

2023

AP<sup>®</sup>



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# AP<sup>®</sup> Physics 1: Algebra-Based

## Sample Student Responses and Scoring Commentary

DRAFT

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Begin your response to **QUESTION 1** on this page.**PHYSICS 1****SECTION II****Time—1 hour and 30 minutes****5 Questions**

**Directions:** Questions 1, 4, and 5 are short free-response questions that require about 13 minutes each to answer and are worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

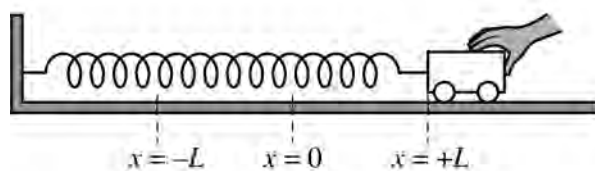


Figure 1

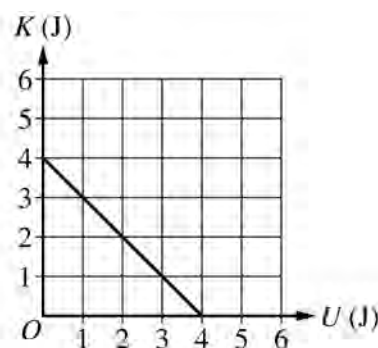


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions  $x = +L$  and  $x = -L$ . Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy  $K$  versus potential energy  $U$  shown in Figure 2, the values for the  $x$ -intercept and  $y$ -intercept are the same. Briefly explain why this is true, using physics principles.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

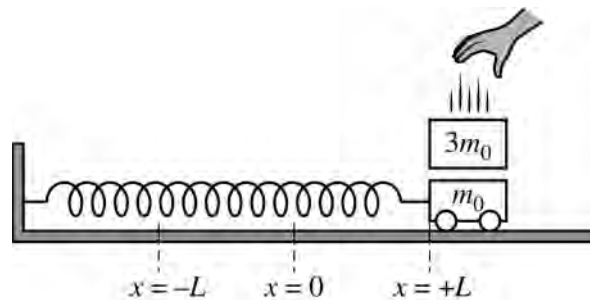


Figure 3

When the cart is at  $+L$  and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between  $-L$  and  $+L$ . The masses of the cart and the block are  $m_0$  and  $3m_0$ , respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is  $f_1$ . The frequency of oscillation after

the block is dropped onto the cart is  $f_2$ . Calculate the numerical value of the ratio  $\frac{f_2}{f_1}$ .

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 1** on this page.

- (c) The dashed line in Figure 4 shows the kinetic energy  $K$  versus potential energy  $U$  of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

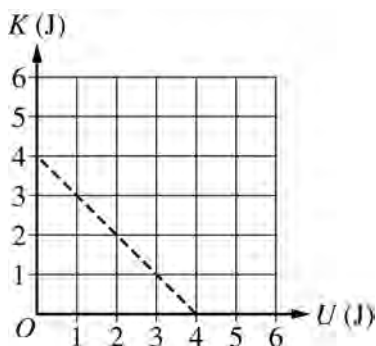


Figure 4

- i. Briefly explain why the two graphs must be the same, using physics principles.
- ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

**GO ON TO THE NEXT PAGE.**



**Question 1: Short Answer****7 points**

- (a) For an explanation that indicates that the maximum kinetic energy and maximum potential energy are the same due to energy conservation **1 point**

**Scoring Note:** This point may be earned for only stating “conservation of energy.”

**Example Response**

*The maximum kinetic energy and maximum potential energy of the car-spring system are both 4 J, because energy is conserved in this system.*

**Total for part (a) 1 point**

- (b) For using the equation for frequency or period in a ratio **1 point**

**Example Responses**

$$\frac{1}{2\pi} \sqrt{\frac{k}{m_2}} \quad \text{OR} \quad \frac{1}{2\pi} \sqrt{\frac{k}{m_1}} \quad \text{OR} \quad \frac{2\pi \sqrt{m_2}}{k} \quad \text{OR} \quad \frac{2\pi \sqrt{m_1}}{k}$$

$$\frac{1}{2\pi} \sqrt{\frac{k}{m_1}} \quad \frac{1}{2\pi} \sqrt{\frac{k}{m_2}} \quad \frac{2\pi \sqrt{m_1}}{k} \quad \frac{2\pi \sqrt{m_2}}{k}$$

**Scoring Note:** Simplified versions of the above ratios also earn this point.

For substituting the total mass  $4m_0$  into the correct ratio:  $\frac{f_2}{f_1}$  or  $\frac{T_1}{T_2}$  **1 point**

**Example Response**

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$\frac{f_2}{f_1} = \frac{\frac{1}{2\pi} \sqrt{\frac{k}{4m_0}}}{\frac{1}{2\pi} \sqrt{\frac{k}{m_0}}}$$

$$\frac{f_2}{f_1} = \frac{1}{2}$$

**Total for part (b) 2 points**

- (c)(i) For a valid explanation in terms of work or energy for why the systems' energies should be the same **1 point**

Accept **one** of the following:

- No work is done on the system
- The maximum spring potential energy is the same
- The force exerted on the system is perpendicular to the direction of motion

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**Example Response**

*The maximum potential energy of the system does not depend upon the mass of the system, therefore there will be no change when the block is added.*

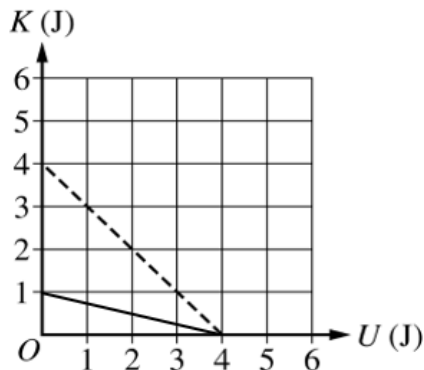
- (c)(ii) For drawing a single straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of 4 J **1 point**

For drawing a line with a vertical intercept that is less than the vertical intercept in the original graph **1 point**

For drawing a line with the correct vertical intercept of 1 J **1 point**

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**Example Response**




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**Total for part (c) 4 points**

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**Total for question 1 7 points**

Question 1

Begin your response to QUESTION 1 on this page.

PHYSICS 1  
SECTION II

Time—1 hour and 30 minutes

5 Questions

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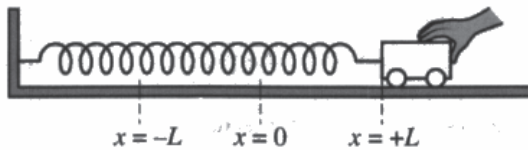


Figure 1

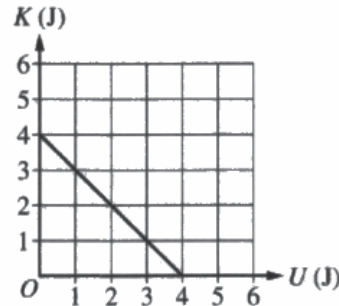


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions  $x = +L$  and  $x = -L$ . Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy  $K$  versus potential energy  $U$  shown in Figure 2, the values for the  $x$ -intercept and  $y$ -intercept are the same. Briefly explain why this is true, using physics principles.

The two values are the same since the system demonstrates the conservation of energy. When the cart has the most kinetic energy,  $U_s$  is at a minimum as  $U_s = \frac{1}{2}kx^2$ , and the peak of kinetic energy occurs when  $x=0$ . The same is true for kinetic energy in that it is at its minimum when potential energy is at its max.

Question 1

Continue your response to QUESTION 1 on this page.

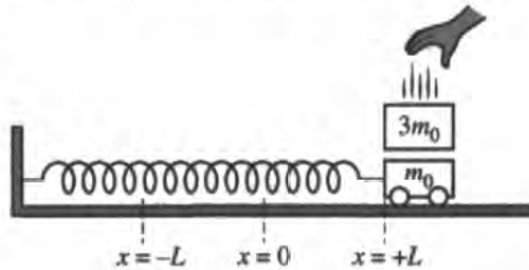


Figure 3

When the cart is at  $+L$  and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between  $-L$  and  $+L$ . The masses of the cart and the block are  $m_0$  and  $3m_0$ , respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is  $f_1$ . The frequency of oscillation after the block is dropped onto the cart is  $f_2$ . Calculate the numerical value of the ratio  $\frac{f_2}{f_1}$ .

$$T_3 = 2\pi\sqrt{\frac{m}{k}} \quad f_3 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$
 ~~$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_0}}$~~ 
 ~~$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{3m_0 + m_0}} = \frac{1}{2\pi}\sqrt{\frac{k}{4m_0}}$~~ 
 ~~$= \frac{1}{2\pi}\sqrt{\frac{1}{4} \cdot \frac{k}{m_0}}$~~ 
 ~~$= \frac{1}{4\pi}\sqrt{\frac{k}{m_0}}$~~ 
 ~~$f_2 = \frac{1}{4\pi}\sqrt{\frac{k}{m_0}}$~~ 
 ~~$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_0}}$~~ 

$$\frac{f_2}{f_1} = \frac{\frac{1}{4\pi}\sqrt{\frac{k}{m_0}}}{\frac{1}{2\pi}\sqrt{\frac{k}{m_0}}} = \frac{2}{4} = \frac{1}{2} = \frac{f_2}{f_1}$$



## Question 1

Continue your response to **QUESTION 1** on this page.

- (c) The dashed line in Figure 4 shows the kinetic energy  $K$  versus potential energy  $U$  of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

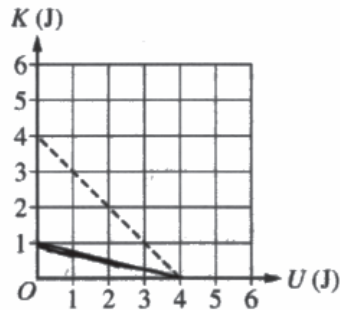


Figure 4

- i. Briefly explain why the two graphs must be the same, using physics principles.

The two graphs must be the same since  $U_s = \frac{1}{2}kx^2$ .  
~~When~~ When the block is dropped on the cart, the spring is still stretched the same distance, so the potential and kinetic energy remain the same.

- ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

Question 1

Begin your response to QUESTION 1 on this page.

PHYSICS 1  
SECTION II

Time—1 hour and 30 minutes  
5 Questions

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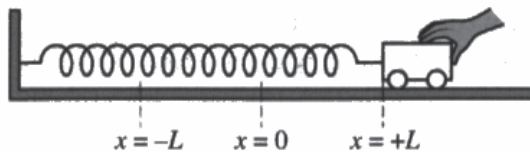


Figure 1

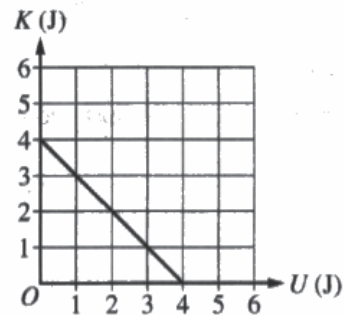


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions  $x = +L$  and  $x = -L$ . Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy  $K$  versus potential energy  $U$  shown in Figure 2, the values for the  $x$ -intercept and  $y$ -intercept are the same. Briefly explain why this is true, using physics principles.

For the cart and spring system the total mechanical energy is conserved as it is a closed system. As the potential energy increases, the kinetic energy decreases and vice-versa. Because of the conservation of energy, the maximum kinetic energy which is  $y=4$ , the  $y$ -intercept is equal to the maximum potential energy at  $x=4$ . The sum of the other points will sum to 4.

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Page 2

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.



Question 1

Continue your response to QUESTION 1 on this page.

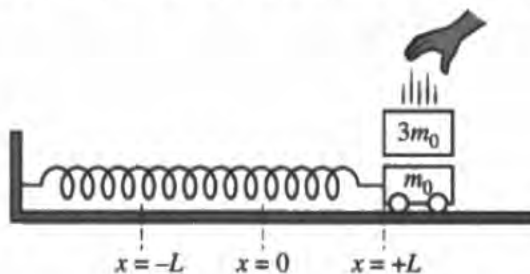


Figure 3

When the cart is at  $+L$  and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between  $-L$  and  $+L$ . The masses of the cart and the block are  $m_0$  and  $3m_0$ , respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is  $f_1$ . The frequency of oscillation after

the block is dropped onto the cart is  $f_2$ . Calculate the numerical value of the ratio  $\frac{f_2}{f_1}$ .

$$f_1 =$$

$$f_2 =$$

$$\frac{\frac{1}{2\pi} \sqrt{\frac{k}{m_0}}}{\frac{1}{2\pi} \sqrt{\frac{k}{3m_0}}} = \frac{\frac{1}{2\pi} \sqrt{\frac{k}{m_0}}}{1} \cdot \frac{1}{\frac{1}{2\pi} \sqrt{\frac{k}{3m_0}}} = \frac{\sqrt{\frac{k}{m_0}}}{\sqrt{\frac{k}{3m_0}}} = \frac{\sqrt{k}}{\sqrt{k}} \cdot \frac{\sqrt{3m_0}}{\sqrt{m_0}} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

$$\frac{f_2}{f_1} = \frac{3m_0}{m_0} = 3$$

Question 1

Continue your response to **QUESTION 1** on this page.

- (c) The dashed line in Figure 4 shows the kinetic energy  $K$  versus potential energy  $U$  of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

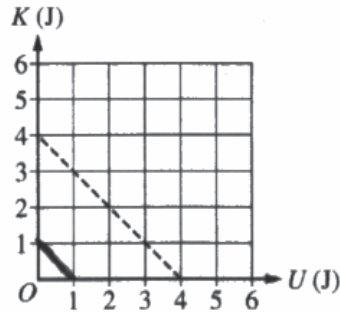


Figure 4

- i. Briefly explain why the two graphs must be the same, using physics principles.

Since potential energy at the point the additional mass is added is  $\frac{1}{2}kx^2$ , the total kinetic energy of the system stays the same as the new mass does no work, being perpendicular, and there is no change in  $\frac{1}{2}mv^2$ .  
 ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

kinetic energy.

$$k = \frac{1}{2}mv^2$$

$$\frac{1}{4}$$



## Question 1

Begin your response to **QUESTION 1** on this page.

## PHYSICS 1

## SECTION II

Time—1 hour and 30 minutes

5 Questions

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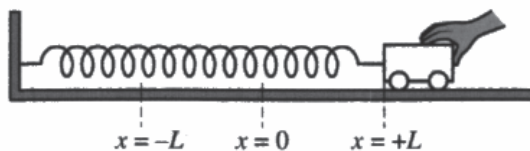


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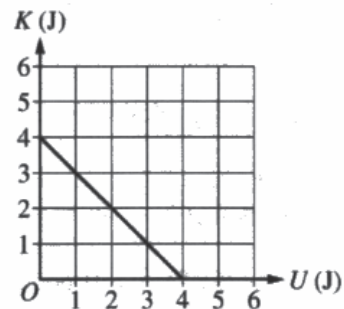


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions  $x = +L$  and  $x = -L$ . Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy  $K$  versus potential energy  $U$  shown in Figure 2, the values for the  $x$ -intercept and  $y$ -intercept are the same. Briefly explain why this is true, using physics principles.

Conservation of Energy means that the total energy of a closed system will remain constant.  
 $K_{\max} = U_{\max}$

Question 1

Continue your response to QUESTION 1 on this page.

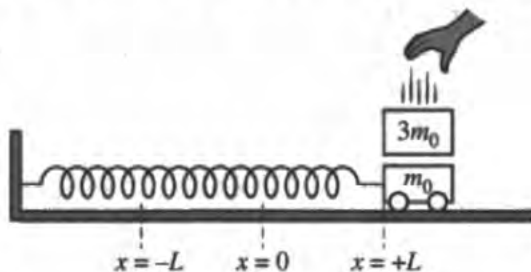


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(b) The frequency of oscillation before the block is dropped onto the cart is  $f_1$ . The frequency of oscillation after

the block is dropped onto the cart is  $f_2$ . Calculate the numerical value of the ratio  $\frac{f_2}{f_1}$ .

$$T_s = 2\pi \sqrt{\frac{m}{k}} \quad F = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f_1 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$f_2 = \frac{1}{2\pi} \sqrt{\frac{k}{4m}}$$

$$\frac{f_2}{f_1} = \frac{\frac{1}{2\pi} \sqrt{\frac{k}{4m}}}{\frac{1}{2\pi} \sqrt{\frac{k}{m}}}$$

$$\frac{f_2}{f_1} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$



## Question 1

Continue your response to **QUESTION 1** on this page.

- (c) The dashed line in Figure 4 shows the kinetic energy  $K$  versus potential energy  $U$  of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

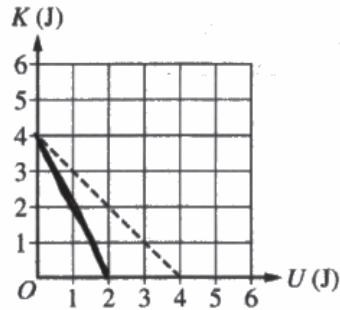


Figure 4

- i. Briefly explain why the two graphs must be the same, using physics principles.

Since the block is part of the system, it is still a closed-system so energy is still conserved.  
No work is done.

- ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

## Question 1

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## PHYSICS 1

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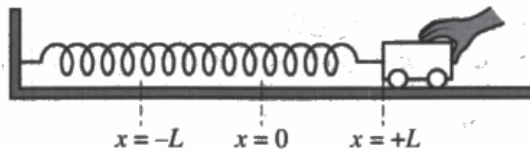


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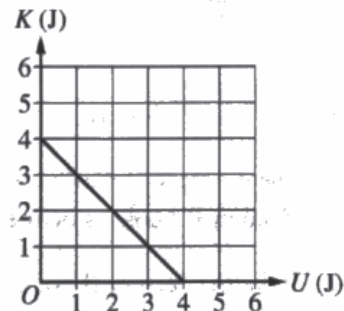


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(a) On the graph of kinetic energy  $K$  versus potential energy  $U$  shown in Figure 2, the values for the  $x$ -intercept and  $y$ -intercept are the same. Briefly explain why this is true, using physics principles.

This is because the total mechanical energy of the system doesn't change, because frictional and outside forces are negligible. The greatest value for kinetic and potential energy will be equal and will occur when the other energy aspect is equal to 0.

Question 1

Continue your response to QUESTION 1 on this page.

