surface and pipe exit) are the same by eliminating the terms.
- 1 point was earned. The response correctly substitutes the speeds and the heights into the expression.

b.ii.

- 1 point was earned. The response uses the continuity equation to derive the relationship between v_p and R.
- 1 point was earned. The response correctly substitutes values for areas and speeds.

b.iii.

- 1 point was earned. The response references conservation principles (comparing areas to compare speeds) to show that surface speed is small (zero) compared to the speed exiting the pipe.
- 1 point was earned. The response indicates that v_s is not part of the equation because v_s is "minisule" compared to v_p.

c.

- 0 points were earned. Although the response references the expression from part (b)(i), the response reaches an incorrect conclusion that the speed of the water exiting the pipe must increase.
- 0 points were earned. Although the response references the equation from (b)(ii), the response claims that v_s increases when R increases, rather than v_s decreasing when R increases.

Sample Identifier: P2 Q3 Sample G
Score: 6

a.i.

- 0 points were earned. The response does not indicate that the water pushes up on the block or that the air pushes down on the block.
- 0 points were earned. The response does not indicate that the force exerted from the water is greater than the force exerted from the air.

a.ii.

- 1 point was earned. The response indicates that the density of Block A is greater because Block A displaces a greater amount of water.

b.i.

- 1 point was earned. The response uses Bernoulli's equation to attempt to derive the relationship between v_p and h.
- 1 point was earned. The response shows that the pressures at the two locations (surface and pipe exit) are the same by eliminating the terms.
- 0 points were earned. The response does not correctly substitute the speeds and heights into the expression. The response uses the fixed value h_0 rather than the changing height h.

b.ii.

- 1 point was earned. The response uses the continuity equation.
- 1 point was earned. The response correctly substitutes values for areas and speeds.

b.iii.

- 0 points were earned. The response does not reference conservation principles to show that surface speed is small compared to the speed exiting the pipe.
- 0 points were earned. Although the response mentions that the speed at the surface can be ignored, this is given information. The response does not indicate the reason the speed at the surface is ignored is because the value is very small/negligible in the context.

c.

- 1 point was earned. The response uses the expression from part (b)(i) to support a claim that as height decreases, the speed of the water exiting the pipe also decreases.
- 0 points were earned. The response does not reference the effect of the changing radius on the speed of the water at the surface.

Sample Identifier: P2 Q3 Sample H
Score: 3

a.i.

- 0 points were earned. Although the response indicates the water particles push upward on the block, the response does not indicate the air particles push downward on the block.
- 0 points were earned. The response does not indicate that the force exerted from the water is greater than the force exerted from the air.

a.ii.

- 0 points were earned. Although the response claims that Block A has a greater density than Block B, the response does not correctly connect the different buoyant forces to the different displaced volumes.

b.i.

- 1 point was earned. The response uses Bernoulli's equation to attempt to derive the relationship between v_p and h.
- 1 point was earned. The response shows that the pressures at the two locations (surface and pipe exit) are the same by eliminating the terms.
- 1 point was earned. The response correctly substitutes the speeds and heights into the expression.

b.ii.

- 0 points were earned. The response does not use the continuity equation to derive the relationship between v_p and R.
- 0 points were earned. The response does not correctly substitute values for areas and speeds.

b.iii.

- 0 points were earned. The response does not use conservation principles to justify that v_s is much smaller than v_p.
- 0 points were earned. Although the response mentions an "almost negligible" quantity, it is not clear whether the response is referencing a height decreasing at a slow rate (corresponding to negligible v_s) or speed decreasing at a negligible rate (constant but non-negligible v_s).

c.

- 0 points were earned. Although the response references the equation from part (b)(i), the response does not correctly connect the decreasing height to the decreasing speed of the water exiting the pipe.
- 0 points were earned. Although the response references the equation from part (b)(ii), the response does not correctly relate the increasing radius to decreasing surface speed.

Sample Identifier: P2 Q3 Sample I
Score: 2

a.i.

- 1 point was earned. The response correctly indicates that the air particles push down on the block and the water particles push up on the block.
- 0 points were earned. The response indicates that the force exerted by the water is greater than the forces exerted by the air and gravity, rather than the force exerted by the water being greater than the force exerted by the air.

a.ii.

- 0 points were earned. Although the response claims that Block A has the greater density, the response does not correctly connect this claim to the buoyant force or the volume of the displaced fluid.

b.i.

- 0 points were earned. The response does not use Bernoulli's equation to derive the relationship between v_p and h.
- 0 points were earned. The response does not show that the pressures at the two locations (surface and pipe exit) are the same.
- 0 points were earned. The response does not correctly substitute the speeds and the heights into an expression.

b.ii.

- 0 points were earned. The response does not use the continuity equation to derive the relationship between v_p and R.
- 0 points were earned. The response does not substitute values for areas and speeds in the expression for v_p.

b.iii.

- 0 points were earned. The response does not reference conservation principles to explain why v_s is much smaller than v_p.
- 1 point was earned. The response correctly indicates that v_s is a negligible value.

c.

- 0 points were earned. The response does not connect the decreasing height to the decreasing v_p as the water exits the pipe.
- 0 points were earned. The response does not connect the increasing radius to the decreasing surface speed of the water.

Sample Identifier: P2 Q3 Sample J
Score: 1

a.i.

- 0 points were earned. The response does not indicate that the air particles push down on the block and that the water particles push up on the block.
- 0 points were earned. The response does not indicate that the force exerted from the water is greater than the force exerted from the air. Although the response does reference a "stronger" force exerted from the water, it is unclear what that force is stronger than (i.e. gravity or air).

a.ii.

- 0 points were earned. Although the response claims that Block A has the greater density, the response does not correctly connect this claim to the buoyant force or the volume of displaced fluid.

b.i.

- 0 points were earned. The response does not use Bernoulli's equation to derive the relationship between v_p and h.
- 0 points were earned. The response does not show that the pressures at both locations (surface and pipe exit) are the same.
- 0 points were earned. The response does not correctly substitute the speeds and the heights into an expression.

b.ii.

- 0 points were earned. The response does not use the continuity equation to derive the relationship between v_p and R.
- 0 points were earned. The response does not substitute values for areas and speeds.

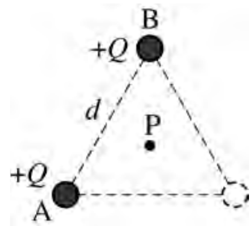
b.iii.