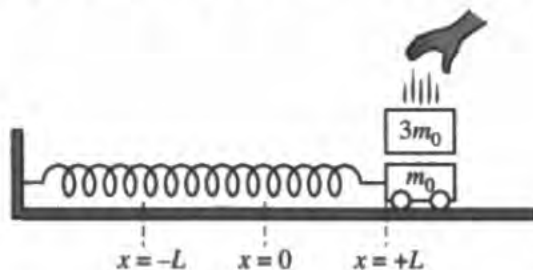


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(b) The frequency of oscillation before the block is dropped onto the cart is  $f_1$ . The frequency of oscillation after

the block is dropped onto the cart is  $f_2$ . Calculate the numerical value of the ratio  $\frac{f_2}{f_1}$ .

$$T_s = 2\pi \sqrt{\frac{1}{k}} \quad T_{s1} = 2\pi \sqrt{\frac{1}{k}} \quad T_{s2} = 2\pi \sqrt{\frac{4}{k}} \quad T_{s2} = 2\pi \sqrt{\frac{4}{k}}$$

$$f_{s1} = \frac{k}{2\pi} \quad f_{s2} = \frac{k}{4\pi}$$

$$\frac{f_{s2}}{f_{s1}} = \frac{k/4\pi}{k/2\pi}$$

$$\frac{k}{4\pi} \cdot \frac{2\pi}{k} = \frac{1}{2}$$

$$\frac{f_2}{f_1} = \frac{1}{2}$$





## Question 1

Continue your response to QUESTION 1 on this page.

- (c) The dashed line in Figure 4 shows the kinetic energy  $K$  versus potential energy  $U$  of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

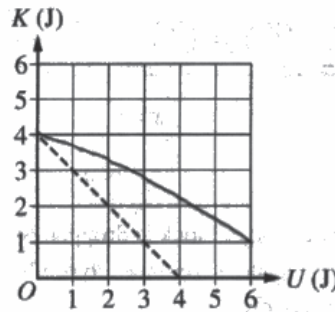


Figure 4

- i. Briefly explain why the two graphs must be the same, using physics principles.

When comparing  $KE_{max}$  and  $PE_{max}$ , mass is a constant that doesn't change and can be cancelled out, therefore not affecting the graph.

- ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

mass would stay the same  
velocity would increase, so KE would increase

Question 1

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PHYSICS 1

SECTION II

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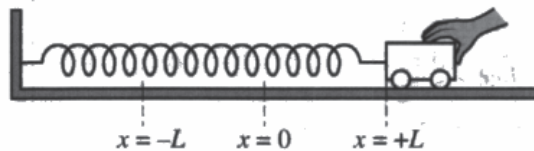


Figure 1

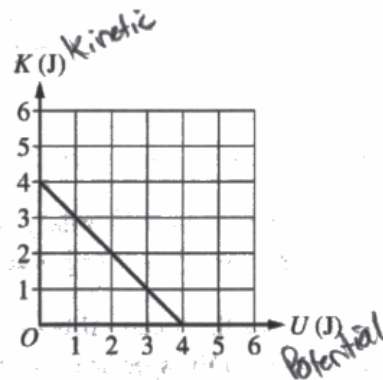


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(a) On the graph of kinetic energy  $K$  versus potential energy  $U$  shown in Figure 2, the values for the  $x$ -intercept and  $y$ -intercept are the same. Briefly explain why this is true, using physics principles.

$$E_{\text{mechanical}} = KE + U_{\text{spring}} + U_{\text{grav}}$$

$$E_{\text{mech A}} = U_{\text{spring}}(+L) \quad E_{\text{mech B}} = KE(\frac{1}{2}mv^2)_{(-L)} \quad E_{\text{mech A}} = E_{\text{mech B}}$$

Since frictional forces are negligible, energy is conserved. The total mechanical energy is equal at all points, so the intercepts for kinetic and potential energy are equal.

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Page 2

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 1

Continue your response to QUESTION 1 on this page.

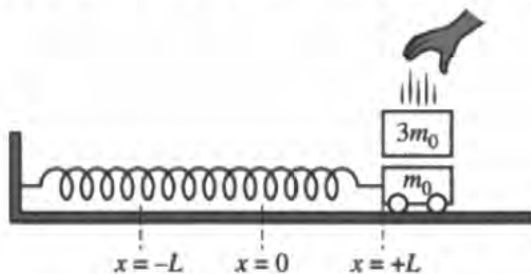


Figure 3

When the cart is at  $+L$  and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between  $-L$  and  $+L$ . The masses of the cart and the block are  $m_0$  and  $3m_0$ , respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is  $f_1$ . The frequency of oscillation after

the block is dropped onto the cart is  $f_2$ . Calculate the numerical value of the ratio  $\frac{f_2}{f_1}$ .

$f_1$        $f_2$   
 $m_0$        $4m_0$

$$|\vec{F}_s| = k\Delta x$$

$$f_2 = \frac{1}{4} f_1$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.





## Question 1

Continue your response to **QUESTION 1** on this page.

- (c) The dashed line in Figure 4 shows the kinetic energy  $K$  versus potential energy  $U$  of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

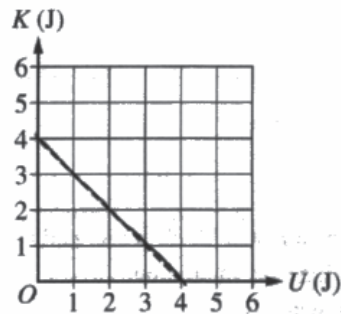


Figure 4

- i. Briefly explain why the two graphs must be the same, using physics principles.

Since friction is negligible, there is no loss of energy when the block is dropped onto the spring. The total mechanical energy before the block is added = total mechanical energy after block is added.

- ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

## Question 1

Begin your response to QUESTION 1 on this page.

## PHYSICS 1

## SECTION II

Time—1 hour and 30 minutes

5 Questions

**Directions:** Questions 1, 4, and 5 are short free-response questions that require about 13 minutes each to answer and are worth 7 points each. Questions 2 and 3 are long free-response questions that require about 25 minutes each to answer and are worth 12 points each. Show your work for each part in the space provided after that part.

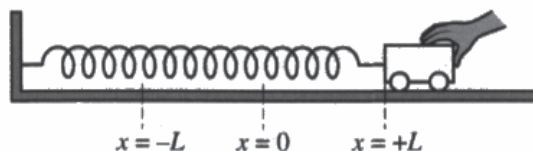


Figure 1

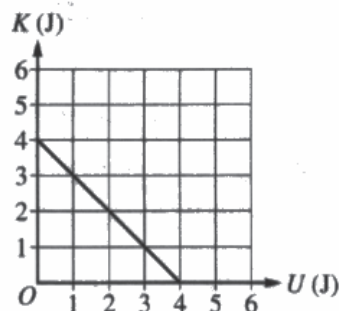


Figure 2

1. (7 points, suggested time 13 minutes)

A cart on a horizontal surface is attached to a spring. The other end of the spring is attached to a wall. The cart is initially held at rest, as shown in Figure 1. When the cart is released, the system consisting of the cart and spring oscillates between the positions  $x = +L$  and  $x = -L$ . Figure 2 shows the kinetic energy of the cart-spring system as a function of the system's potential energy. Frictional forces are negligible.

(a) On the graph of kinetic energy  $K$  versus potential energy  $U$  shown in Figure 2, the values for the  $x$ -intercept and  $y$ -intercept are the same. Briefly explain why this is true, using physics principles.

This is true because after the cart starts moving toward  $x = -L$ , it travels the equal amount of distance, and soon it bounces back and travels back. On the graph,  $K(J)$  of 2 and  $U(J)$  of 2 is  $x = 0$  in figure 1. That's the 0 point that the cart passes through.

Question 1

Continue your response to QUESTION 1 on this page.

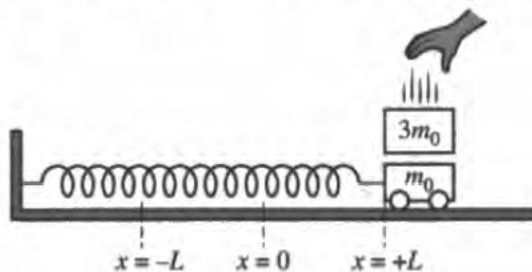


Figure 3

When the cart is at  $+L$  and momentarily at rest, a block is dropped onto the cart, as shown in Figure 3. The block sticks to the cart, and the block-cart-spring system continues to oscillate between  $-L$  and  $+L$ . The masses of the cart and the block are  $m_0$  and  $3m_0$ , respectively.

(b) The frequency of oscillation before the block is dropped onto the cart is  $f_1$ . The frequency of oscillation after

the block is dropped onto the cart is  $f_2$ . Calculate the numerical value of the ratio  $\frac{f_2}{f_1}$ .

$$\frac{3m_0}{m_0} = 2m_0$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.



## Question 1

Continue your response to **QUESTION 1** on this page.

- (c) The dashed line in Figure 4 shows the kinetic energy  $K$  versus potential energy  $U$  of the block-cart-spring system after the block is dropped onto the cart. This graph is identical to the graph shown in Figure 2 for the cart-spring system before the block is dropped onto the cart.

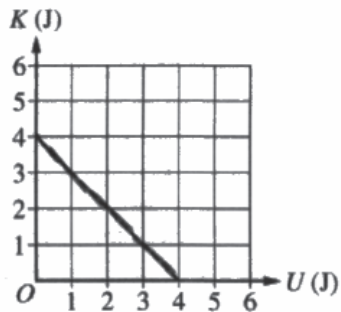


Figure 4

- i. Briefly explain why the two graphs must be the same, using physics principles.

This two graphs must be the same because they traveled the same equal amount of distance. mass does not affect the distance that the cart travels.

- ii. After the block is dropped onto the cart, consider a system that consists only of the cart and the spring. On Figure 4, sketch a solid line that shows the kinetic energy of the system that consists of the cart and the spring but not the block after the block is dropped onto the cart.

**Question 1****Sample Identifier: P1 Q1 Sample A****Score: 7**

- a.
- 1 point was earned. The response contains an explanation that indicates that the maximum kinetic energy and maximum potential energy are the same due to energy conservation.
- b.
- 1 point was earned. The response uses the equation for frequency or period in a ratio.
  - 1 point was earned. The response substitutes the total mass  $4m_0$  into the correct ratio:  $\frac{f_2}{f_1}$  or  $\frac{T_1}{T_2}$ .
- c.i.
- 1 point was earned. The response contains a valid explanation in terms of work or energy for why the systems' energies should be the same.
- c.ii.
- 1 point was earned. The response contains a straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of 4 J.
  - 1 point was earned. The response contains a vertical intercept that is less than the vertical intercept in the original graph.
  - 1 point was earned. The response contains a correct vertical intercept of 1 J.

**Sample Identifier: P1 Q1 Sample B**

**Score: 5**

a.

- 1 point was earned. The response contains an explanation that indicates that the maximum kinetic energy and maximum potential energy are the same due to energy conservation.

b.

- 1 point was earned. The response uses the equation for frequency or period in a ratio.
- 0 points were earned. The response does not substitute the total mass  $4m_0$  into the correct ratio:  $\frac{f_2}{f_1}$  or

$$\frac{T_1}{T_2}.$$

c.i.

- 1 point was earned. The response contains a valid explanation in terms of work or energy for why the systems' energies should be the same.

c.ii.

- 0 points were earned. The response does not contain a straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of 4 J.
- 1 point was earned. The response contains a vertical intercept that is less than the vertical intercept in the original graph.
- 1 point was earned. The response contains a correct vertical intercept of 1 J.

**Sample Identifier: P1 Q1 Sample C**

**Score: 4**

- a.
- 1 point was earned. The response contains an explanation that indicates that the maximum kinetic energy and maximum potential energy are the same due to energy conservation.
- b.
- 1 point was earned. The response uses the equation for frequency or period in a ratio.
  - 1 point was earned. The response substitutes the total mass  $4m_0$  into the correct ratio:  $\frac{f_2}{f_1}$  or  $\frac{T_1}{T_2}$ .
- c.i.
- 1 point was earned. The response contains a valid explanation in terms of work or energy for why the systems' energies should be the same.
- c.ii.
- 0 points were earned. The response does not contain a straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of 4 J.
  - 0 points were earned. The response does not contain a vertical intercept that is less than the vertical intercept in the original graph.
  - 0 points were earned. The response does not contain a correct vertical intercept of 1 J.

**Sample Identifier: P1 Q1 Sample D**

**Score: 3**

- a.
- 1 point was earned. The response contains an explanation that indicates that the maximum kinetic energy and maximum potential energy are the same due to energy conservation.
- b.
- 1 point was earned. The response uses the equation for frequency or period in a ratio.
  - 1 point was earned. The response substitutes the total mass  $4m_0$  into the correct ratio:  $\frac{f_2}{f_1}$  or  $\frac{T_1}{T_2}$ .
- c.i.
- 0 points were earned. The response does not contain a valid explanation in terms of work or energy for why the systems' energies should be the same.
- c.ii.
- 0 points were earned. The response does not contain a straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of 4 J.
  - 0 points were earned. The response does not contain a vertical intercept that is less than the vertical intercept in the original graph.
  - 0 points were earned. The response does not contain a correct vertical intercept of 1 J.



**Sample Identifier: P1 Q1 Sample E****Score: 2**

a.

- 1 point was earned. The response contains an explanation that indicates that the maximum kinetic energy and maximum potential energy are the same due to energy conservation.

b.

- 0 points were earned. The response does not use the equation for frequency or period in a ratio.
- 0 points were earned. The response does not substitute the total mass  $4m_0$  into the correct ratio:  $\frac{f_2}{f_1}$  or

$$\frac{T_1}{T_2}.$$

c.i.

- 0 points were earned. The response does not contain a valid explanation in terms of work or energy for why the systems' energies should be the same.
  - Note: The response states that no energy is lost because there is no friction. The response needs to address the fact that no work is done on the system.

c.ii.

- 1 point was earned. The response contains a straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of 4 J.
- 0 points were earned. The response does not contain a vertical intercept that is less than the vertical intercept in the original graph.
- 0 points were earned. The response does not contain a correct vertical intercept of 1 J.

**Sample Identifier: P1 Q1 Sample F****Score: 1**

a.

- 0 points were earned. The response does not contain an explanation that indicates that the maximum kinetic energy and maximum potential energy are the same due to energy conservation.

b.

- 0 points were earned. The response does not use the equation for frequency or period in a ratio.
- 0 points were earned. The response does not substitute the total mass  $4m_0$  into the correct ratio:  $\frac{f_2}{f_1}$  or

$$\frac{T_1}{T_2}.$$

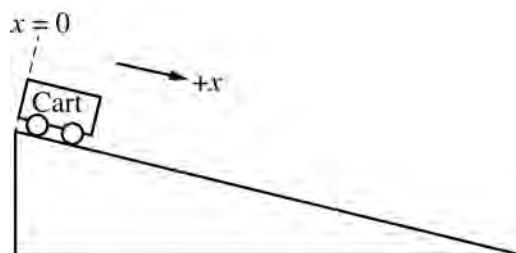
c.i.

- 0 points were earned. The response does not contain a valid explanation in terms of work or energy for why the systems' energies should be the same.

c.ii.

- 1 point was earned. The response has a solid line drawn on top of the dotted line. The response contains a straight line with a horizontal intercept that is the same as the horizontal intercept of the original graph of 4 J.
- 0 points were earned. The response does not contain a vertical intercept that is less than the vertical intercept in the original graph.
- 0 points were earned. The response does not contain a correct vertical intercept of 1 J.

Begin your response to **QUESTION 2** on this page.



2. (12 points, suggested time 25 minutes)

- (a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position $x$ (m)	Time $t$ (s)	
	0.06	0.39	
	0.14	0.59	
	0.24	0.77	
	0.37	0.96	
	0.55	1.20	

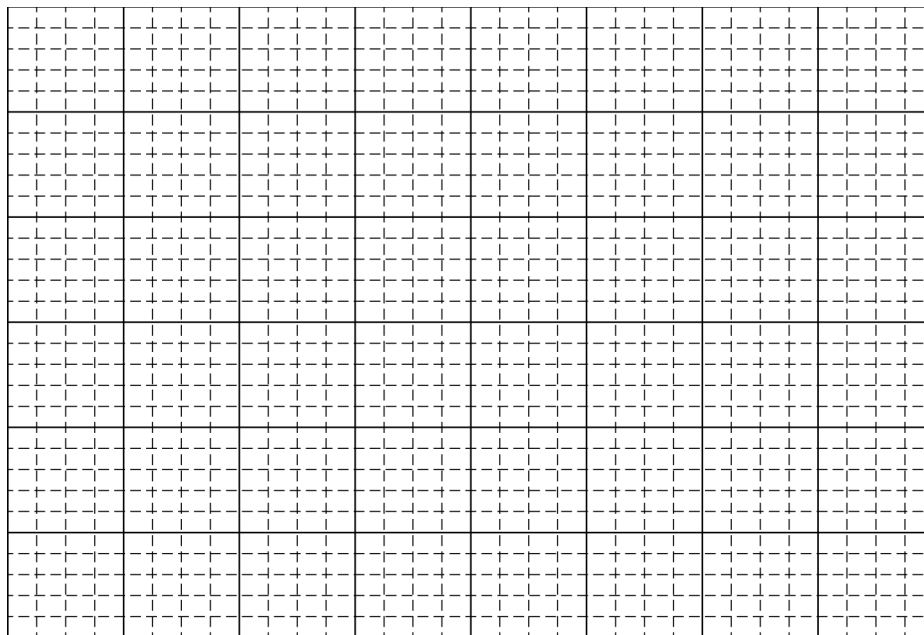
- i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

Vertical axis: \_\_\_\_\_ Horizontal axis: \_\_\_\_\_

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 2** on this page.

- ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



- iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

- (b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

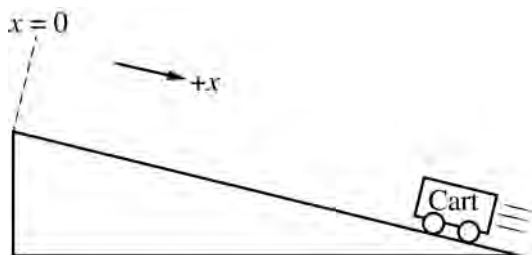
i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$  ?

ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .

**GO ON TO THE NEXT PAGE.**

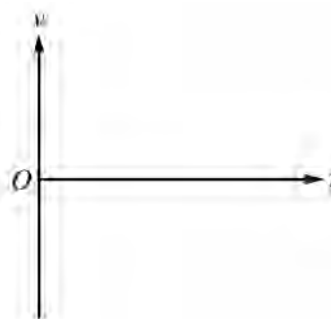
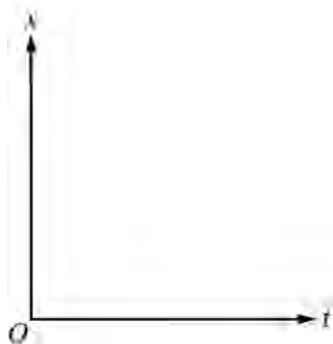
Continue your response to **QUESTION 2** on this page.

- (c) The students calculate the value of  $g_{\text{exp}}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .
- What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{\text{exp}}$ ?
  - Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{\text{exp}}$ .



The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

- (d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



**GO ON TO THE NEXT PAGE.**

**Question 2: Experimental Design****12 points**

- (a)(i) For indicating two quantities that, when graphed together, produce a straight line whose slope can be used to determine the acceleration  $a$  **1 point**

**Example Response**

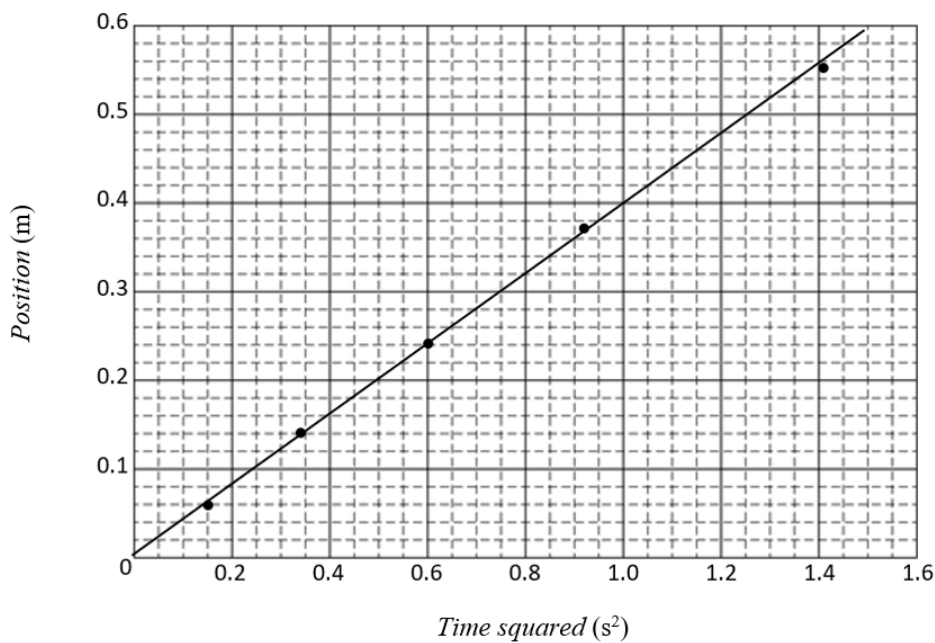
Vertical Axis : Position      Horizontal Axis : Time squared

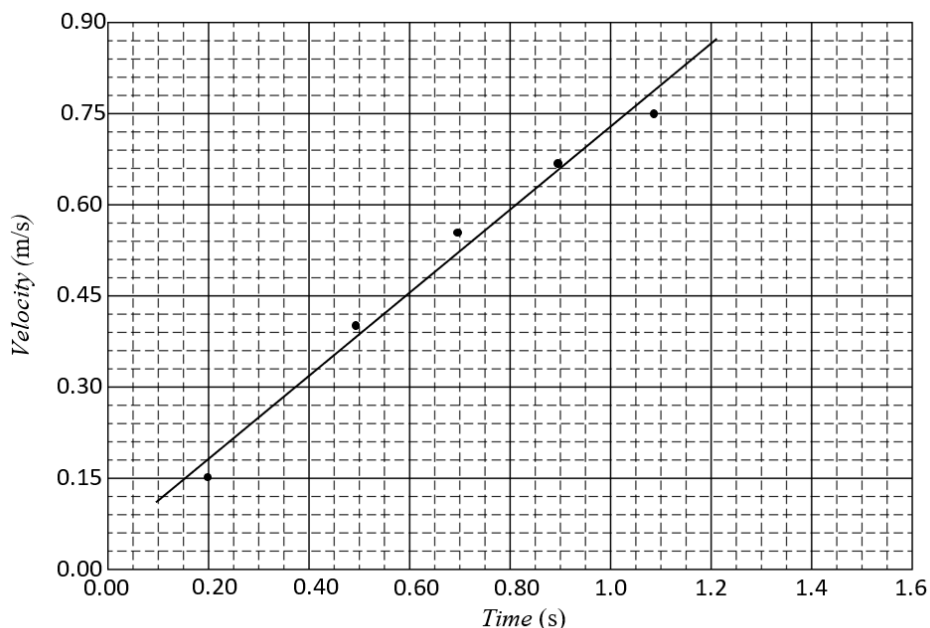
Position $x$ (m)	Time $t$ (s)	Time squared $t^2$ ( $s^2$ )
0.06	0.39	0.15
0.14	0.59	0.35
0.24	0.77	0.59
0.37	0.96	0.92
0.55	1.20	1.44

- (a)(ii) The axes have a linear scale and are identified (labels **OR** units) so that when graphed correctly, the data will span more than half of the horizontal and vertical axes **1 point**

For plotting at least 4 of the data points correctly **1 point**

For drawing a best-fit line that approximates the trend of the data **1 point**

**Example Response**

*Alternate Example Response*

**Scoring Note:** The following tables represent the most common linearized graphs with the data that were used to determine the acceleration.

Graph: $v$ vs. $t$	
$v \left( \frac{\text{m}}{\text{s}} \right)$	$t$ (s)
0.15	0.20
0.40	0.49
0.56	0.68
0.68	0.87
0.75	1.08

Graph: $2x$ vs. $t^2$	
$2x$ (m)	$t^2$ ( $\text{s}^2$ )
0.12	0.15
0.28	0.35
0.48	0.59
0.74	0.92
1.10	1.44

Graph: $2v_{\text{avg}}$ vs. $t$	
$2v_{\text{avg}} \left( \frac{\text{m}}{\text{s}} \right)$	$t$ (s)
0.31	0.39
0.47	0.59
0.62	0.77
0.77	0.96
0.92	1.20

Graph: $x$ vs. $\frac{1}{2}t^2$	
$x$ (m)	$\frac{1}{2}t^2$ ( $\text{s}^2$ )
0.06	0.08
0.14	0.17
0.24	0.30
0.37	0.46
0.55	0.72

Graph: $v_{\text{avg}}^2$ vs. $x$	
$v_{\text{avg}}^2 \left( \frac{\text{m}^2}{\text{s}^2} \right)$	$x$ (m)
0.02	0.06
0.06	0.14
0.10	0.24
0.15	0.37
0.21	0.55

Graph: $\sqrt{x}$ vs. $t$	
$\sqrt{x}$ ( $\sqrt{\text{m}}$ )	$t$ (s)
0.24	0.39
0.37	0.59
0.49	0.77
0.61	0.96
0.74	1.20

- 
- (a)(iii) For attempting to find the slope,  $\left(\frac{\text{rise}}{\text{run}}\right)$  or  $\left(\frac{\Delta y}{\Delta x}\right)$ , of the best-fit line drawn in part (a)(ii) **1 point**

**Scoring Note:** An indication that a calculator was used for linear regression to determine the value of the slope may earn this point.

- 
- For using the slope in a valid kinematic equation to calculate the acceleration **1 point**

**Scoring Note:** This point can be earned if evidence of a kinematic equation exists in graphed quantities (e.g., a graph of position as a function of  $\frac{1}{2}t^2$ ).

---

**Example Response**

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta \text{position}}{\Delta \text{time}^2} = \frac{0.48 \text{ m} - 0.18 \text{ m}}{1.2 \text{ s}^2 - 0.4 \text{ s}^2} = 0.375 \frac{\text{m}}{\text{s}^2}$$

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$\frac{\Delta x}{t^2} = \frac{1}{2} a$$

$$\text{slope} \times 2 = a$$

$$a = 0.75 \frac{\text{m}}{\text{s}^2}$$

---

**Total for part (a) 6 points**

---



**(b)(i)** For indicating a quantity to be measured

**1 point**

Accept **one** of the following:

- The angle  $\theta$  with the horizontal
- The height  $h$  and length  $L$  of the ramp

**Scoring Note:** Stating only the height needs to be measured can earn this point if an energy approach is used.

**(b)(ii)** For providing a correct expression relating the acceleration of gravity to the acceleration measured

**1 point**

**Scoring Note:** If  $\cos\theta$  is used, the response must specify that  $\theta$  was measured from the vertical.

---

**Example Response**

$$mg_{\text{exp}} \sin \theta = ma$$

$$g_{\text{exp}} = \frac{a}{\sin \theta}$$

**OR**

$$\sin \theta = \frac{h}{L}$$

$$g_{\text{exp}} = \left(\frac{L}{h}\right)a$$

**OR**

$$mg_{\text{exp}}h = \frac{1}{2}mv^2$$

$$g_{\text{exp}}h = \frac{1}{2}v^2$$

$$v = \sqrt{2g_{\text{exp}}h}$$

$$v = at$$

$$at = \sqrt{2g_{\text{exp}}h}$$

$$g_{\text{exp}} = \frac{a^2t^2}{2h}$$

---

**Total for part (b) 2 points**

---

---

(c)(i) For identifying a physical factor that could have affected the result

**1 point**

Accept **one** of the following:

- A physical factor in the materials used (e.g., the wheels have nonnegligible rotational inertia, the ramp was bumpy, the wheels were wobbly or not perfectly round, the base of the ramp was not level, the floor was not level)
- A physical factor in the environment (e.g., the room was being accelerated (elevator), the experiment was performed at high elevation or on a different planet)
- A physical error in measurement collection (e.g., time, position, or angle was measured incorrectly)

**Scoring Note:** A statement of “Human error” does not earn this point.

---

(c)(ii) For correctly indicating the functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$

**1 point**

Accept **one** of the following:

- Correctly indicating the functional dependence between the physical factor in the materials used and  $g_{\text{exp}}$  (e.g., if the rotational inertia of the rotating wheels is nonnegligible, the cart will have a smaller acceleration and  $g_{\text{exp}}$  will be smaller.)
- Correctly indicating the functional dependence between the physical factor in the environment and  $g_{\text{exp}}$  (e.g., if the experiment was performed at a high elevation, the acceleration will be smaller and  $g_{\text{exp}}$  will be smaller.)
- Correctly indicating the functional dependence between the physical error in the measurement collection and  $g_{\text{exp}}$  (e.g., if the angle of the ramp is smaller than the measured value, the cart will have a smaller acceleration and  $g_{\text{exp}}$  will be smaller.)

---

**Example Response**

*The expression I derived for the value for  $g_{\text{exp}}$  did not take into consideration that the wheels had any rotational inertia. If the wheels have rotational inertia and are rotating, the acceleration of the cart would be less than  $g \sin \theta$ , so the value of  $g_{\text{exp}}$  would be less than  $9.8 \frac{\text{m}}{\text{s}^2}$ .*

---

**Total for part (c) 2 points**

---

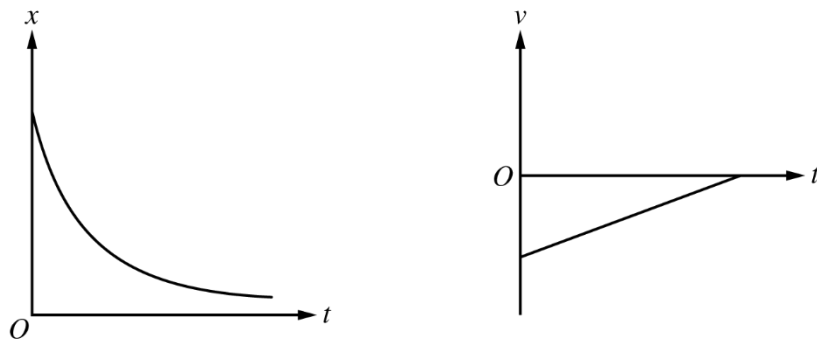
- 
- (d) For sketching a concave up curve with an initially negative slope for the graph of position as a function of time **1 point**
- 

For **one** of the following:

**1 point**

- Drawing a line with a positive slope and a negative vertical intercept for the  $v$  vs  $t$  graph
  - Drawing a  $v$  vs  $t$  graph that is consistent with the  $x$  vs  $t$  graph that shows acceleration
- 

**Example Response**



**Scoring Note:** Alternate example graphs with the points the response would earn.

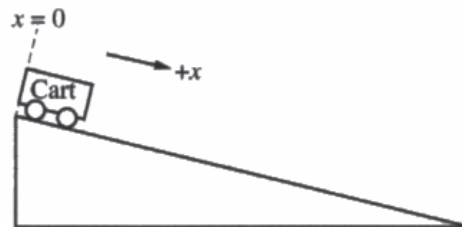
		<p>Earns 2 points</p>
		<p>Earns 1 point</p>
		<p>Earns 1 point</p>
		<p>Earns 1 point</p>
		<p>Earns 0 points</p>

**Total for part (d) 2 points**

**Total for question 2 12 points**

## Question 2

Begin your response to **QUESTION 2** on this page.



2. (12 points, suggested time 25 minutes)

- (a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position $x$ (m)	Time $t$ (s)	
	0.06	0.39	
	0.14	0.59	
	0.24	0.77	
	0.37	0.96	
	0.55	1.20	

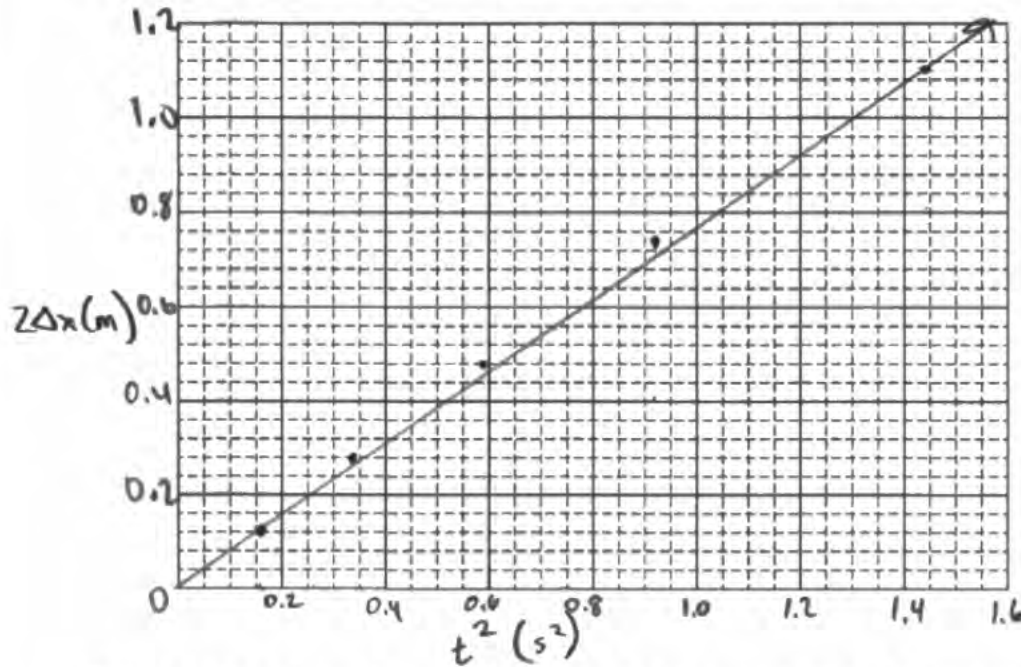
- i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

Vertical axis:  $2\Delta x$       Horizontal axis:  $t^2$

Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

$$a = \frac{0.96 - 0.2}{1.25 - 0.25} = \frac{0.76}{1} = 0.76$$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

The angle of the ramp from the ground ( $\theta$ )

ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .

$$a = g_{\text{exp}} \sin \theta$$

$$\frac{a}{\sin \theta} = g_{\text{exp}}$$

**Question 2**

Continue your response to **QUESTION 2** on this page.

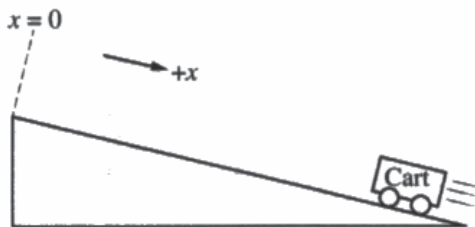
(c) The students calculate the value of  $g_{\text{exp}}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .

i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{\text{exp}}$ ?

The measured value of  $\theta$  was greater than the actual value of  $\theta$ .

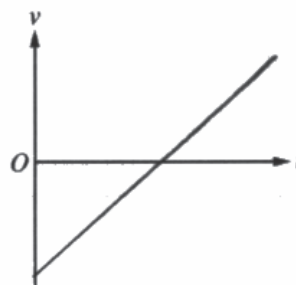
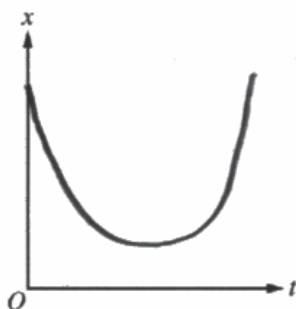
ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{\text{exp}}$ .

A greater than accurate  $\theta$  would result in a greater than accurate  $\sin \theta$ , which would result in a lower than accurate  $g_{\text{exp}}$ .



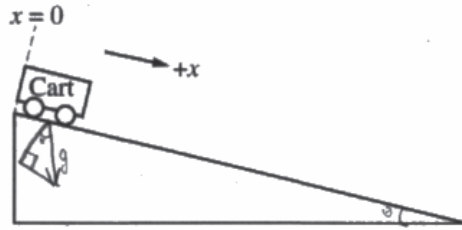
The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

(d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



Question 2

Begin your response to QUESTION 2 on this page.



2. (12 points, suggested time 25 minutes)

$$x = \frac{1}{2}at^2$$

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position $x$ (m)	Time $t$ (s)	Time $t^2$ (s <sup>2</sup> )
	0.06	0.39	0.15
	0.14	0.59	0.35
	0.24	0.77	0.59
	0.37	0.96	0.92
	0.55	1.20	1.44

i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

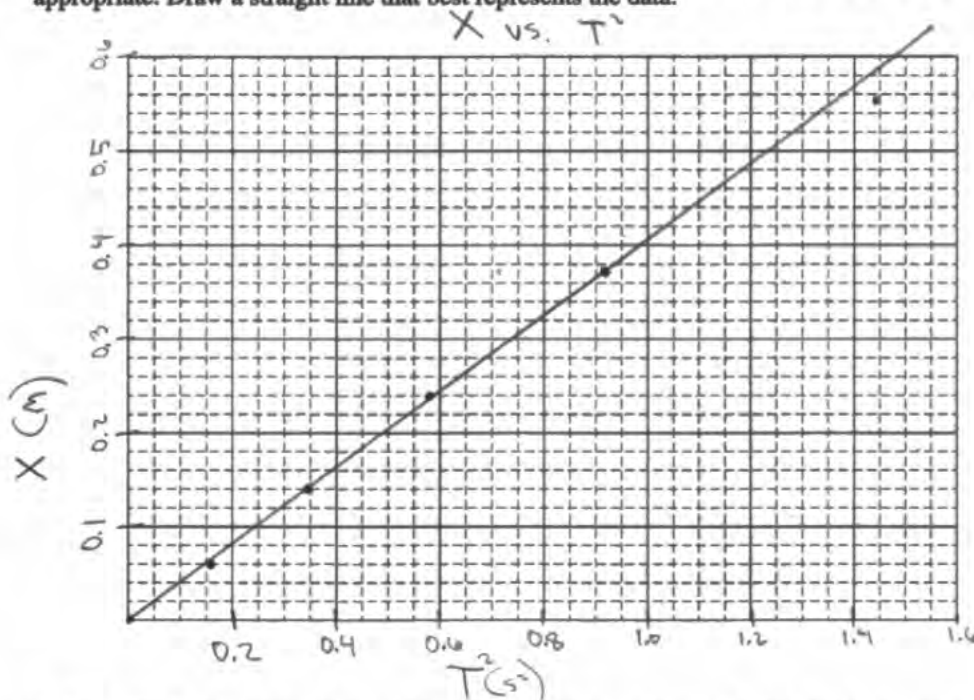
Vertical axis:  $x$  (m)      Horizontal axis:  $t^2$  (s<sup>2</sup>)



Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

$$\text{Slope} = \frac{0.37 - 0.14}{0.92 - 0.35} = \frac{0.23}{0.57} \approx 0.40 \text{ m/s}^2$$

$$a = 2 \cdot 0.40 \text{ m/s}^2 = 0.80 \text{ m/s}^2$$

$x = \frac{1}{2}at^2$   
 $a = 2 \cdot x/t^2$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

The angle with the horizontal  $\theta$ .

ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .

$$g_{\text{exp}} \sin \theta = a$$

$$g_{\text{exp}} = \frac{a}{\sin \theta}$$

**Question 2**

Continue your response to **QUESTION 2** on this page.

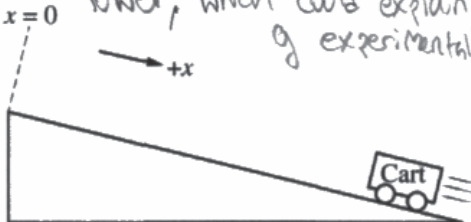
(c) The students calculate the value of  $g_{\text{exp}}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .

i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{\text{exp}}$ ?

The ramp could be stationed significantly above the Earth's surface, at a high elevation.

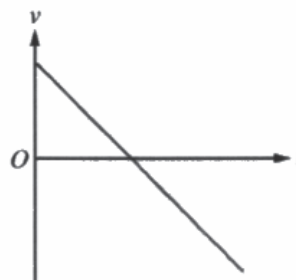
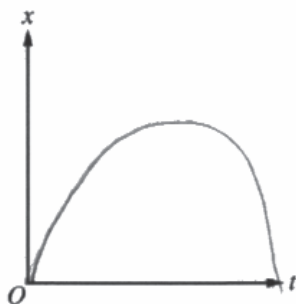
ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{\text{exp}}$ .

As two objects move further apart, their gravitational attraction and thus acceleration caused by gravity decreases. So, as an object such as the cart moves into a higher elevation, the value for  $g$  would be lower, which could explain the lower value of  $g$  experimentally determined.



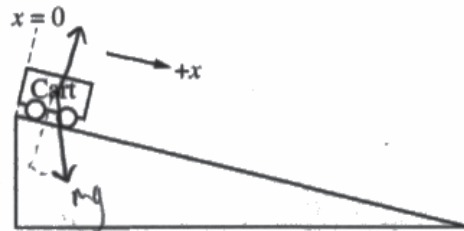
The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

(d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



Question 2

Begin your response to QUESTION 2 on this page.



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position $x$ (m)	Time $t$ (s)	time $t^2$ (s <sup>2</sup> )
	0.06	0.39	0.15
	0.14	0.59	0.35
	0.24	0.77	0.59
	0.37	0.96	0.92
	0.55	1.20	1.44

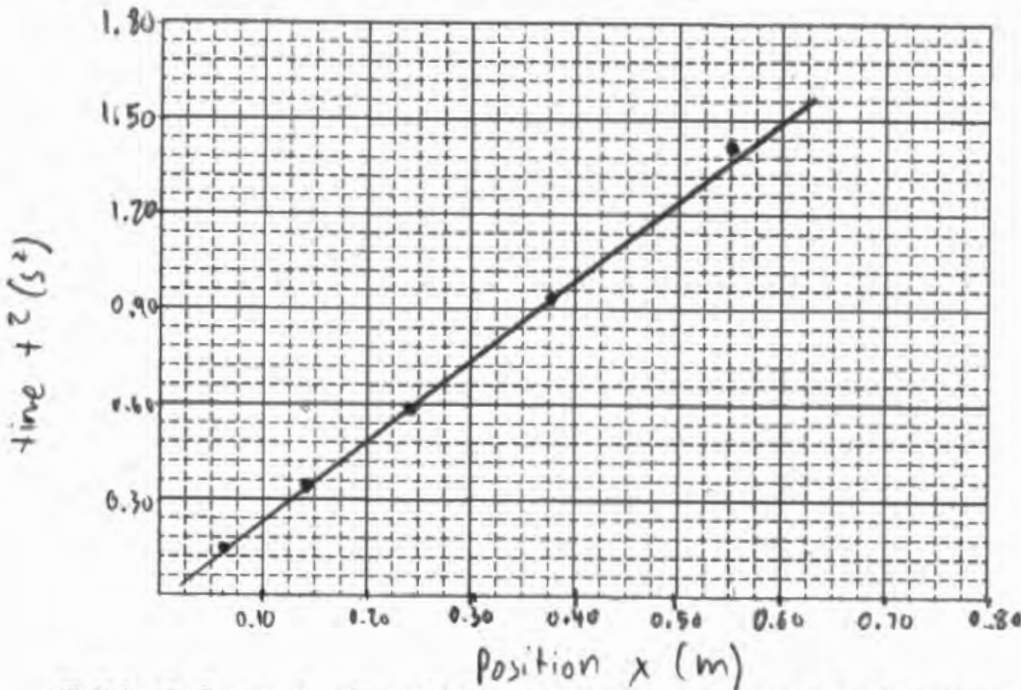
i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

Vertical axis: time  $t^2$  (s<sup>2</sup>)      Horizontal axis: position  $x$  (m)

Question 2

Continue your response to QUESTION 2 on this page.

- ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



- iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

$$\text{slope} = \frac{0.59 - 0.35}{0.24 - 0.14} = 2.4 \quad \left| \quad a = \frac{1}{\text{slope}} = \frac{1}{2.4} = 0.42 \text{ m/s}^2$$

- (b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

- i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

$$\begin{aligned} \Sigma F &= ma \\ mg \sin \theta &= ma \end{aligned} \quad \left| \quad \text{Students need to measure angle of ramp}$$

- ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .

$$\begin{aligned} \Sigma F &= ma \\ mg \sin \theta &= ma \\ g_{\text{exp}} &= \frac{a}{\sin \theta} \end{aligned}$$



**Question 2**

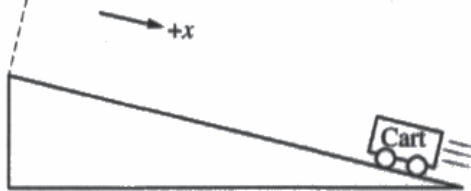
Continue your response to **QUESTION 2** on this page.

- (c) The students calculate the value of  $g_{\text{exp}}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .
- (i) What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{\text{exp}}$ ?

The students may have measured the angle of the ramp to be too large.

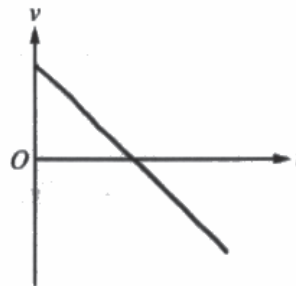
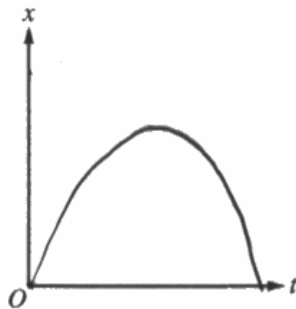
- (ii) Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{\text{exp}}$ .

since  $g_{\text{exp}} = \frac{a}{\sin \theta}$ , if  $\theta$  is too large  $\sin \theta$  becomes large and since  $a$  is inversely proportional to  $g_{\text{exp}}$  the large  $\theta$  leads to smaller  $g_{\text{exp}}$ .



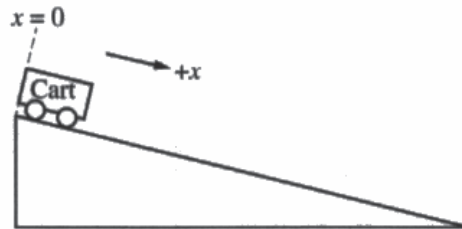
The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

- (d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



Question 2

Begin your response to QUESTION 2 on this page.



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position $x$ (m)	Time $t$ (s)	velocity $v$ (m/s)
	0.06	0.39	0.31
	0.14	0.59	0.48
	0.24	0.77	0.62
	0.37	0.96	0.77
	0.55	1.20	0.92

$= v_{avg} \cdot t = \frac{\Delta x}{t} \cdot t$

i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

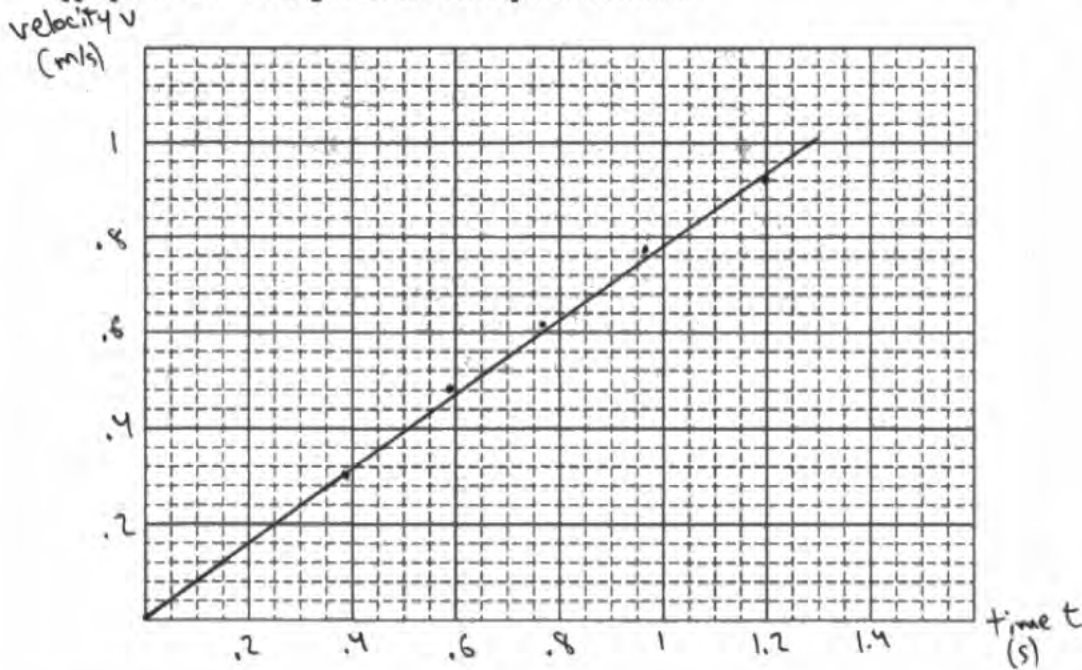
Vertical axis: velocity  $v$  (m/s)  
 (at time  $t$ )

Horizontal axis: Time  $t$  (s)

Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

Slope =  $a$

$$a = \frac{\Delta v}{\Delta t} = \frac{1.0 - 0}{1.2 - 0} = .78 \text{ m/s}^2$$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

The angle of the incline  $\theta$ .

ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .

$$W_{\parallel} = mg \sin \theta = ma$$

$$g = \frac{a}{\sin \theta}$$

$$g_{\text{exp}} = \frac{a}{\sin \theta}$$



**Question 2**

Continue your response to **QUESTION 2** on this page.

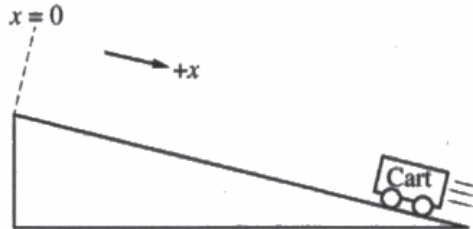
(c) The students calculate the value of  $g_{exp}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .

i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{exp}$ ?

If the students plotted velocity only as  $\frac{\Delta x}{\Delta t}$ , then they calculated average velocity, which is less than instantaneous velocity at a time  $t$  if the object is accelerating and could change the values of  $v$  and therefore  $g_{exp}$ .

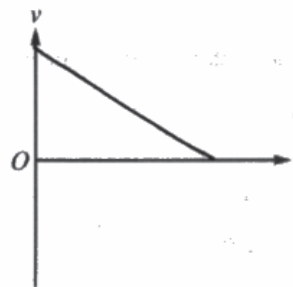
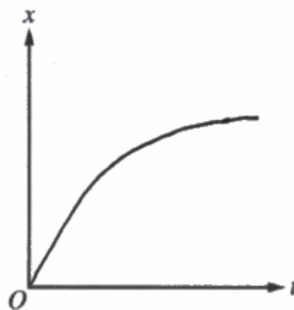
ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{exp}$ .

For an accelerating object,  $v_{average} < v_{instant}$  if time  $t$  is the end of an interval. Therefore, the slope of the graph would be less if  $v_{average}$  is calculated. Since  $g_{exp}$  is proportional to  $a$ , this would result in a lower  $g_{exp}$ .



The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

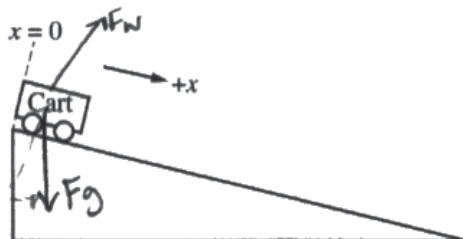
(d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.





Question 2

Begin your response to QUESTION 2 on this page.



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position $x$ (m)	Time $t$ (s)	Velocity
	0.06	0.39	$0.1538 \frac{m}{s}$
	0.14	0.59	$0.237 \frac{m}{s}$
	0.24	0.77	$0.3116 \frac{m}{s}$
	0.37	0.96	$0.3854 \frac{m}{s}$
	0.55	1.20	$0.458 \frac{m}{s}$

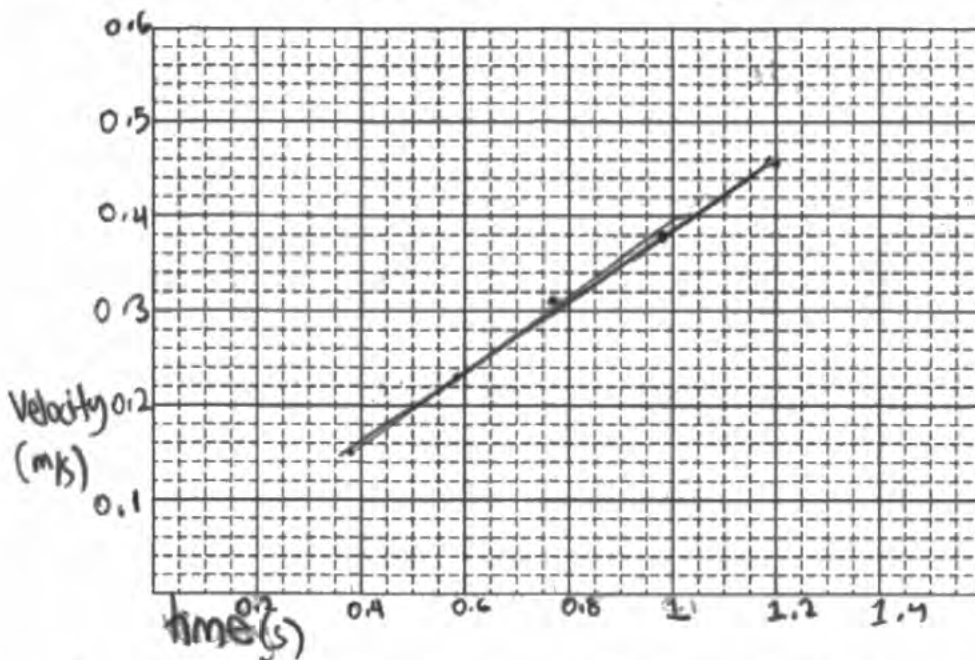
i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

Vertical axis: velocity      Horizontal axis: time

Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

$$a = \frac{0.458 - 0.1538}{1.2 - 0.39} = 0.375 \frac{m}{s^2}$$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{exp}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{exp}$  from  $a$ ?

- mass of cart
- angle of ramp

ii. Write an expression for the value of  $g_{exp}$  in terms of  $a$ .

Let  $\theta$  be angle of ramp

$$F_{net \parallel} = ma$$

$$mg_{exp} \sin \theta = ma$$

$$g_{exp} = \frac{a}{\sin \theta}$$

**Question 2**

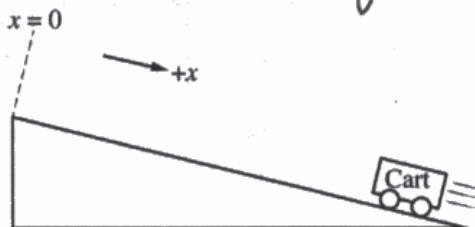
Continue your response to **QUESTION 2** on this page.

- (c) The students calculate the value of  $g_{exp}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .
- What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{exp}$ ?

A physical reason that could have significantly lower  $g_{exp}$  be the cart wheels not slipping

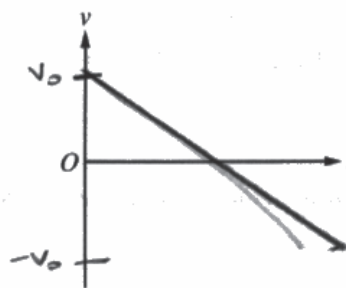
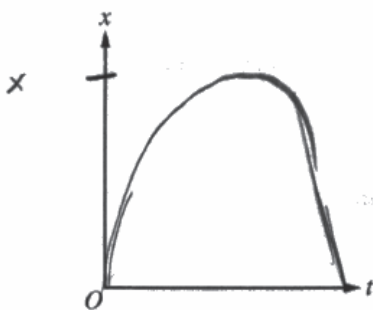
- Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{exp}$ .

The cart wheels not slipping would have caused more friction causing the acceleration to lower.



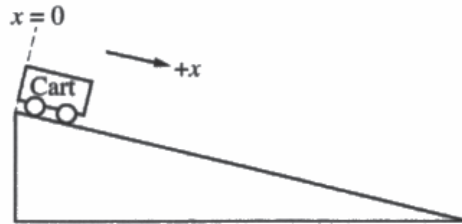
The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

- (d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



Question 2

Begin your response to **QUESTION 2** on this page.