- 0 points were earned. Although the response does attempt to reference conservation principles, the response does not make it clear that v_s is much smaller than v_p, but rather makes it seem as if v_p is small.
- 0 points were earned. The response does not correctly indicate that v_s is not part of the expression because v_s is a negligible value.

c.

- 1 point was earned. The response uses the expression from part (b)(i) to support a claim that as height decreases, the speed of the water exiting the pipe also decreases.
- 0 points were earned. Although the response references the expression from part (b)(ii), the response connects the increasing radius to a decreased speed at the valve, not surface.

Begin your response to **QUESTION 4** on this page.



4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

GO ON TO THE NEXT PAGE.

Continue your response to **QUESTION 4** on this page.

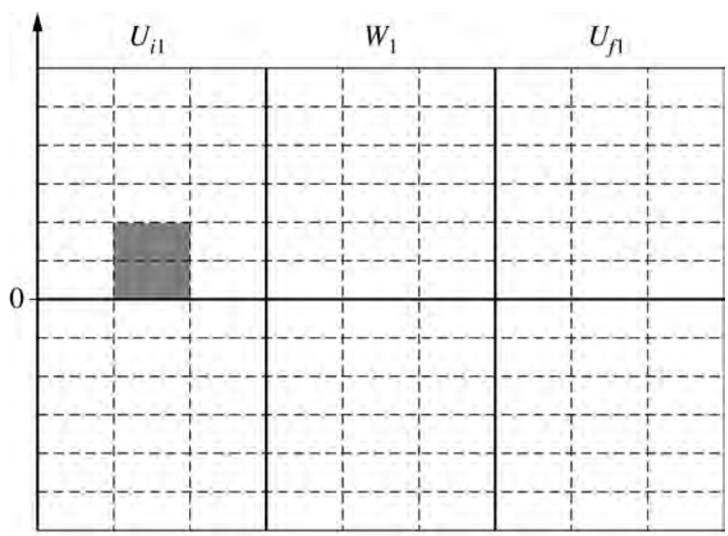
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

GO ON TO THE NEXT PAGE.

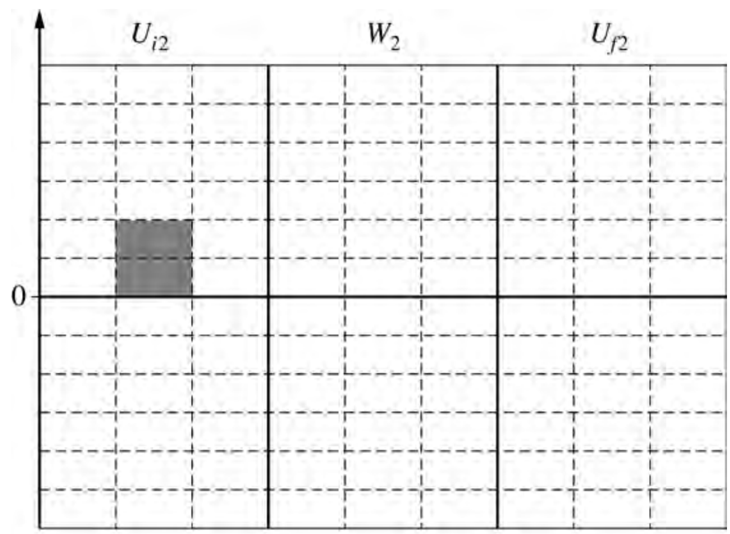
Continue your response to **QUESTION 4** on this page.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

GO ON TO THE NEXT PAGE.

Question 4: Short Answer Paragraph Argument 10 points

(a)	For an evaluation of Student Y's statement that correctly includes the vector nature of electric field	**1 point**
	For indicating that Student Y should have stated that the third particle must have charge $+Q$ for the electric field at Point P to be zero OR For a statement indicating what the resultant magnitude of the electric field at Point P would be for a particle with charge $+2Q$	**1 point**
	For an evaluation of Student Z's statement that correctly includes the scalar nature of electric potential	**1 point**
	For indicating that zero electric potential at Point P would require the third particle having charge $-2Q$	**1 point**
	For a logical, relevant, and internally consistent argument that addresses the required argument or question asked, and follows the guidelines described in the published requirements for the paragraph-length response	**1 point**

Example Response

Student Y is incorrect. Before the third particle is placed at the bottom-right vertex, the electric field from particles A and B at Point P is down and to the right. The electric field from a positively charged particle placed at the bottom-right vertex is up and to the left. The third particle needs to have charge $+Q$, rather than $+2Q$, in order to have the correct magnitude to make the resultant field zero at Point P.

Student Z is incorrect. Before the third particle is placed at the bottom-right vertex, the value of the electric potential at Point P is positive. Because Point P is equidistant from all three particles, the electric potential at Point P is proportional to the total charge of the system. If the total charge of the system is zero, the electric potential at Point P will be zero. This requires the third particle to have charge $-2Q$.

OR

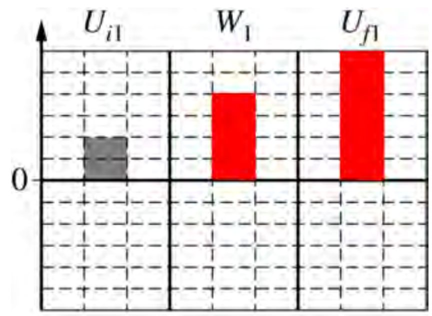
Student Y is incorrect that a particle with charge $+2Q$ placed at the bottom-right vertex will result in no electric field at Point P. The horizontal component of the electric field from Particle A is less than the horizontal component of the electric field from the particle with charge $+2Q$. The sum of the vertical components of the fields from particles A and B is less than the vertical component of the field from the particle with charge $+2Q$. Therefore, the resulting electric field at Point P is nonzero and points in a direction between particles A and B.

Student Z is incorrect. Before the third particle is placed at the bottom-right vertex, the value of the electric potential at Point P is positive. Electric potential is a scalar quantity, so if the third particle has charge $-2Q$ rather than $-Q$, the electric potential at Point P will be zero.