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

Velocity (m/s)	Position $x$ (m)	Time $t$ (s)	
0.1638	0.06	0.39	
0.2373	0.14	0.59	
0.3117	0.24	0.77	
0.3854	0.37	0.96	
0.4583	0.55	1.20	

i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

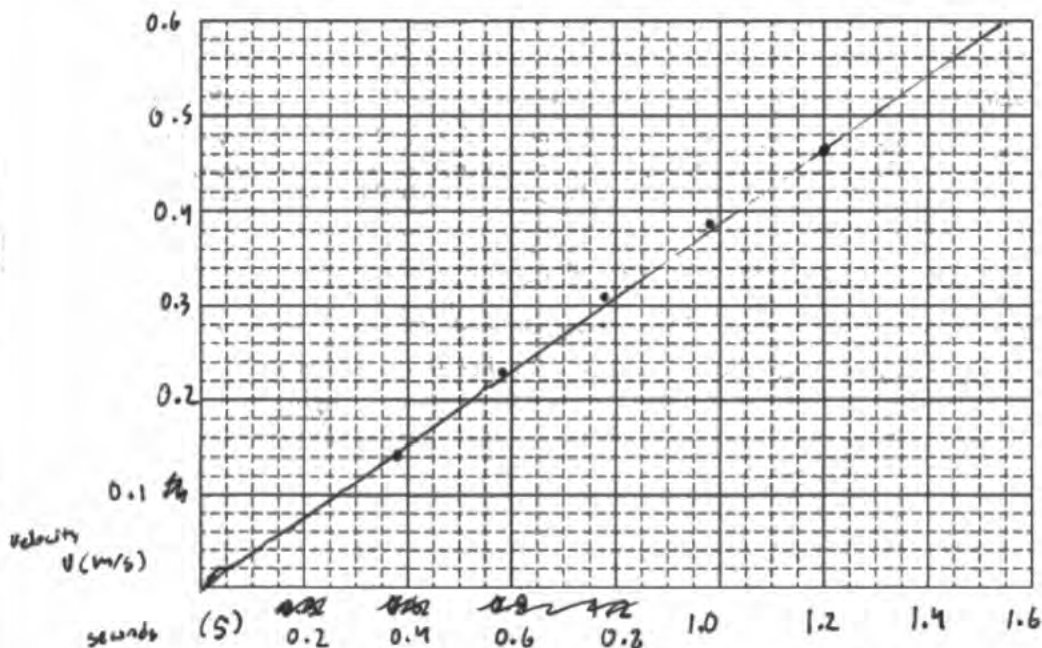
Vertical axis: velocity                      Horizontal axis: time



Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

$$a = 0.420$$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

angle of the ramp ( $\theta$ ) and the mass ( $m$ ) of the cart in kg

ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .



$$a = \frac{F_g \cos \theta}{m}$$

Question 2

Continue your response to QUESTION 2 on this page.

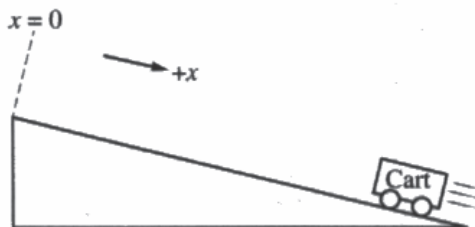
(c) The students calculate the value of  $g_{exp}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .

i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{exp}$ ?

The Normal Force that reacts from the  $F_g$  against the ramp reacts in equal magnitude but at an angle. This means that the reactive  $F_N$  force in  $F_N$  cancels out the vertical components but leaves a <sup>small</sup> horizontal component that is less than  $9.8 \text{ m/s}^2$  but uncountered by any friction.

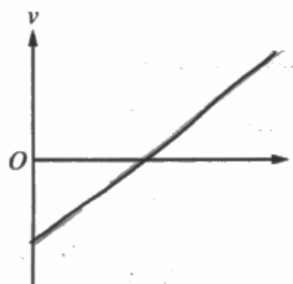
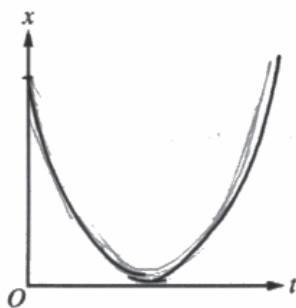
ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{exp}$ .

The vertical component of the angled Normal Force negates the vertical component of  $F_g$ , causing only the horizontal part to be uncountered leaving the value to be less than the standard acceleration due to gravity of  $9.8$ .



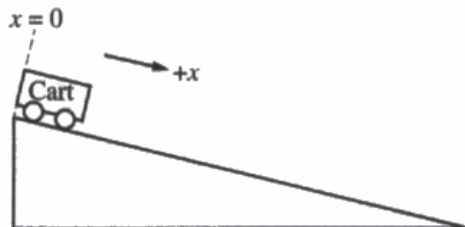
The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

(d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



Question 2

Begin your response to QUESTION 2 on this page.



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

Velocity (m/s)	Position $x$ (m)	Time $t$ (s)	
0.15	0.06	0.39	
0.24	0.14	0.59	
0.31	0.24	0.77	
0.39	0.37	0.96	
0.46	0.55	1.20	

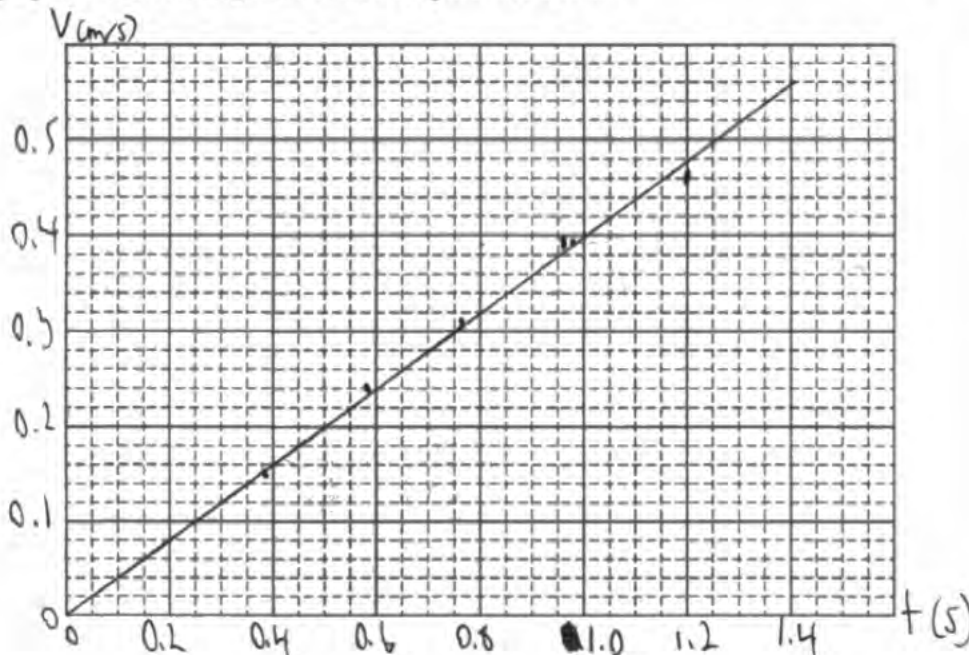
i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

Vertical axis: velocity (m/s)      Horizontal axis: time (s)

Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

$$a = 0.4 \text{ m/s}^2$$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

~~The angle of the ramp from the horizontal~~  
~~The height of the ramp of the angle of the ramp from the horizontal~~  
 ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .

$$g_{\text{exp}} = a \sin \theta$$

$\theta$  = angle of the ramp from horizontal



**Question 2**

Continue your response to **QUESTION 2** on this page.

(c) The students calculate the value of  $g_{exp}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .

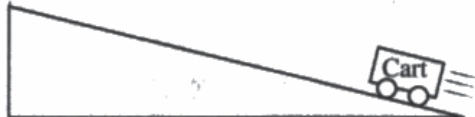
i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{exp}$ ?

The normal force ~~of~~ the ramp is exerting on the cart

ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{exp}$ .

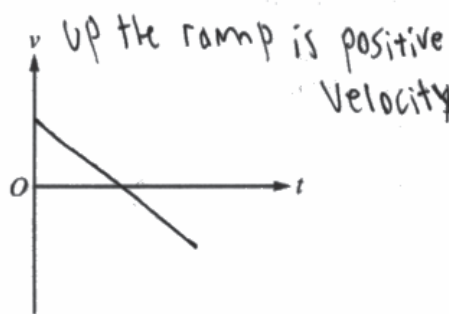
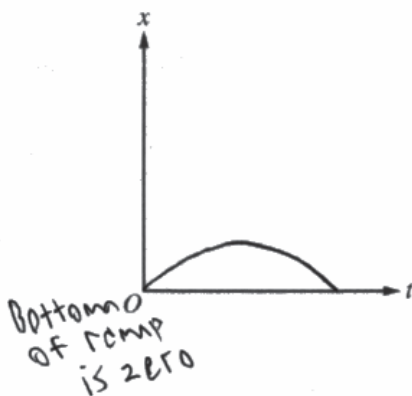
the y component of the normal force being exerted on the cart would be opposing the force of gravity, so without taking account of the normal force it would seem as if

the experimentally determined value of  $g_{exp}$  would be significantly less than the accepted value of  $9.8 \text{ m/s}^2$ .



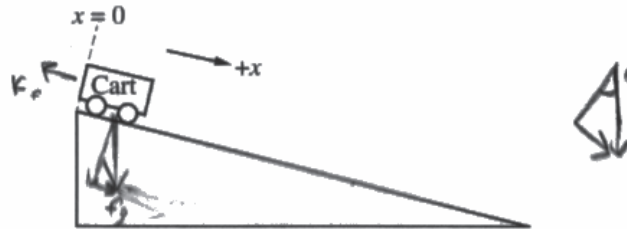
The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

(d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



Question 2

Begin your response to QUESTION 2 on this page.



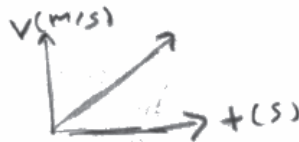
2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

Velocity $v$ (m/s)	Position $x$ (m)	Time $t$ (s)	
$\approx 0.154$	0.06	0.39	
$\approx 0.247$	0.14	0.59	
$\approx 0.312$	0.24	0.77	
$\approx 0.395$	0.37	0.96	
$\approx 0.46$	0.55	1.20	

i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

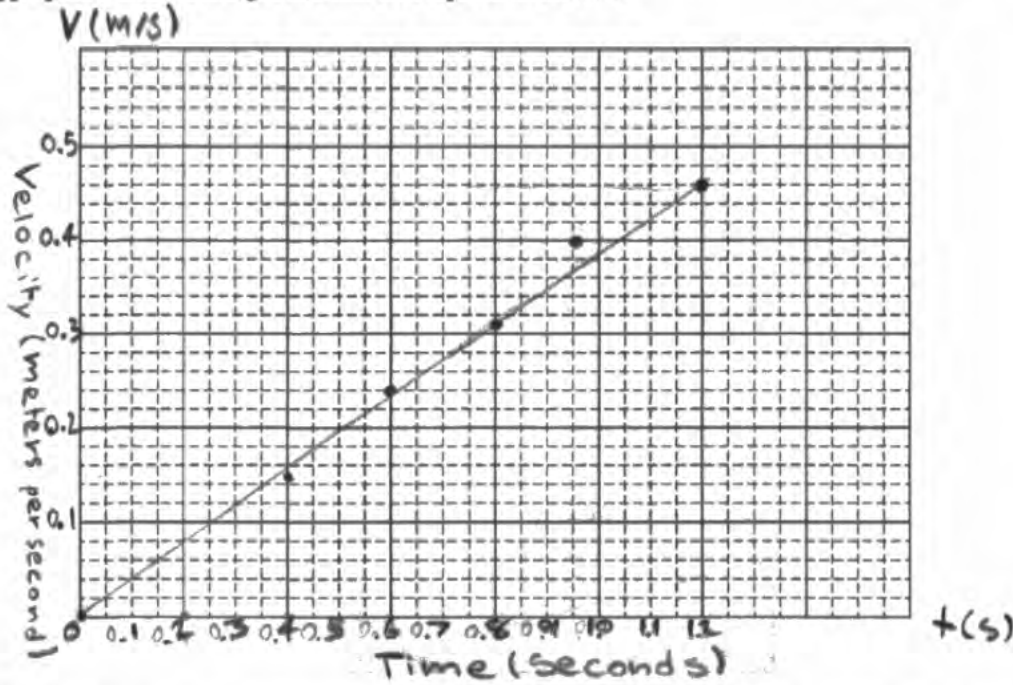
Vertical axis: Velocity (m/s) Horizontal axis: Time (s)



Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

$$a = \frac{.46 \left(\frac{m}{s}\right)}{1.2 (s)} = 0.383\bar{3} \approx \boxed{0.383 \text{ m/s}^2} = a$$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

none, as provided friction is negligible,  $a$  and  $g_{\text{exp}}$  are the same value as it is the only force acting on the system

ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ . that isn't canceled out

$$a = F_g \sin \theta = g_{\text{exp}}$$

$$\Rightarrow \boxed{a = g_{\text{exp}}}$$

or

$$a + F_f = g_{\text{exp}}$$

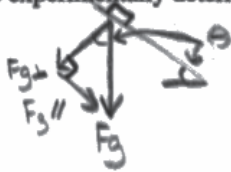
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 2

Continue your response to QUESTION 2 on this page.

(c) The students calculate the value of  $g_{exp}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .

i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{exp}$ ?

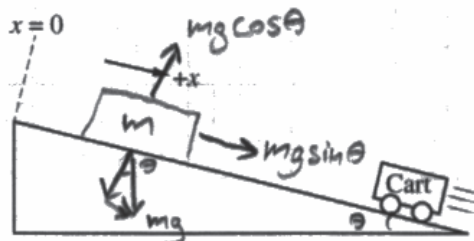


$g_{exp}$  is lower than  $9.8 \text{ m/s}^2$  because  $F_g \sin \theta$  is the force causing the acceleration of the cart, which is not equal to the entire force of gravity.

ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{exp}$ .

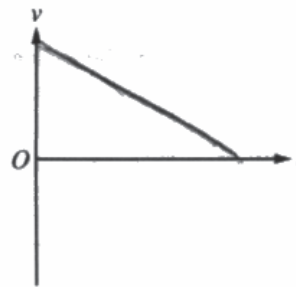
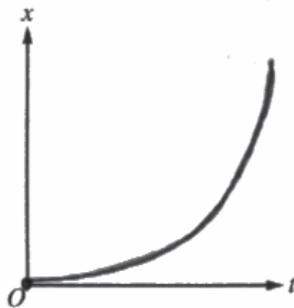
$$\sqrt{(mg \sin \theta)^2 + (mg \cos \theta)^2} = mg$$

$$mg \sin(\theta) < mg$$



The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

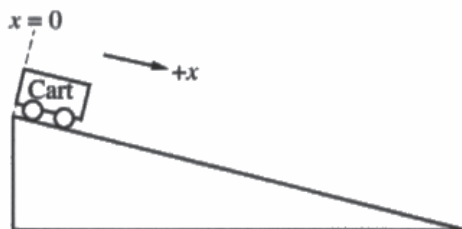
(d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.





Question 2

Begin your response to QUESTION 2 on this page.



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

	Position $x$ (m)	Time $t$ (s)	
	0.06	0.39	
	0.14	0.59	
	0.24	0.77	
	0.37	0.96	
	0.55	1.20	

i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

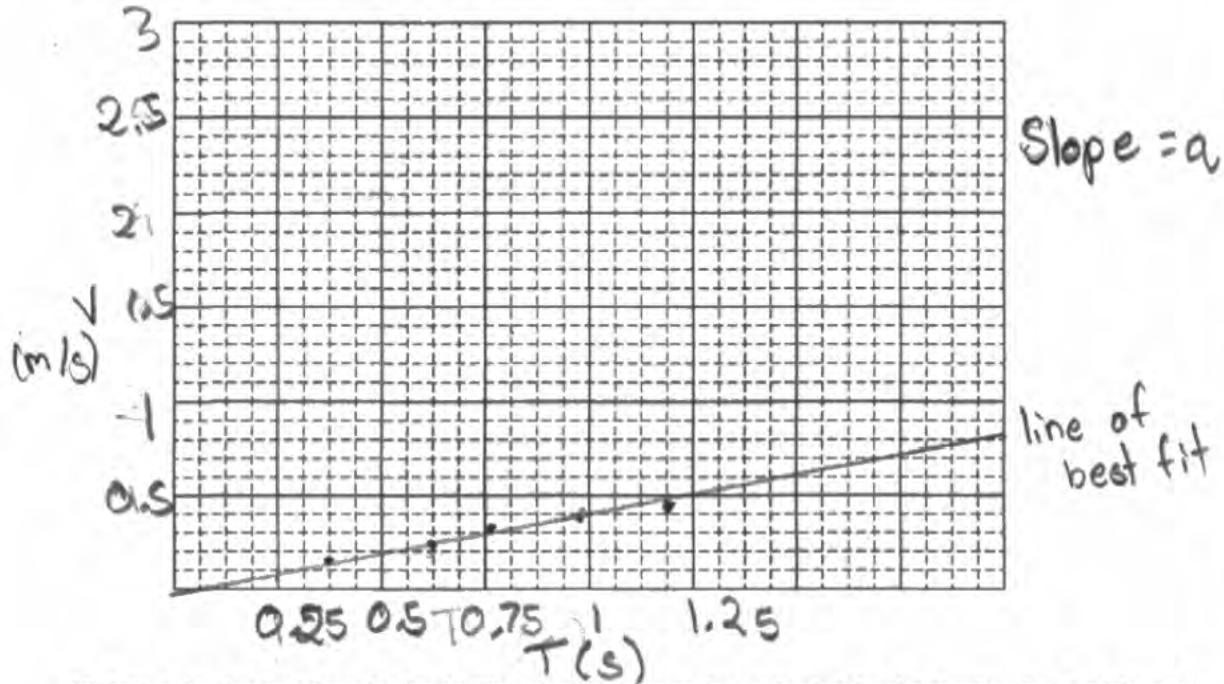
Vertical axis: \_\_\_\_\_ Horizontal axis:  \_\_\_\_\_

$v = 0.153$   
 $0.237$   
 $0.31$   
 $0.38$

Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

$$S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.38 - 0.153}{1.2 - 0.39} = \frac{0.227}{0.81} = 0.28 \text{ m/s}^2$$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

mass of the cart

ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .

$$F = ma$$

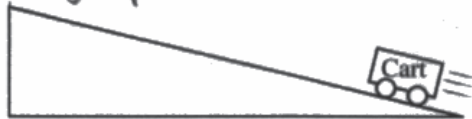
Question 2

Continue your response to QUESTION 2 on this page.

- (c) The students calculate the value of  $g_{exp}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .
- i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{exp}$ ?

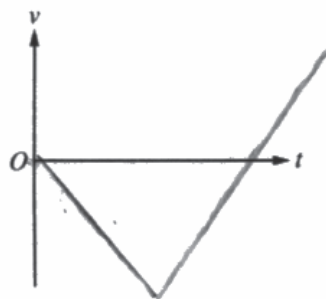
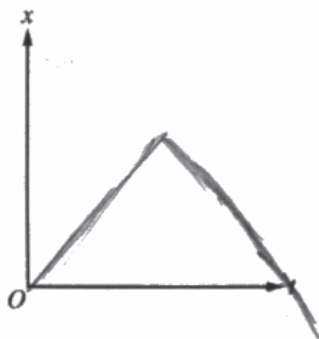
This is because only a small component of the  $9.8 \text{ m/s}^2$  acts on the  $x$  axis, the rest acts on the  $y$  axis and does not affect acceleration in the  $x$  axis.

- ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{exp}$ .
- The force exerted by gravity is split into two components as the  $x$  axis is parallel to the ramp, the majority of the force is still applied in the  $y$  axis causing the  $g_{exp}$  value to be lower.



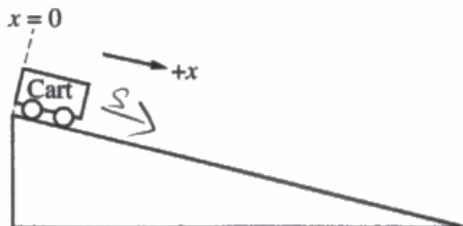
The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

- (d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



Question 2

Begin your response to QUESTION 2 on this page.



2. (12 points, suggested time 25 minutes)

(a) Students conduct an experiment to determine the acceleration  $a$  of a cart. The cart is released from rest at the top of the ramp at time  $t = 0$  and moves down the ramp. The  $x$ -axis is defined to be parallel to the ramp with its origin at the top, as shown in the figure. The students collect the data shown in the following table.

Velocity (m/s)	Position $x$ (m)	Time $t$ (s)	
1.153	0.06	0.39	
1.237	0.14	0.59	
1.311	0.24	0.77	
1.385	0.37	0.96	
1.458	0.55	1.20	

i. Indicate which quantities could be graphed to yield a straight line whose slope could be used to determine the acceleration  $a$  of the cart. You may use the remaining columns in the table, as needed, to record any quantities (including units) that are not already in the table.

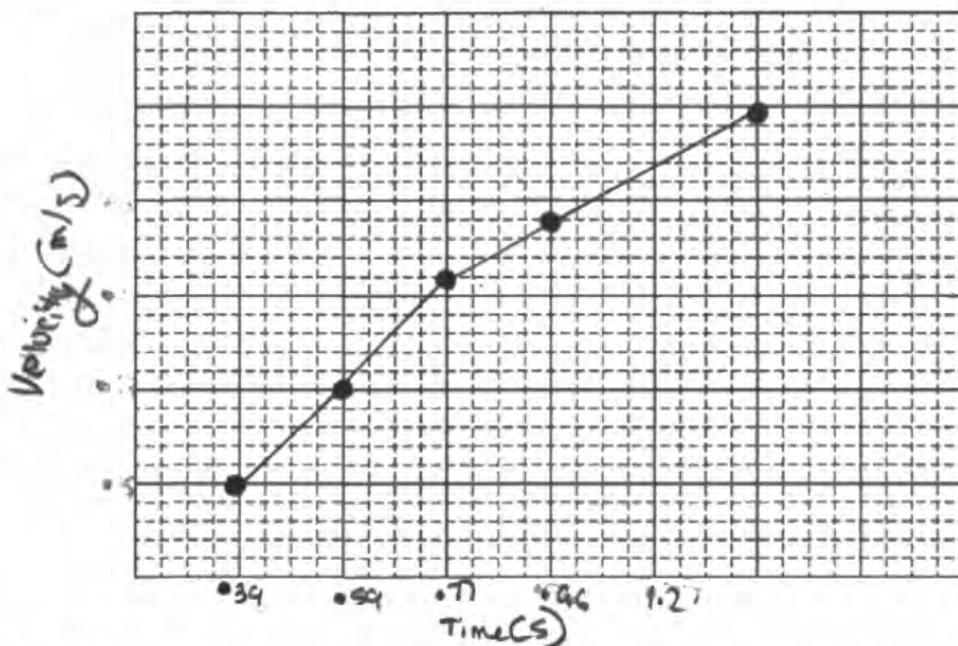
Vertical axis: Velocity (m/s)      Horizontal axis: time.



Question 2

Continue your response to QUESTION 2 on this page.

ii. On the following grid, plot the appropriate quantities to create a graph that can be used to determine the acceleration  $a$  of the cart as it rolls down the ramp. Clearly scale and label all axes (including units), as appropriate. Draw a straight line that best represents the data.



iii. Using the line you drew in part (a)(ii), calculate an experimental value for the acceleration  $a$  of the cart as it rolls down the ramp.

An experimental value would be  $a = \frac{2.37}{1.50} = a$   
 $a = 1.40 \text{ m/s}^2$

(b) The students are asked to determine an experimental value for the acceleration due to gravity  $g_{\text{exp}}$  using their data.

i. What additional quantities do the students need to measure in order to calculate  $g_{\text{exp}}$  from  $a$ ?

The student would need to measure the net force.

ii. Write an expression for the value of  $g_{\text{exp}}$  in terms of  $a$ .

$$g_{\text{exp}} = F_{\text{net}} (m)$$



**Question 2**

Continue your response to **QUESTION 2** on this page.

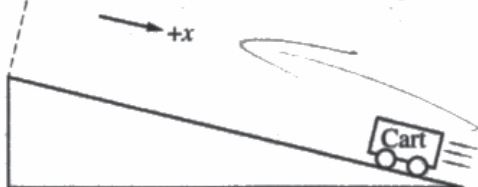
(c) The students calculate the value of  $g_{\text{exp}}$  to be significantly lower than the accepted value of  $9.8 \text{ m/s}^2$ .

i. What is a physical reason, other than friction or air resistance, that could lead to a significant difference in the experimentally determined value of  $g_{\text{exp}}$ ?

The significance reason needs to be the total mass of the system that is greater than earth. Higher the mass  $\rightarrow$  the gravity pull causing the acceleration to be higher.

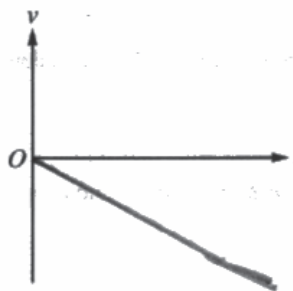
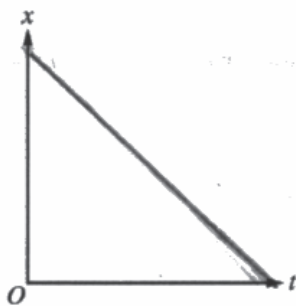
ii. Briefly explain how the physical reason you identified in part (c)(i) would lead to the decrease in the experimentally determined value of  $g_{\text{exp}}$ .

Since the gravity acceleration or  $g$  is lower in this system. A physical reason would be since the mass of cart is similar to the earth the gravitational pull would be lower making the acceleration of the cart to be lower.



The students want to confirm that the acceleration is the same whether the cart rolls up or down the ramp. The students start the cart at the bottom and give the cart a quick push so that it rolls up the ramp and momentarily comes to rest. The  $x$ -axis is still defined to be parallel to the ramp with the origin at the top.

(d) On the following graphs, sketch the position  $x$  and velocity  $v$  as functions of time  $t$  that correspond to the scenario shown while the cart moves up the ramp.



**Question 2****Sample Identifier: P1 Q2 Sample A****Score: 12**

a.i.

- 1 point was earned. The response correctly identifies two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 1 point was earned. The response has axes that are scaled so that the data spans more than half the horizontal and vertical axes. Both axes are appropriately identified.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 1 point was earned. The response uses two points from the best fit line to calculate the slope of the line.
- 1 point was earned. The response graphs variables that give evidence of the use of kinematic equations.

b.i.

- 1 point was earned. The response correctly identifies a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 1 point was earned. The response provides a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 1 point was earned. The response identifies a physical factor that might have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 1 point was earned. The response correctly indicates the functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$ .

d.

- 1 point was earned. The response shows an  $x$  vs  $t$  curve that is concave up and has an initially negative slope. The second half of this graph does not affect this point.
- 1 point was earned. The response shows a  $v$  vs  $t$  line with a positive slope and a negative intercept. The second half of this graph does not affect this point.

**Sample Identifier: P1 Q2 Sample B**

**Score: 11**

a.i.

- 1 point was earned. The response correctly identifies two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 1 point was earned. The response has axes that are scaled so that the data spans more than half the horizontal and vertical axes. Both axes are appropriately identified.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 1 point was earned. The response uses two points from the best fit line to calculate the slope of the line.
- 1 point was earned. The response uses the slope in a valid kinematic equation to calculate the acceleration.

b.i.

- 1 point was earned. The response correctly identifies a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 1 point was earned. The response provides a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 1 point was earned. The response identifies a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 1 point was earned. The response correctly indicates the functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$ .

d.

- 0 points were earned. The response does not show an  $x$  vs  $t$  curve that is concave up and has an initially negative slope.
- 1 point was earned. The response shows a  $v$  vs  $t$  line that is consistent with the  $x$  vs  $t$  graph. The second half of this graph does not affect this point.

**Sample Identifier: P1 Q2 Sample C****Score: 10**

a.i.

- 1 point was earned. The response indicates two variables that will yield a straight line that could be used to determine a value for acceleration. If there are multiple conflicting labels, in which at least one is correct, look at the data table for clarification. If the data in the data table does not help clarify, use the following priority, if present: quantity, variable, and lastly units. The response initially is unclear if time or  $t$  squared was graphed, but it was verified that the response is graphing time squared by the values in their data table.

a.ii.

- 1 point was earned. The response has axes that are scaled so that the data spans more than half the horizontal and vertical axes. Both axes are appropriately identified.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 1 point was earned. The response uses two points from the best fit line to calculate the slope of the line.
- 0 points were earned. The response does not use the slope in a valid kinematic equation to calculate acceleration.

b.i.

- 1 point was earned. The response correctly identifies a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 1 point was earned. The response provides a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 1 point was earned. The response identifies a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 1 point was earned. The response correctly indicates the functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$ .

d.

- 0 points were earned. The response does not show an  $x$  vs  $t$  curve that is concave up and has an initially negative slope.
- 1 point was earned. The response shows a  $v$  vs  $t$  line that is consistent with the  $x$  vs  $t$  graph. The second half of this graph does not affect this point.

**Sample Identifier: P1 Q2 Sample D**

**Score: 9**

a.i.

- 1 point was earned. The response correctly identifies two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 1 point was earned. The response has axes that are scaled so that the data spans more than half the horizontal and vertical axes. Both axes are appropriately identified.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 1 point was earned. The response uses two points from the best fit line to calculate the slope of the line.
- 1 point was earned. The response graphs variables that give evidence of the use of kinematic equations. The average velocity values were doubled in the table.

b.i.

- 1 point was earned. The response correctly identifies a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 1 point was earned. The response provides a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 0 points were earned. The response does not identify a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 0 points were earned. Because the response does not provide a physical reason in part (c)(i), a functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$  cannot be indicated.

d.

- 0 points were earned. The response does not show an  $x$  vs  $t$  curve that is concave up and has an initially negative slope.
- 1 point was earned. The response shows a  $v$  vs  $t$  line that is consistent with the  $x$  vs  $t$  graph.

**Sample Identifier: P1 Q2 Sample E**

**Score: 8**

a.i.

- 1 point was earned. The response correctly identifies two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 1 point was earned. The response has axes that are scaled so that the data spans more than half the horizontal and vertical axes. Both axes are appropriately identified.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 1 point was earned. The response uses two points from the best fit line to calculate the slope of the line.
- 0 points were earned. The response does not use the slope in a valid kinematic equation to calculate acceleration.

b.i.

- 1 point was earned. The response correctly identifies a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 1 point was earned. The response provides a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 0 points were earned. The response does not identify a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 0 points were earned. The response does not indicate the functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$ .

d.

- 0 points were earned. The response does not show an  $x$  vs  $t$  curve that is concave up and has an initially negative slope.
- 1 point was earned. The response shows a  $v$  vs  $t$  line that is consistent with the  $x$  vs  $t$  graph. The second half of this graph does not affect this point.

**Sample Identifier: P1 Q2 Sample F**

**Score: 7**

a.i.

- 1 point was earned. The response correctly identifies two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 1 point was earned. The response has axes that are scaled so that the data spans more than half the horizontal and vertical axes. Both axes are appropriately identified.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 0 points were earned. The response does not use two points from the best fit line to calculate the slope of the line.
- 0 points were earned. The response does not use the slope in a valid kinematic equation to calculate acceleration.

b.i.

- 1 point was earned. The response correctly identifies a quantity that is needed to calculate  $g_{\text{exp}}$  from *a*. Additional listed quantities did not affect this point.

b.ii.

- 0 points were earned. The response does not provide a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 0 points were earned. The response does not identify a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 0 points were earned. The response does not indicate the functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$ .

d.

- 1 point was earned. The response shows an  $x$  vs  $t$  curve that is concave up and has an initially negative slope. The second half of this graph does not affect this point.
- 1 point was earned. The response shows a  $v$  vs  $t$  line with a positive slope and a negative intercept. The second half of this graph does not affect this point.



**Sample Identifier: P1 Q2 Sample G**

**Score: 6**

a.i.

- 1 point was earned. The response correctly identifies two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 1 point was earned. The response has axes that are scaled so that the data spans more than half the horizontal and vertical axes. Both axes are appropriately identified.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 0 points were earned. The response does not use two points from the best fit line to calculate the slope of the line.
- 0 points were earned. The response does not use the slope in a valid kinematic equation to calculate acceleration.

b.i.

- 1 point was earned. The response correctly identifies a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 0 points were earned. The response does not provide a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 0 points were earned. The response does not identify a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 0 points were earned. The response does not indicate the functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$ .

d.

- 0 points were earned. The response does not show an  $x$  vs  $t$  curve that is concave up and has an initially negative slope.
- 1 point was earned. The response shows a  $v$  vs  $t$  line that is consistent with the  $x$  vs  $t$  graph. The second half of this graph does not affect this point.

**Sample Identifier: P1 Q2 Sample H**

**Score: 5**

a.i.

- 1 point was earned. The response correctly identifies two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 1 point was earned. The response has axes that are scaled so that the data spans more than half the horizontal and vertical axes. Both axes are appropriately identified.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 1 point was earned. The response uses the origin and another point from the best fit line to calculate the slope of the line.
- 0 points were earned. The response does not use the slope in a valid kinematic equation to calculate acceleration.

b.i.

- 0 points were earned. The response does not correctly identify a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 0 points were earned. The response does not provide a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 0 points were earned. The response does not identify a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 0 points were earned. Because the response does not provide a physical reason in part (c)(i), a functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$  cannot be indicated.

d.

- 0 points were earned. The response does not show an  $x$  vs  $t$  curve that is concave up and has an initially negative slope.
- 0 points were earned. The response shows a  $v$  vs  $t$  graph that is neither a line with a positive slope and a negative intercept nor a line consistent with the  $x$  vs  $t$  graph.

**Sample Identifier: P1 Q2 Sample I**

**Score: 2**

a.i.

- 0 points were earned. The response does not correctly identify two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 0 points were earned. The response has axes that are not scaled so that the data spans more than half the horizontal and vertical axes.
- 1 point was earned. The response plots at least 4 points correctly.
- 1 point was earned. The response has a best fit line that approximates the trend of the data.

a.iii.

- 0 points were earned. The response does not use correct  $y$  values from the best fit line.
- 0 points were earned. The response does not use the slope in a valid kinematic equation to calculate acceleration.

b.i.

- 0 points were earned. The response does not correctly identify a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 0 points were earned. The response does not provide a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 0 points were earned. The response does not identify a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

c.ii.

- 0 points were earned. Because the response does not provide a physical reason in part (c)(i), a functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$  cannot be indicated.

d.

- 0 points were earned. The response does not show an  $x$  vs  $t$  curve that is concave up and has an initially negative slope.
- 0 points were earned. The response shows a  $v$  vs  $t$  graph that is neither a line with a positive slope and a negative intercept nor a line consistent with the  $x$  vs  $t$  graph.

**Sample Identifier: P1 Q2 Sample J**

**Score: 1**

a.i.

- 1 point was earned. The response correctly identifies two variables that will yield a straight line that could be used to determine a value for acceleration.

a.ii.

- 0 points were earned. The response uses a nonlinear scaling, and one axis is unscaled.
- 0 points were earned. The response does not plot at least 4 points correctly.
- 0 points were earned. The response does not have a best fit line that approximates the trend of the data.

a.iii.

- 0 points were earned. The response does not use two points from the best fit line, and to calculate the slope of the line.
- 0 points were earned. The response does not use the slope in a valid kinematic equation to calculate acceleration.

b.i.

- 0 points were earned. The response does not correctly identify a quantity that is needed to calculate  $g_{\text{exp}}$  from  $a$ .

b.ii.

- 0 points were earned. The response does not provide a correct expression relating the acceleration of gravity to the acceleration measured.

c.i.

- 0 points were earned. The response does not identify a physical factor that could have affected the experimentally determined value of  $g_{\text{exp}}$ .

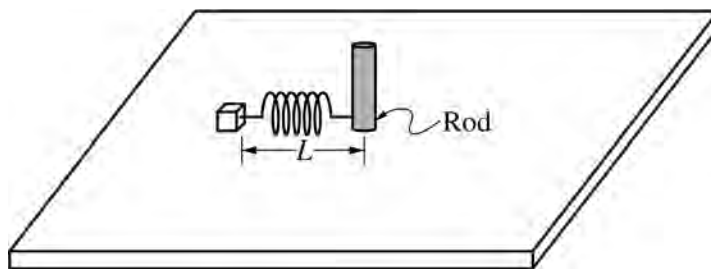
c.ii.

- 0 points were earned. The response does not indicate the functional dependence between the reason listed in part (c)(i) and  $g_{\text{exp}}$ .

d.

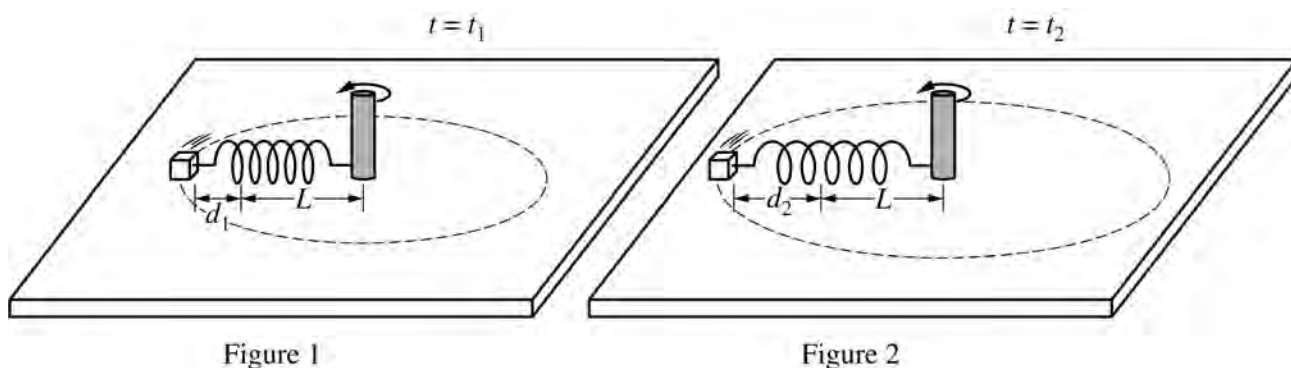
- 0 points were earned. The response does not show an  $x$  vs  $t$  curve that is concave up and has an initially negative slope.
- 0 points were earned. The response shows a  $v$  vs  $t$  graph that is neither a line with a positive slope and a negative intercept nor a line consistent with the  $x$  vs  $t$  graph.