Total for part (a) 5 points

(b)(i)	For drawing a bar on the grid that shows a positive value for W_1	**1 point**
	For drawing a bar on the grid that shows $U_{f1} = 3U_{i1}$	**1 point**
	For drawing bars on the grid so that the work done on the system is equal to the change in energy, $U_{i1} + W_1 = U_{f1}$	**1 point**

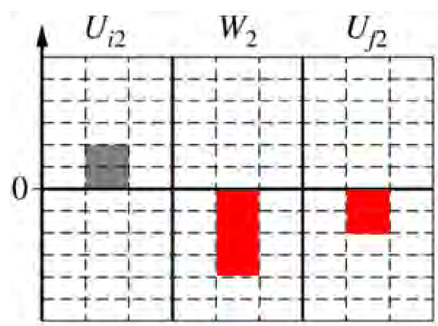
Example Response



Scenario 1

(b)(ii)	For drawing a bar on the grid that shows $U_{f2} = -U_{i2}$	**1 point**
	For drawing a bar on the grid that shows a negative value of W_2 so that the work done on the system is equal to the change in energy $U_{i2} + W_2 = U_{f2}$	**1 point**

Example Response



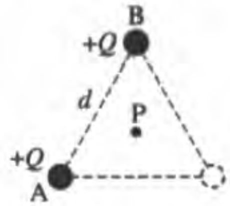
Scenario 2

Total for part (b) 5 points

Total for question 4 10 points

Question 4

Begin your response to QUESTION 4 on this page.



4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

Student Y's statement is incorrect, to make the electric field at P 0, we should instead place a particle w/ +Q charge. Because w/ a particle w/ +Q charge at the third vertexe, a particle @ P would experice 3 electric force in such configuration

, which would cancel e/o out and give P an electric field of 0. Student Z's stantment is incorrect, to make the electric potential at P 0, we should instead place a particle w/ −2Q charge. Because the current V at P is $2 \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{\frac{2}{\sqrt{3}}d}$. The only way to negative this V is by placing a charge of −2Q, which changes the V by $\frac{1}{4\pi\varepsilon_0} \cdot \frac{-2Q}{\frac{2}{\sqrt{3}}d}$, giving a ΣV of 0 –

Question 4

Continue your response to QUESTION 4 on this page.

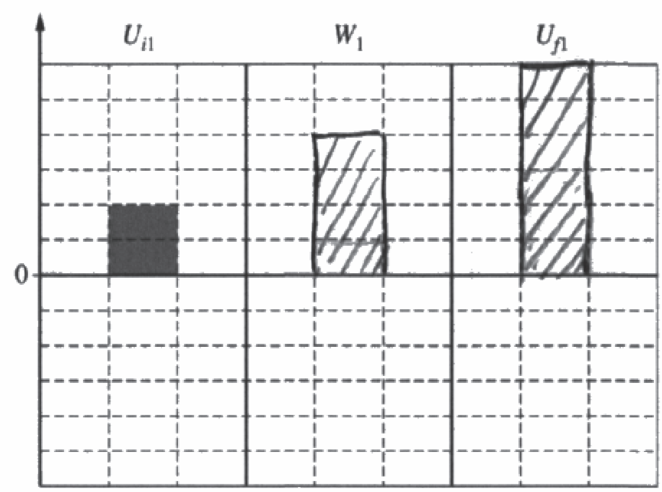
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

Question 4

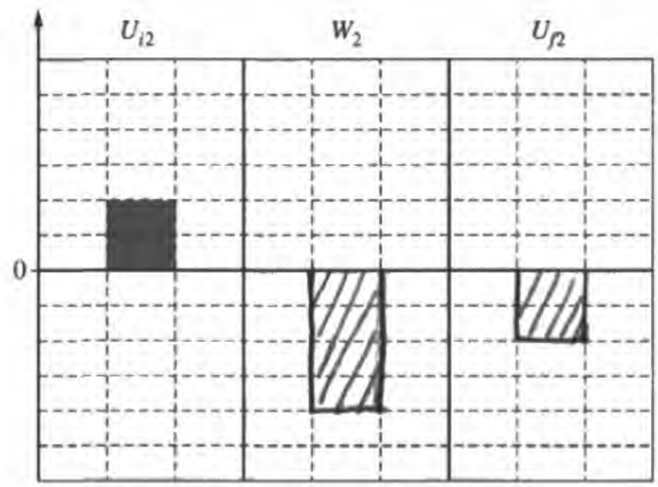
Continue your response to **QUESTION 4** on this page.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

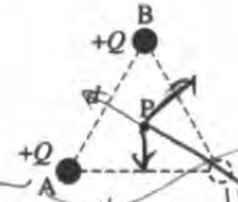
The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

Question 4

Begin your response to QUESTION 4 on this page.



also, all of the particles are equidistant from point P, so they have the same effect on P proportional to their charges

4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P." ~~+~~ yes

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex." – No / −2Q

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

Student Y's statement is correct. The +Q charge from particle B generates an electric field that points down while the +Q charge from particle A generates an electric field that points to the top right. Together, when the vectors from these two electric fields are added together, the resulting electric field is directed to the bottom left vertex. lets say at point P each charge generates an electric field strength of 2E. Now, when the new charge of 2Q is placed at the bottom right, it generates an electric field pointing to the top left, again since electric fields point away from positive charges. This new electric field is twice as strong as either of the other two fields, and equal to the combined strength of the fields. Thus, when all 3 fields are combined from the 3 particles, the net electric field is 0, making student Y correct. However, Student Z is wrong. Electric potential is additive regardless of direction. The existing 2 charges each give point P an electric potential proportional to +Q. Together, the resulting electric potential is ~~equal~~ proportional to +2Q. Thus, to oppose that electric potential and get a net of 0, the new electric ~~[illegible]~~ must be proportional to −2Q when generated by the new particle. Thus a −Q charge is not enough. A −2Q charge is required to get −2Q + 2Q = 0.

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5196/14

Question 4

Continue your response to **QUESTION 4** on this page.

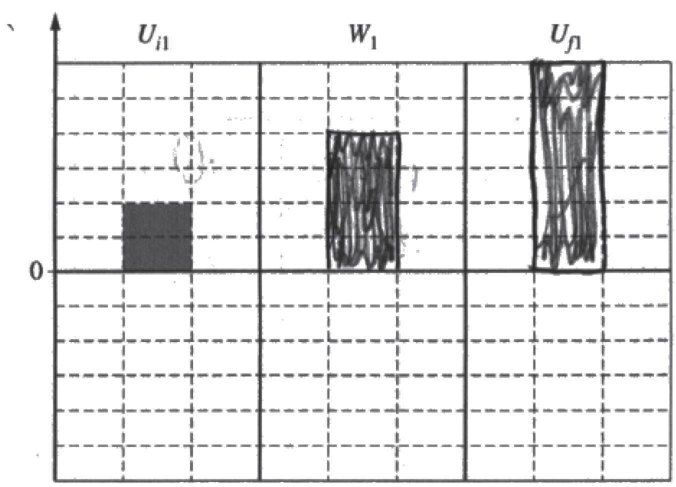
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

Question 4

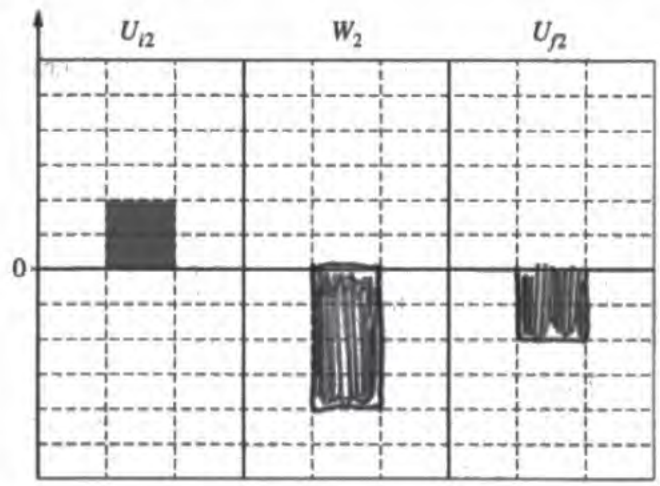
Continue your response to **QUESTION 4** on this page.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

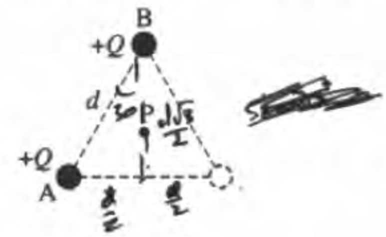
The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

Question 4

Begin your response to QUESTION 4 on this page.



4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

+Q instead

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

−2Q instead

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

We know that the field at P due to charge Q at A is up and right, while the field at P due to charge Q at B is down. Additionally, if we add a charge 2Q at the bottom-right vertex, the field due to it at P will be left and up. Thus by splitting into x and y components, we can find the field at P:

$$E_x = \frac{kQ}{x^2}\cos 30^\circ - \frac{2kQ}{x^2}\cos 30^\circ = -\frac{kQ\sqrt{3}}{2x^2}$$

$$E_y = \frac{kQ}{x^2}\sin 30^\circ + \frac{2kQ}{x^2}\sin 30^\circ - \frac{kQ}{x^2} = \frac{kQ}{2x^2}$$

where x is the distance from each charge to P.

Thus, the magnitude of the electric field at P is not zero, contradicting student Y's statement. To make the field 0 at P, a charge +Q should be added. The potential would be $\frac{kQ}{x} + \frac{kQ}{x} - \frac{kQ}{x} = \frac{kQ}{x} \neq 0$, contradicting student Z. To make the potential 0, a charge −2Q should be added.

GO ON TO THE NEXT PAGE.

Question 4

Continue your response to **QUESTION 4** on this page.

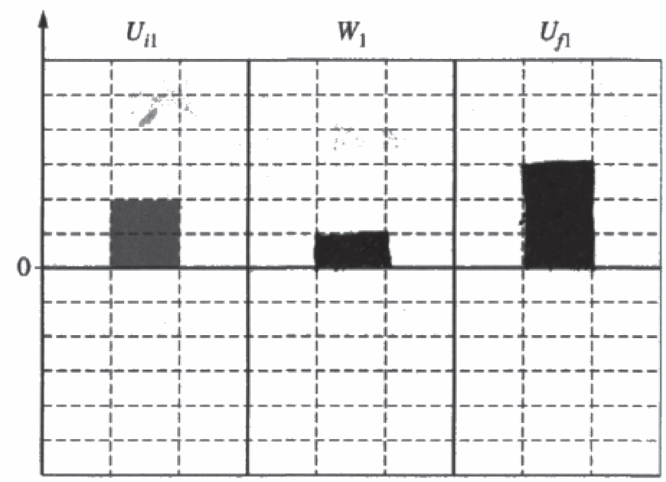
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

Question 4

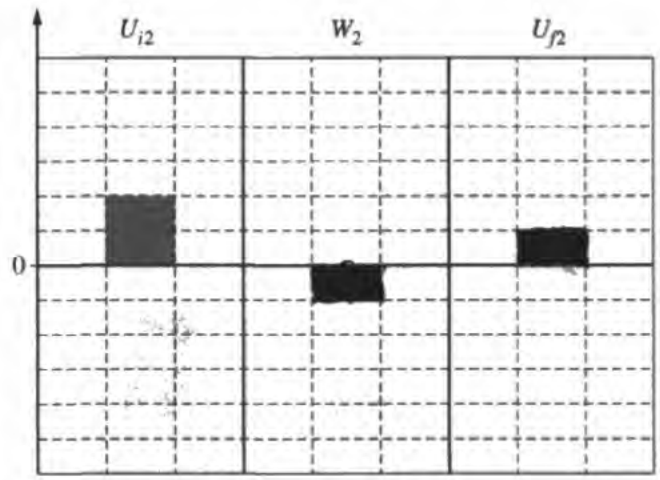
Continue your response to **QUESTION 4** on this page.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

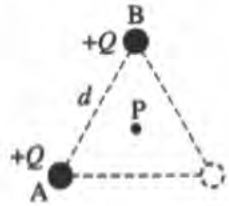
The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

Question 4

Begin your response to QUESTION 4 on this page.



4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

All the particles are equal distanced from point P, so if any one of them has a greater charge than the others, it would have a bigger affect on point P. This is why Student Y is wrong as the particle having a larger charge would affect P with a larger force but the same angle as A and B so the net force could not be zero. Studen Z is wrong as having a negative particle would make the electric field at point P towards itself. The other two particles are already making it point that direction so the net force wouldn't be zero. Instead, the charge should be +Q as three charges of the same sign and magnitude placed the same distance and angle away from point P would cancel out and make the net electric field zero.

Question 4

Continue your response to QUESTION 4 on this page.