Begin your response to **QUESTION 3** on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.



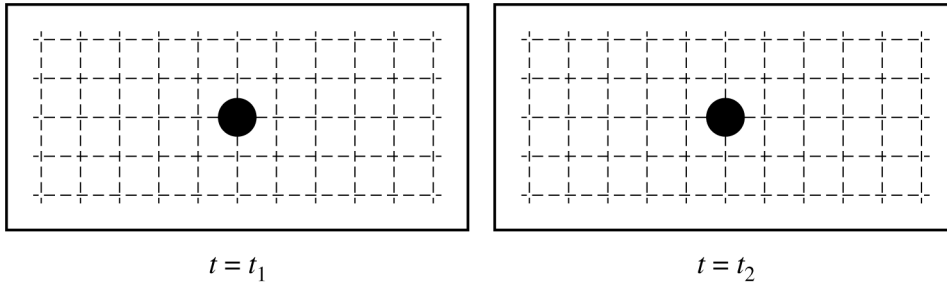
- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 3** on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

\_\_\_\_\_  $v_1 > v_2$       \_\_\_\_\_  $v_1 < v_2$       \_\_\_\_\_  $v_1 = v_2$

Justify your answer without using equations.

**GO ON TO THE NEXT PAGE.**





**Question 3: Quantitative/Qualitative Translation****12 points**

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(a)(i) For drawing rightward arrows in both diagrams **1 point**

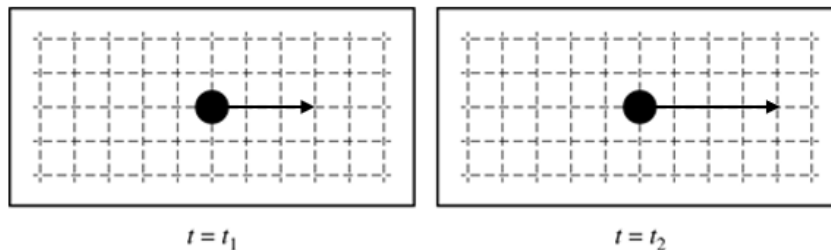
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For the length of the arrow at  $t = t_2$  being longer than the arrow at  $t = t_1$  **1 point**

---

**Scoring Notes:**

- A maximum of 1 point can be earned if extraneous unlabeled arrows are drawn.
  - A maximum of 1 point can be earned if incorrect labeled forces are drawn.
- 

**Example Response**

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(a)(ii) For an explanation that refers to the difference in the stretch length and indicates that the magnitude of the spring force is (or is not) related to the stretch length, consistent with the force diagram drawn in part (a)(i) **1 point**

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**Example Response**

*The spring force arrow drawn at  $t = t_2$  is longer because the spring is stretched a greater distance at that time and the spring force is related to the stretch distance.*

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(a)(iii) For a correct selection with an attempt at a relevant justification, or a selection and justification consistent with the response in part (a)(ii) **1 point**

---

For indicating that the spring force is the net force **1 point**

**Scoring Note:** Stating  $F = kx$  earns this point.

---

For indicating that the net force is related to the speed (or acceleration) **1 point**

**Scoring Note:** The relationship does not need to be defined to earn this point.

---

**Example Response**

\_\_\_\_\_  $v_1 > v_2$     X  $v_1 < v_2$     \_\_\_\_\_  $v_1 = v_2$

*The net force is the spring force. When the spring is stretched a greater length, the spring force is greater, so the net force is greater, and therefore the tangential speed is greater at  $t = t_2$ .*

---

**Total for part (a) 6 points**

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**(b)(i)** For the correct answer:  $F_{\text{net}} = k_0d$  **1 point**

**Scoring Notes:**

- An answer of  $kx$  does not earn this point.
- Points for part (b)(i) may be earned if correct in (b)(ii).

**Example Response**

$$F_{\text{net}} = \Sigma F = F_s$$

$$F_{\text{net}} = \Sigma F = k_0d$$

**(b)(ii)** For a multistep derivation that begins with Newton's second law:  $\Sigma F = ma$  **1 point**

For **one** of the following: **1 point**

- Substituting  $kx$  for force into Newton's second law
- Substituting  $\frac{v^2}{r}$  for acceleration into Newton's second law
- Substituting  $(L + d)$  for the radius

For the consistent answer in terms of the given variables:  $v = \sqrt{\frac{k_0d(L + d)}{m_0}}$  **1 point**

**Scoring Notes:**

- Subscripts for  $m$  and  $k$  are not required to earn this point.
- Points in (b)(ii) can be earned if correct in (b)(i).

**Example Response**

$$\Sigma F = ma_c$$

$$kx = \frac{mv^2}{r}$$

$$k_0d = \frac{m_0v^2}{L + d}$$

$$v = \sqrt{\frac{k_0d(L + d)}{m_0}}$$

**Total for part (b) 4 points**

- 
- (c) For an answer that attempts to use functional dependence to relate the tangential speed with stretched distance **1 point**

**Scoring Note:** It is not necessary to use the functional dependence correctly to earn this point.

- 
- For a correct explanation for why the derived equation in part (b)(ii) does or does not support the reasoning in part (a) **1 point**

---

**Example Response**

*My equation from part (b)(ii) agrees with my reasoning in part (a). The tangential speed of the block as it travels in a horizontal circle is related to the distance the spring is stretched. The greater the tangential speed of the block, the greater distance the spring is stretched. The equation shows this because the  $d$  is in the numerator.*

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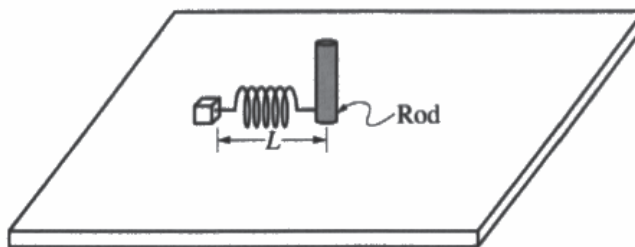
**Total for part (c) 2 points**

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**Total for question 3 12 points**

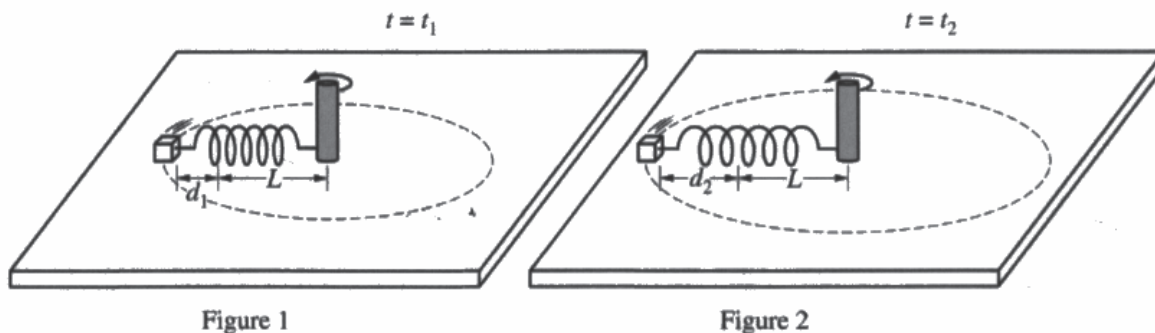
Question 3

Begin your response to QUESTION 3 on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.



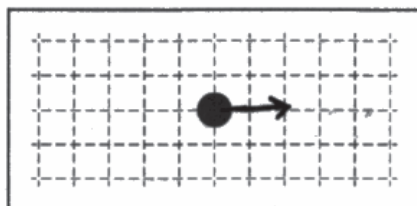
(a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

Question 3

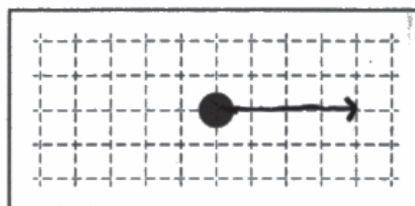
Continue your response to **QUESTION 3** on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



$t = t_1$



$t = t_2$

ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

$d_2 > d_1$ , so the spring at  $t_2$  is stretched further and applies a greater force compared to  $t_1$ . The further stretched spring at  $t_2$  applies a greater force on the block, so the force on the block by the spring is greater and has a higher magnitude.

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

$v_1 > v_2$       $v_1 < v_2$       $v_1 = v_2$

Justify your answer without using equations.

When the tangential velocity of the block is greater, the centripetal acceleration and force also will be greater. The greater force the spring applies to the block, the further it stretches, such as at  $t_2$ .

## Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$F = k|x|$$

$$F_{\text{net}} = k_0 d$$

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$F_{\text{net}} = ma$$

$$F_{\text{net}} = m_0 a_c$$

$$a_c = \frac{v^2}{r}$$

$$k_0 d = \frac{m_0 v^2}{L+d}$$

$$r = L+d$$

$$F_{\text{net}} = k_0 d$$

$$\sqrt{\frac{(L+d)(k_0 d)}{m_0}} = v$$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

Yes  No

Explain your reasoning.

If  $L$ ,  $k_0$ , and  $m_0$  are held constant,

$$\text{so } \sqrt{\frac{(L+d)(k_0 d)}{m_0}} = v,$$

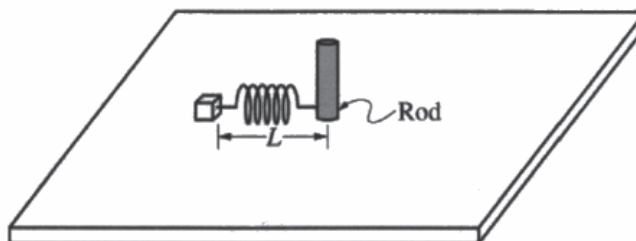
if  $v$  increases, so does  $d$ , and vice versa



## Question 3

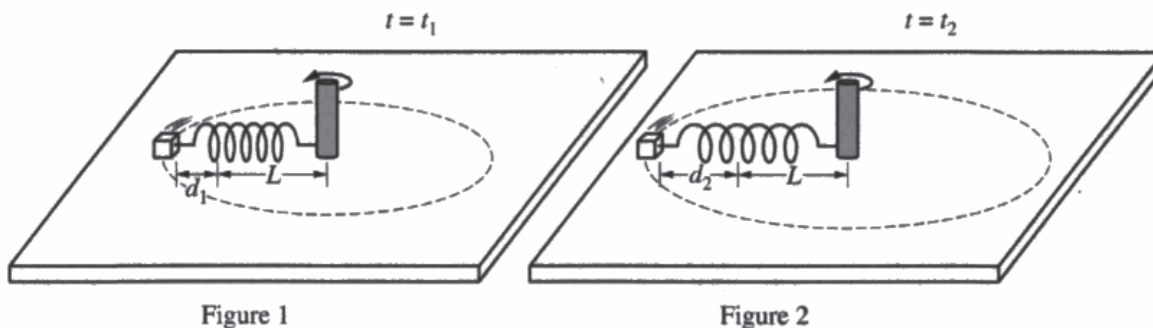
Begin your response to QUESTION 3 on this page.

3:10  
2:35



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.



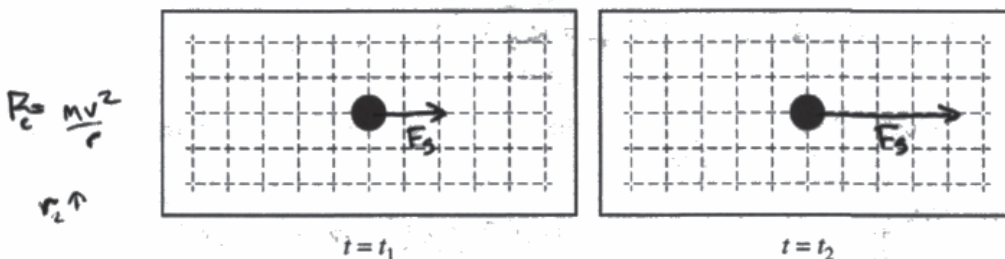
- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

Question 3

Continue your response to QUESTION 3 on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

The ~~force~~ force is greater at  $t = t_2$  because  $d_2 > d_1$ , and the further the distance from equilibrium, the stronger the force of the spring, according to Hooke's law.

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

$v_1 > v_2$       $v_1 < v_2$       $v_1 = v_2$

$F_{c2} > F_{c1}$   
~~that~~

Justify your answer without using equations.

~~$F_c = \frac{mv^2}{r}$~~      ~~$F_{c2}$  at  $t_2$ ;  $F_{c1}$  at  $t_1$~~

Because the block is further away from the axis of rotation, it is receiving a higher centripetal force at  $t_2$ , then it must be moving at a higher speed to maintain a circular path, as if the block's speed were greater at  $t_1$  than  $t_2$ , then the scenario at  $t_2$  wouldn't follow a uniform circular path.

Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

~~$F_{\text{net}} = ma$~~   ~~$F_{\text{net}} = ma$~~   $F_{\text{net}} = kx = F_{\text{spring}}$   
 $F_{\text{spring}} = k_0 d$ , so  $F_{\text{net}} = k_0 d$

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$F_{\text{net}} = ma_c$   
 $\Sigma F = \frac{m_0 v^2}{r} \rightarrow \Sigma F = k_0 d \rightarrow k_0 d = \frac{m_0 v^2}{r} \rightarrow r = L + d$   
 $v^2 = \frac{k_0 d}{m_0 (L + d)} \leftarrow \frac{k_0 d}{L + d} = m_0 v^2 \leftarrow k_0 d = \frac{m_0 v^2}{L + d}$   
 $v = \sqrt{\frac{k_0 d}{m_0 (L + d)}}$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

Yes     No

Explain your reasoning.

Because the  $d$  on the top of the fraction is multiplying while the  $d$  on the bottom is adding first, the  $d$  on the top will have a larger impact when it is increased, so the final velocity will be greater if the  $d$  value is increased, as all other values are constant.

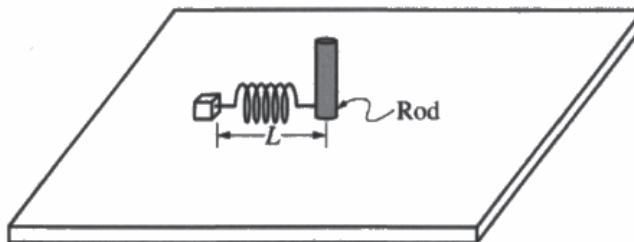
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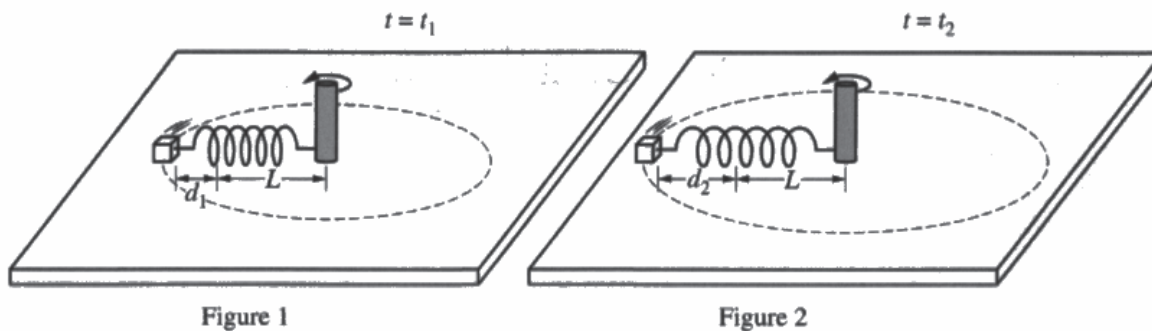
## Question 3

Begin your response to QUESTION 3 on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.



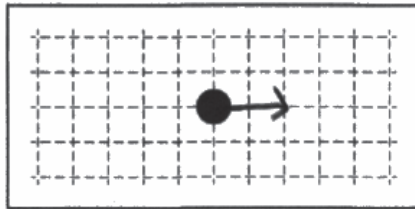
- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

Question 3

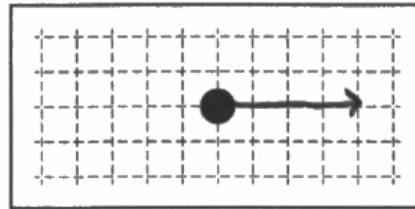
Continue your response to **QUESTION 3** on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



$t = t_1$



$t = t_2$

ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

There was more force toward the center of the circle at  $t = t_2$ .  $F_s = kx$   
 $F_s$  increased so  $t_2$  also increased the stretch length of the spring.

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

$v_1 > v_2$       $v_1 < v_2$       $v_1 = v_2$

Justify your answer without using equations.

As the force toward the center of the circle increases, so does the acceleration. Therefore, the velocity must also increase.

## Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

~~$F = ma$~~   $F_s = k_0(d)$   $d = d$  distance stretched

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$F = ma_c$   $a_c = \frac{v^2}{R}$   ~~$F = ma_c$~~

$\frac{F}{m} = a_c$   $\frac{F}{m} = \frac{v^2}{R}$   $\sqrt{\frac{FR}{m}} = v$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

Yes  No

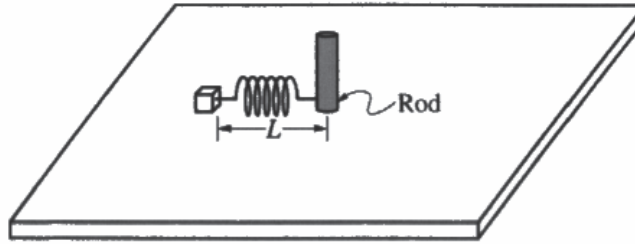
Explain your reasoning.

The centripetal force is increased when acceleration increases causing velocity to increase.



Question 3

Begin your response to QUESTION 3 on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.