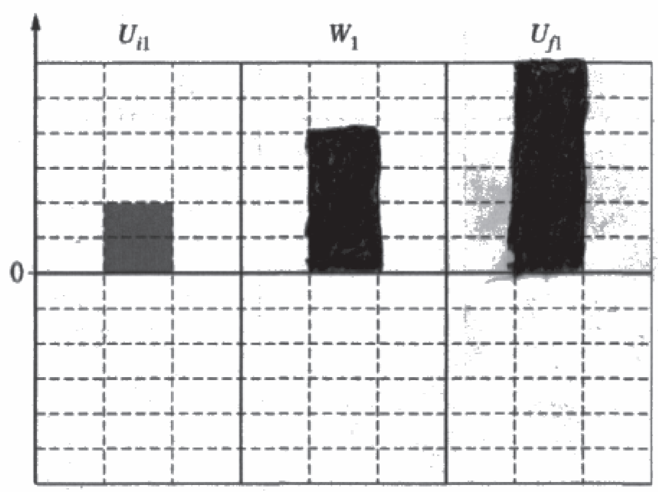
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

Question 4

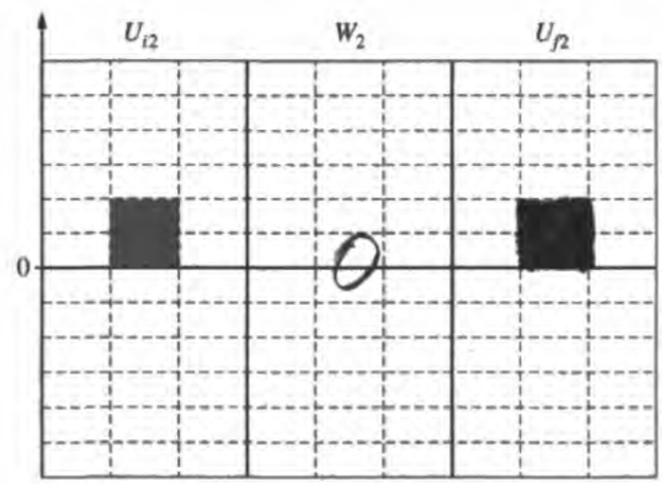
Continue your response to **QUESTION 4** on this page.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

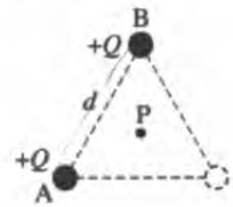
The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

Question 4

Begin your response to QUESTION 4 on this page.



4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

The charge would have to be -2Q. Student Y was correct about the charge being 2Q but it would have to be negative. Student Z understood it was a negative charge but not the magnitude of 2Q. The goal is to get the charges to sum up to 0 because the The charges would have to sum up to 0 to have an electric potential and field of 0 so the charge would have to be -2Q because $E = \frac{F_e}{q}$ and $\Delta V = \frac{\Delta U_e}{q}$

Electric Field $= \frac{F_e}{q}$

GO ON TO THE NEXT PAGE.

Question 4

Continue your response to **QUESTION 4** on this page.

(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

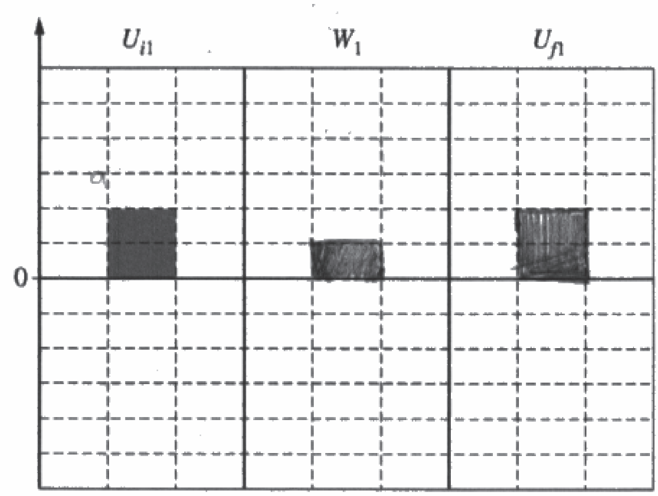
In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.

$W = F \cdot d = Eqd$

$\frac{1}{2} qE$



Scenario 1

$U = qV$

Question 4

Continue your response to **QUESTION 4** on this page.

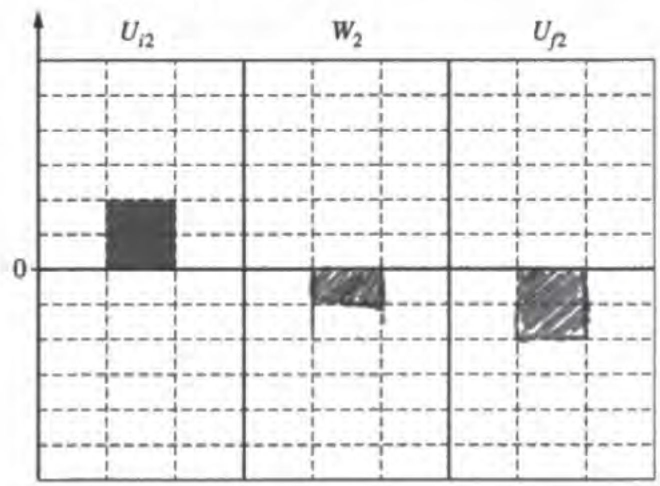
ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.

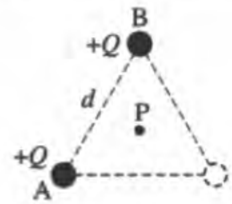
$U = W + Q$



Scenario 2

Question 4

Begin your response to **QUESTION 4** on this page.



4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

Student Y is wrong. The electric field would not be zero if a charge of +2Q were to be placed in the bottom right vertex. The charge would need to be -2Q as adding more positive charge would only make the point more positive, not nuetral. Student Z is also wrong. The charge would need to be -2Q as -Q is not enough to make point p equal to zero.

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 4

Continue your response to **QUESTION 4** on this page.

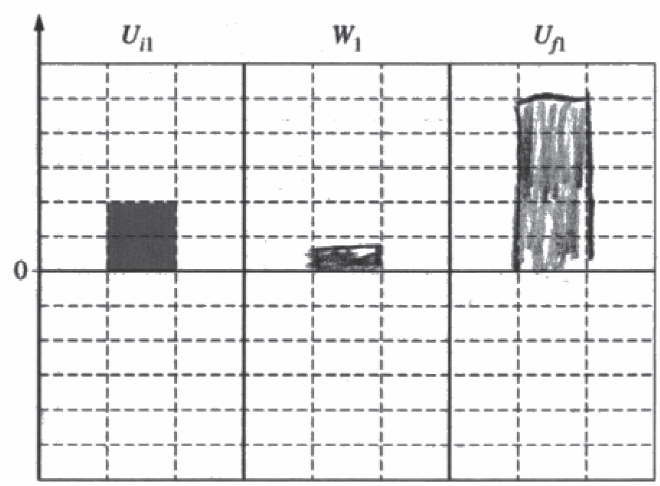
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

Question 4

Continue your response to **QUESTION 4** on this page.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

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- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
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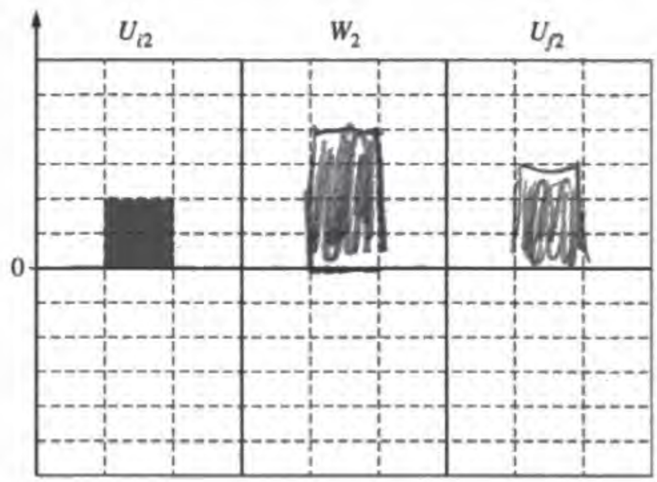
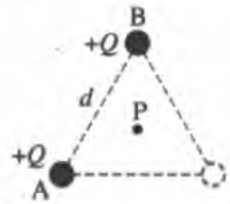
The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

Question 4

Begin your response to **QUESTION 4** on this page.



4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

Student Y is less accurate than student ~~Is~~ Z.

$$E = \frac{Q}{\varepsilon_0 A} \quad F_E = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$\Delta V = \frac{Q}{C} \quad \Delta U = q\Delta V$$

Question 4

Continue your response to **QUESTION 4** on this page.

(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

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- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
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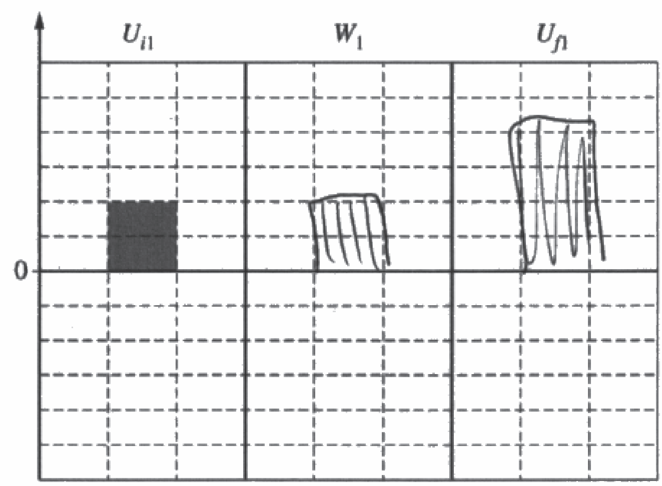
The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

Question 4

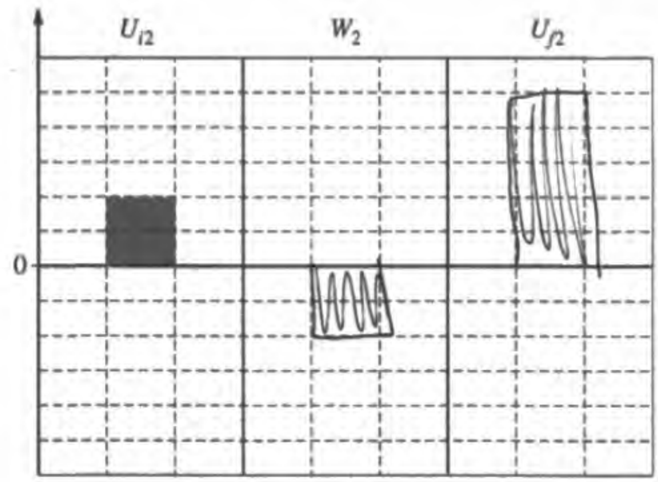
Continue your response to QUESTION 4 on this page.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

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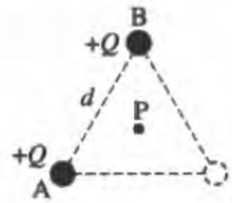
Question 4

Begin your response to QUESTION 4 on this page.



4. (10 points, suggested time 20 minutes)

Particles A and B each have positive charge $+Q$ and are held fixed at two vertices of an equilateral triangle of side length d, as shown. Point P is located equidistant from each vertex of the triangle.

Students Y and Z discuss the electric field and the electric potential at Point P after a third charged particle is placed at the bottom-right vertex. The students make the following statements.

Student Y: "If a particle with positive charge $+2Q$ is placed at the bottom-right vertex, the magnitude of the electric field will be zero at Point P."

Student Z: "To make the value of the electric potential zero at Point P, a particle with negative charge $-Q$ should be placed at the bottom-right vertex."

(a) In a coherent, paragraph-length response, evaluate the accuracy of each student's statement. If any aspect of either student's statement is inaccurate, explain how to correct the student's statement. Support your evaluations using appropriate physics principles.

Y: If the charge at the bottom right vertex was negative, point P would be zero

Z: the charge at the bottom needs to be a stronger negative charge to make point P zero

Question 4

Continue your response to **QUESTION 4** on this page.

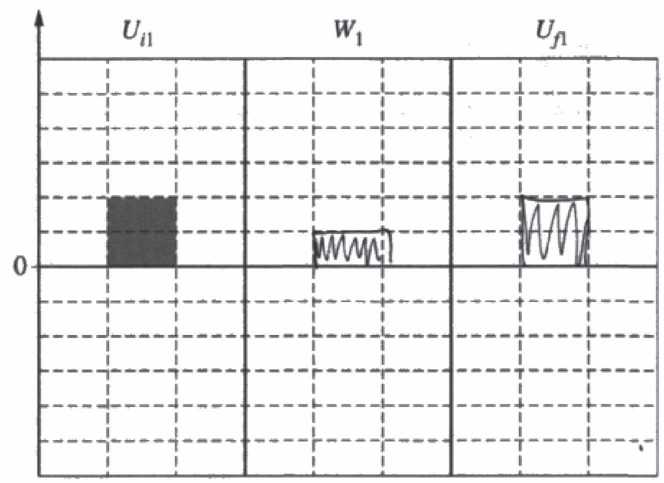
(b) Particles A and B are once again held in place at two vertices of the equilateral triangle. The students want to represent the electric potential energy of a system of particles when a third charged particle is brought from very far away to the bottom-right vertex. Scenarios 1 and 2 are considered.

i. In Scenario 1, a third particle with positive charge $+Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i1} of the system consisting of all three particles when the third particle with positive charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_1 required to move the third particle with positive charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f1} of the system consisting of all three particles when the third particle with positive charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 1

Question 4

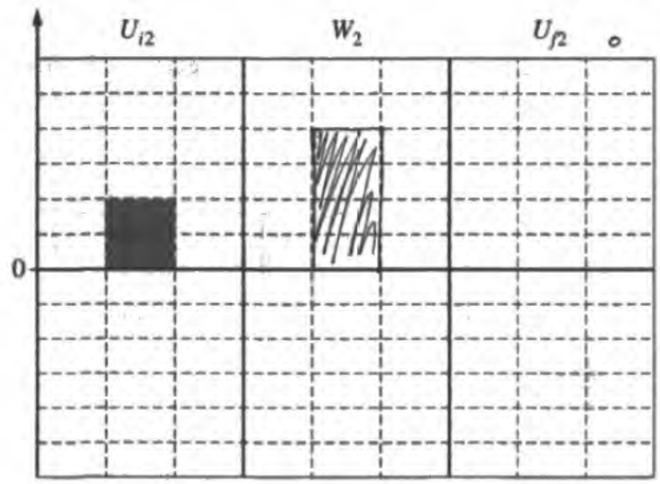
Continue your response to **QUESTION 4** on this page.

ii. In Scenario 2, a particle with negative charge $-Q$ is moved from very far away to the bottom-right vertex and then held in place. A bar is shown on the following chart that represents the electric potential energy U_{i2} of the system consisting of all three particles when the particle with negative charge is very far away from the other particles.

In the grid provided, complete the bar chart.

- Draw a bar to represent the work W_2 required to move the particle with negative charge from very far away to the bottom-right vertex.
- Draw another bar to represent the electric potential energy U_{f2} of the system consisting of all three particles when the particle with negative charge is held in place at the bottom-right vertex.

The height of each bar should be proportional to the energy represented. If the quantity is zero, write a "0" in that column.



Scenario 2

Question 4

Sample Identifier: P2 Q4 Sample A
Score: 10

a.

- 1 point was earned. The response correctly indicates the vector nature of the electric field.
- 1 point was earned. The response corrects Student Y 's statement. The response correctly identifies that a particle of charge $+Q$ placed at the bottom-right vertex will create an electric field contribution at Point P that cancels the contributions to the field from the particles at points A and B and, thus, the electric field at Point P will be zero. Although the response shows force cancellation, the response relates the force on a particle placed at Point P with the electric field at P .
- 1 point was earned. The response correctly refers to the scalar nature of electric potential shown by the use of the equation for the electric potential at Point P due to the two particles at points A and B .
- 1 point was earned. The response correctly states that placing a particle of charge $-2Q$ at the vertex would create an electric potential of zero at Point P .
- 1 point was earned for a logical, relevant, and internally consistent paragraph-length response.

b.i.

- 1 point was earned. The response shows a bar in the positive region of the grid indicating that W_1 is a positive quantity.
- 1 point was earned. The response shows a bar indicating U_{f1} is three times the size of U_{i1}, such that $U_{f1} = 3U_{i1}$.
- 1 point was earned. The response shows a bar indicating that U_{f1} is the sum of W_1 and U_{i1}, so that $U_{i1} + W_1 = U_{f1}$.

b.ii.

- 1 point was earned. The response shows a bar indicating that U_{f2} is equal but opposite in sign to U_{i2}, such that $U_{f2} = -U_{i2}$.
- 1 point was earned. The response shows a bar in the negative region of the grid indicating that W_2 is a negative quantity AND the bar drawn for U_{f2} is the sum of U_{i2} so that $U_{i2} + W_2 = U_{f2}$.

Sample Identifier: P2 Q4 Sample B
Score: 9

a.

- 1 point was earned. The response correctly indicates the vector nature of the electric field by saying the direction of the electric field is to the "top right".
- 0 points were earned. The response incorrectly indicates that placing a particle of charge $+2Q$ at the vertex would create a net electric field of zero at Point P. The response correctly identifies the direction of each particle contribution to the electric field at Point P but incorrectly adds the vectors together giving an incorrect magnitude. The response incorrectly concludes the electric field at Point P is zero.
- 1 point was earned. The response correctly refers to the scalar nature of electric potential by stating that "potential is additive regardless of direction".
- 1 point was earned. The response correctly states that placing a particle of charge $-2Q$ at the vertex would create an electric potential of zero at Point P.
- 1 point was earned for a logical, relevant, and internally consistent paragraph-length response.

b.i.

- 1 point was earned. The response shows a bar in the positive region of the grid indicating that W_1 is a positive quantity.
- 1 point was earned. The response shows a bar indicating that U_{f1} is three times the size of U_{i1}, such that $U_{f1} = 3U_{i1}$.
- 1 point was earned. The response shows a bar indicating U_{f1} is the sum of W_1 and U_{i1}, such that $U_{i1} + W_1 = U_{f1}$.

b.ii

- 1 point was earned. The response shows a bar indicating that U_{f2} is equal in magnitude to but opposite in sign to U_{i2}, such that $U_{f2} = -U_{i2}$.
- 1 point was earned. The response shows a bar in the negative region of the grid indicating that W_2 is a negative quantity AND the bar drawn for U_{f2} is the sum of W_2 and U_{i2}, such that $U_{i2} + W_2 = U_{f2}$.

Sample Identifier: P2 Q4 Sample C
Score: 8

a.

- 1 point was earned. The response correctly indicates the vector nature of the electric field by stating the direction of the electric field from the particles.
- 1 point was earned. The response correctly states that placing a particle of charge $+Q$ at the bottom-right vertex would create an electric field of zero at Point P. Additionally, the response states a particle of charge $+2Q$ would have created an electric field at Point P that is toward the top left of the page. The response shows a mathematical representation of the vector components.
- 1 point was earned. The response correctly refers to the scalar nature of the electric potential shown by the use of the equation for the net electric potential at Point P due to the two particles at points A and B.
- 1 point was earned. The response correctly states that placing a particle of charge $-2Q$ at the vertex would create an electric potential of zero at Point P.
- 1 point was earned for a logical, relevant, and internally consistent paragraph-length response.

b.i.

- 1 point was earned. The response shows a bar in the positive region of the grid indicating that W_1 is a positive quantity.
- 0 points were earned. The response shows an incorrectly sized bar for U_{i1}.
- 1 point was earned. The response shows a bar indicating that U_{f1} is the sum of W_1 and U_{i1}, such that $U_{i1} + W_1 = U_{f1}$.

b.ii

- 0 points were earned. The response shows a bar indicating that U_{f2} is positive on the grid.
- 1 point was earned. The response shows a bar in the negative region of the grid indicating that W_2 is a negative quantity AND the bar drawn for U_{f2} is the sum of W_2 and U_{i2}, such that $U_{i2} + W_2 = U_{f2}$.

Sample Identifier: P2 Q4 Sample D
Score: 6

a.

- 1 point was earned. The response correctly indicates the vector nature of the electric field by stating the direction of the electric field.
- 1 point was earned. The response correctly states that placing a particle of charge $+Q$ at the vertex would result in an electric field of zero at Point P. Although this argument is made when correcting Student Z's statement, the response reiterates the effect of this particle on the electric field rather than the electric potential.
- 0 points were earned. The response makes no reference to electric potential.
- 0 points were earned. The response indicates that "Student Z is wrong" but does not suggest adding a third particle of charge $-2Q$.
- 1 point was earned for a logical, relevant, and internally consistent paragraph-length response.

b.i.

- 1 point was earned. The response shows a bar in the positive region of the grid indicating that W_1 is a positive quantity.
- 1 point was earned. The response shows a bar indicating that U_{f1} is three times the size of U_{i1}, such that $U_{f1} = 3U_{i1}$.
- 1 point was earned. The response shows a bar indicating that U_{f1} is the sum of W_1 and U_{i1}, such that $U_{i1} + W_1 = U_{f1}$.

b.ii.

- 0 points were earned. The response shows a positive value of U_{f2}.
- 0 points were earned. The response shows no work done on the system.

Sample Identifier: P2 Q4 Sample E
Score: 5

a.

- 0 points were earned. The response does not correctly indicate the vector nature of the electric field.
- 0 points were earned. The response makes no statement about changing the charge of the particle to $+Q$ in order for the electric field at Point P to be zero. Additionally, the response does not describe that adding a particle of charge $+2Q$ at the bottom-right vertex would have created an electric field at Point P that is directed toward the top left of the page.
- 1 point was earned. The response indicates the scalar nature of electric potential by discussing the proportionality between the sum of charges and the net electric potential.
- 1 point was earned. The response correctly states that placing a particle of charge $-2Q$ at the vertex would create an electric potential of zero at Point P.
- 1 point was earned for a logical, relevant, and internally consistent paragraph-length response.

b.i.

- 1 point was earned. The response correctly shows a bar drawn on the grid that represents positive W_1.
- 0 points were earned. The response shows an incorrectly sized bar for U_{f1}.
- 0 points were earned. The response shows a bar for W_1 and for U_{f1} such that $U_{i1} + W_1 \neq U_{f1}$.

b.ii.

- 1 point was earned. The response correctly shows a bar drawn on the grid that represents $U_{f2} = -U_{i2}$.
- 0 points were earned. The response does show a negative bar drawn on the grid for W_2, but the response does not show bars that correctly represent $U_{i2} + W_2 = U_{f2}$. For the point to be earned, both parts of the response must be correct.

Sample Identifier: P2 Q4 Sample F
Score: 3

a.

- 0 points were earned. The response does not indicate the vector nature of the electric field.
- 0 points were earned. The response incorrectly states that changing the charge of the particle to $-2Q$ would result in the electric field at Point P to be zero.
- 0 points were earned. The response makes no reference to the scalar nature of electric potential. The response only discusses the addition of charge magnitude.
- 1 point was earned. The response correctly states that a particle of charge $-2Q$ should be placed at the vertex to create an electric potential of zero at Point P.
- 1 point was earned for a logical, relevant, and internally consistent paragraph-length response.

b.i.

- 1 point was earned. The response correctly shows a bar drawn on the grid that represents positive W_1.
- 0 points were earned. The response shows an incorrectly sized bar for U_{f1}.
- 0 points were earned. The response shows a bar for W_1 and a bar for U_{f1} such that $U_{i1} + W_1 \neq U_{f1}$.

b.ii.

- 0 points were earned. The response shows the same sign for the values of U_{f2} and U_{i2}.
- 0 points were earned. The response shows a positive value for W_2 and the response does not show bars that correctly represent $U_{i2} + W_2 = U_{f2}$. For the point to be earned, both parts of the response must be correct.

Sample Identifier: P2 Q4 Sample G
Score: 2

a.

- 0 points were earned. The response does not correctly indicate the vector nature of the electric field.
- 0 points were earned. The response makes no statement of what the resulting electric field would be at Point P due to a particle of charge $+2Q$ added to the system. Alternately, the response does not make a correct statement requiring a particle of charge $+Q$ at the vertex to yield zero electric field at Point P.
- 0 points were earned. The response makes no reference to the scalar nature of electric potential.
- 0 points were earned. The response makes no correct statement that a particle of charge $-2Q$ should be placed at the vertex to create an electric potential of zero at Point P.
- 0 points were earned. There was not a logical, relevant, nor internally consistent paragraph-length argument.

b.i.

- 1 point was earned. The response correctly shows a bar drawn on the grid that represents positive W_1.
- 0 points were earned. The response incorrectly shows a bar drawn on the grid demonstrating $U_{f1} = 2U_{i1}$.
- 1 point was earned. The response shows bars drawn on the grid that correctly show that $U_{i1} + W_1 = U_{f1}$.

b.ii.

- 0 points were earned. The response shows a positive value of U_{f2}.
- 0 points were earned. The response does show a bar drawn on the grid for $W_2 < 0$, but the response does not show bars that correctly represent $U_{i2} + W_2 = U_{f2}$. For the point to be earned, both parts of the response must be correct.

Sample Identifier: P2 Q4 Sample H
Score: 1

a.

- 0 points were earned. The response does not correctly indicate the vector nature of the electric field.
- 0 points were earned. The response incorrectly states that the charge of the particle would need to be negative in order for the electric field at Point P to be zero.
- 0 points were earned. The response makes no reference to electric potential.
- 0 points were earned. Although the response states that a "stronger negative charge" is needed to make the electric potential zero, the response does not say that the value would need to be $-2Q$.
- 0 points were earned. There was not a logical, relevant, or internally consistent paragraph-length response.

b.i.

- 1 point was earned. The response correctly shows a bar drawn on the grid that represents a positive W_1.
- 0 points were earned. The response incorrectly shows that $U_{f1} = U_{i1}$.
- 0 points were earned. The response shows a bar for work W_1 and a bar for U_{f1} such that $U_{i1} + W_1 = U_{f1}$.

b.ii.

- 0 points were earned. The response shows a value of zero for U_{f2}.
- 0 points were earned. The response incorrectly shows a positive value drawn on the grid for W_2.