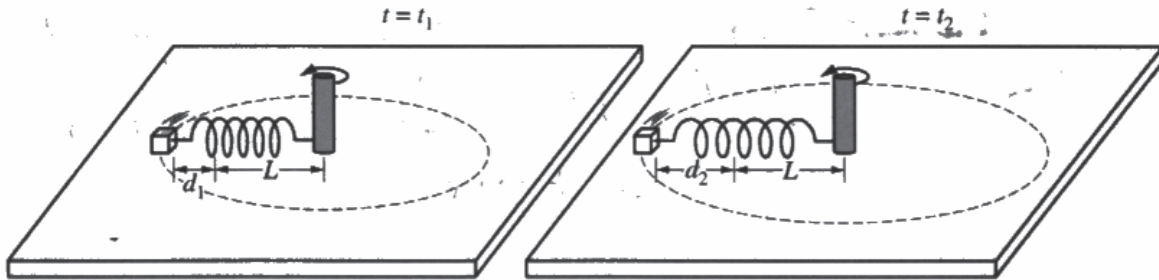


Figure 1

Figure 2

- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

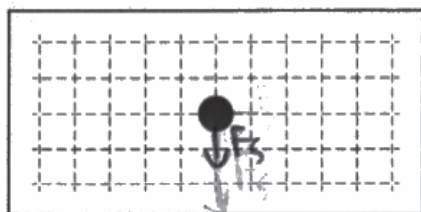


Question 3

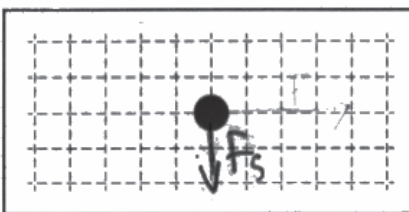
Continue your response to QUESTION 3 on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



$t = t_1$



$t = t_2$

ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

The force from the spring equals the spring constant multiplied by the distance or length of the spring. Since both spring constants are the same, the distance is the main factor in determining force of spring and due to  $d_2$  being greater than  $d_1$ , the force of the spring is greater at  $t = t_2$  than  $t = t_1$  due to the longer arrow in  $t = t_2$ .

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

$v_1 > v_2$       $v_1 < v_2$       $v_1 = v_2$

Justify your answer without using equations.

Due to the greater spring force (centripetal force) in  $t = t_2$ , in order to keep the block in its circular motion, the velocity has to be greater in  $v_2$  than in  $v_1$  because more force is required to keep the block in circular motion.

Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{net}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$\Sigma F_{net} = m_0 a_c$   
 $\Sigma F_s = F_{net}$   
 $|F_s| = v^2/r$   
 $F_{net} = k_0(L+d)$

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$F_s = F_{net}$   
 $F_s = F_{centrifugal}$   
 $F_c = \frac{mv^2}{r}$   
 $r = L+d$   
 $F_s = k_0(L+d)$   
 $\frac{mv^2}{L+d} = k_0(L+d)$   
 $\frac{mv^2}{L+d} = k_0(L+d)$   
 $mv^2 = k_0(L+d)^2$   
 $v^2 = \frac{k_0(L+d)^2}{m_0}$   
 $v = \sqrt{\frac{k_0(L+d)^2}{m_0}}$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

Yes     No

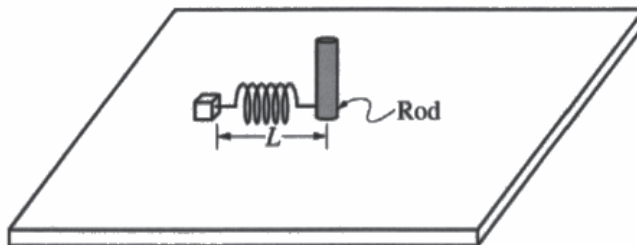
Explain your reasoning.

Due to the mass of the block and spring constant being equal, the radius/distance of the spring is the only constant that determines the tangential speed. Due to the radius being a part of the numerator of the derived equation, the greater the distance (radius)  $(L+d)$  is, the greater the velocity thus making the velocity  $v$  greater than the velocity in part (a) that  $v > v_0$  and that the force of  $F_s$  is greater than the force in part (a).



## Question 3

Begin your response to **QUESTION 3** on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.

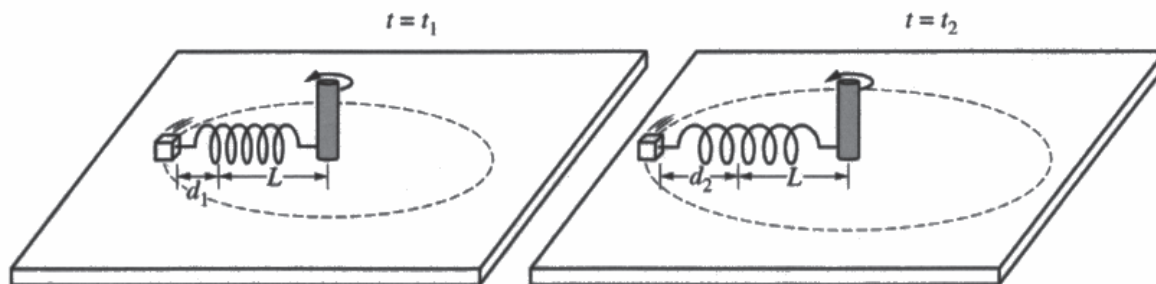


Figure 1

Figure 2

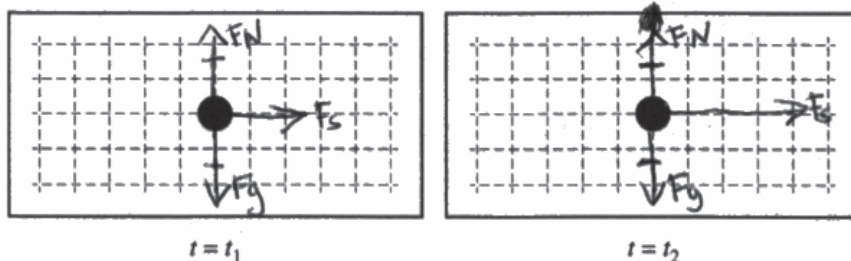
- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

Question 3

Continue your response to QUESTION 3 on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

I drew  $F_s$  for figure 1 half the length of  $F_s$  for figure 2 because the more a spring is stretched the greater the force it exerts on an object. At  $t = t_1$ , the block doesn't stretch the spring as much, only by  $d_1$ . At  $t = t_2$ , the block stretches the spring a distance of  $2d_1 = d_2$ . Using this formula  $F_s = kx$  I knew that an increase in length is directly proportional to an increase in force. Since one stretched a greater distance  $d_2$ , I

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

$v_1 > v_2$       $v_1 < v_2$       $v_1 = v_2$

Justify your answer without using equations.

$v = \omega \cdot r$  shows that radius length is directly proportional to tangential speed. We can assume that they both have the same amount of angular speed so the only differentiating factor is the radius length.  $d_2 + L$  serves as a greater radius for  $t = t_2$  than  $d_1 + L$  for  $t = t_1$ . Since at  $t = t_2$  the radius is bigger because of  $d_2 + L$ , that means the tangential speed is bigger because of this equation:  $v = \omega \cdot r$ .

I knew that it's force had to be greater than the block at  $t = t_1$ .

Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$F_s = k_0 x$$

$$F_s = k_0 (L + d)$$

$$F_{\text{net}} = m_0 a$$

$$k_0 (L + d) = m_0 a$$

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$v = \omega \cdot r$$

$$\omega = \frac{2\pi}{T_s} \quad \omega = \frac{2\pi}{2\pi \sqrt{\frac{m_0}{k_0}}} \Rightarrow \omega = \sqrt{\frac{m_0}{k_0}}$$

$$r = L + d$$

$$v = \sqrt{\frac{m_0}{k_0}} (L + d)$$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

Yes     No

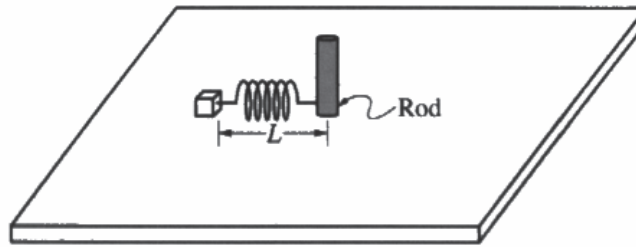
Explain your reasoning.

It does agree because what allocates to  $\omega$  is  $\sqrt{\frac{m_0}{k_0}}$ , which means that the situations have the same angular velocity since they ~~both~~ ~~have~~ blocks have the same mass and the springs have the same spring constant. I was right because I said only the radius will affect tangential velocity. Since  $L + d$  is bigger than  $L$ ,  $v$  will be bigger like I said before. My equation shows that multiplying  $(L + d)$  will increase value of  $v$ . My equation shows that  $\omega = \sqrt{\frac{m_0}{k_0}}$  never changes and  $r$  determines  $v$ .



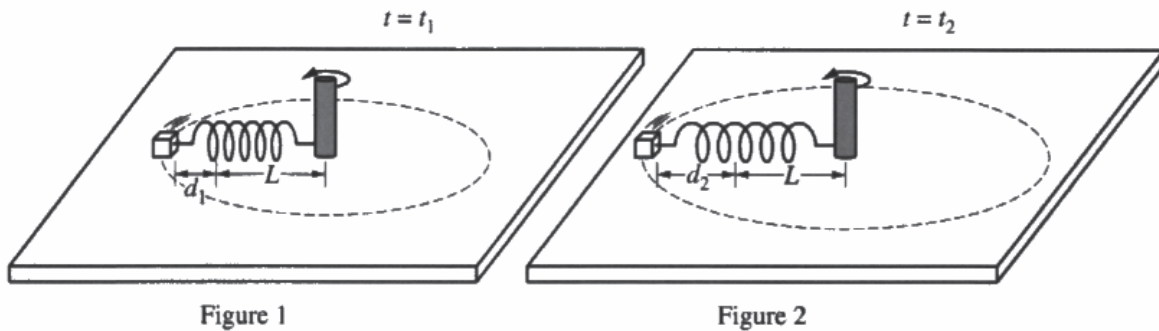
## Question 3

Begin your response to QUESTION 3 on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.



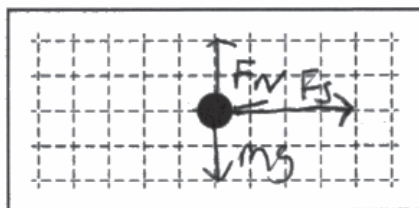
- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

**Question 3**

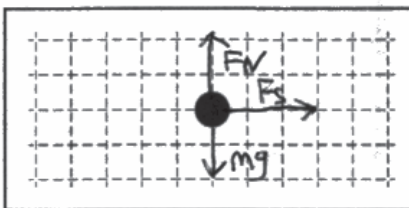
Continue your response to **QUESTION 3** on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



$t = t_1$



$t = t_2$

ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

As the distance grows between the block and the spring force will get weaker and weaker since it is further away from the center

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

$v_1 > v_2$       $v_1 < v_2$       $v_1 = v_2$

Justify your answer without using equations.

Tangential speed ~~is~~ is affected by the distance from the center. The lower the distance the higher the tangential speed



Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$\Sigma F = ma \quad F_{\text{net}} = m_0 \frac{v^2}{r} \quad F_{\text{net}} = \frac{k_0 \cdot F_N}{L+d} m_0 \frac{(k_0 \cdot F_N)^2}{L+d}$$

$$F_{\text{net}} = m_0 \frac{(k_0 \cdot F_N)^2}{L+d}$$

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

~~$v = k_0 \cdot F_N$~~

$$v = k_0 \cdot F_N$$

$$k_0 \cdot F_N = \frac{F_{\text{net}} \cdot (L+d)}{m_0}$$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

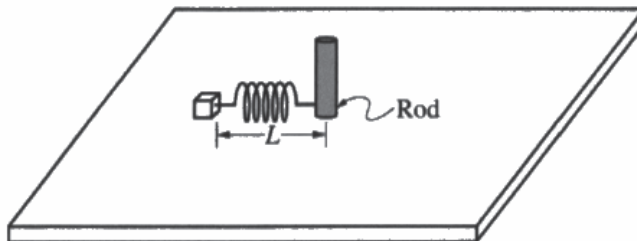
Yes     No

Explain your reasoning.

As  $L+d$  is in the denominator it means that an increase in the distance would actually increase the tangential speed of the block.

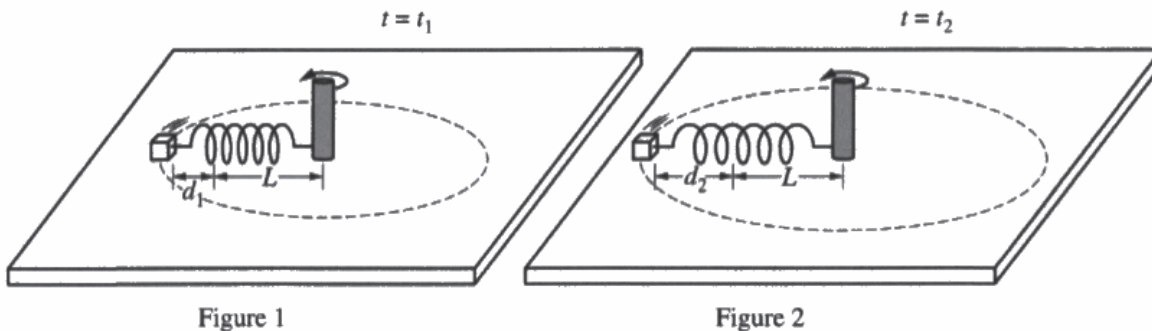
Question 3

Begin your response to QUESTION 3 on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.



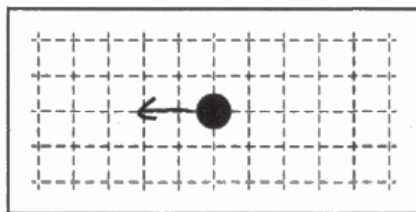
- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

Question 3

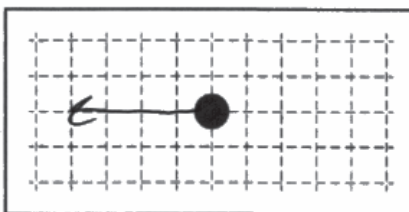
Continue your response to QUESTION 3 on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



$t = t_1$



$t = t_2$

ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

when the length of the spring increases, the force of the spring also increases.

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

$v_1 > v_2$       $v_1 < v_2$       $v_1 = v_2$

Justify your answer without using equations.

The rotational inertia increases as the block moves further from the center, so the velocity decreases, therefore making  $v_2$  less than  $v_1$ .



## Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$\Sigma F = ma_c$$

$$\Sigma F = \frac{m_0 v^2}{L+d}$$

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$\Sigma F = ma_c$$

$$F_s = \frac{m_0 v^2}{L+d}$$

$$k_0(L+d) = \frac{m_0 v^2}{L+d} (L+d)$$

$$\frac{m_0 v^2}{m_0} = \frac{k_0(L+d)^2}{m_0}$$

$$\sqrt{v^2} = \sqrt{\frac{k_0(L+d)^2}{m_0}}$$

$$v = \sqrt{\frac{k_0(L+d)^2}{m_0}}$$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

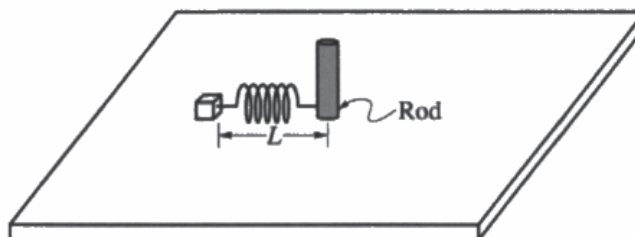
Yes  No

Explain your reasoning.

The distance of the block from the center is in the numerator of the equation, making the velocity greater as the distance increases, rather than smaller.

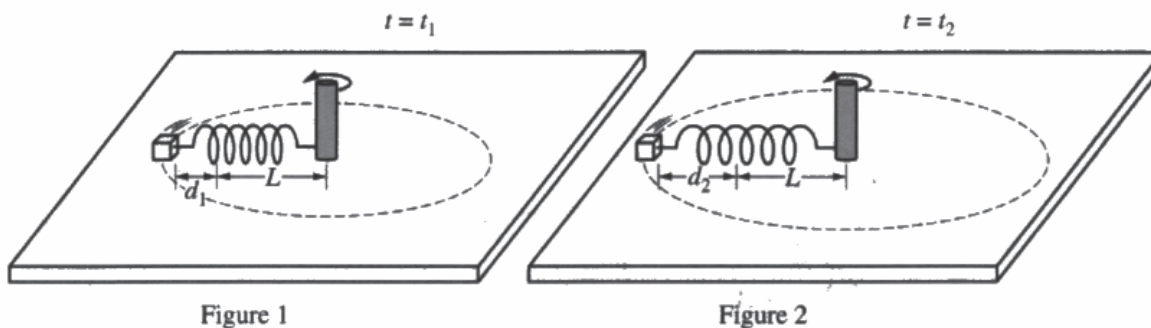
## Question 3

Begin your response to QUESTION 3 on this page.



3. (12 points, suggested time 25 minutes)

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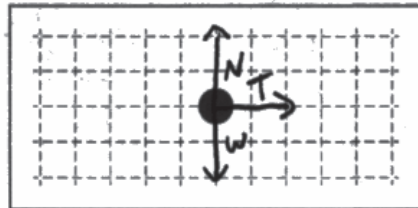
- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

**Question 3**

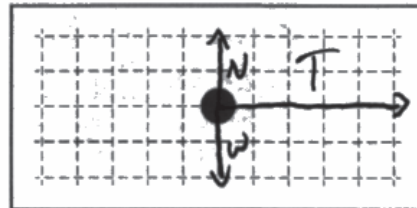
Continue your response to **QUESTION 3** on this page.

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Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



$t = t_1$



$t = t_2$

ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

*I drew the arrow for tension at  $t = t_1$  smaller than  $t = t_2$  because  $d_1$  is smaller at  $t = t_1$ , meaning tension does not have to pull as hard as it does when the block is  $d_2$ .*

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

~~$v_1 > v_2$~~   $v_1 < v_2$   ~~$v_1 = v_2$~~

Justify your answer without using equations.

*The rod is spinning the block faster at  $t_2$  meaning that it has a greater velocity.*



Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$F_{\text{net}} = I \alpha \quad f(L+d) = I \alpha$$

$$F_{\text{net}} = m_0 \cdot \frac{1}{2} k d^2$$

$$f(L) = I \frac{1}{2} k d^2$$

$$f(L+d) = \frac{m_0}{L+d} \cdot \frac{1}{2} k d^2$$

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$R = L+d$$

$$T = \frac{1}{2} k d^2$$

$$v_T = \omega \cdot R$$

$$v_T = \omega \cdot L+d$$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

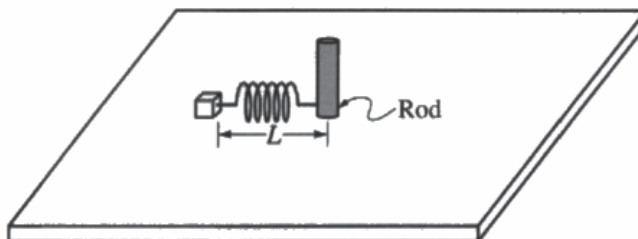
Yes       No

Explain your reasoning.



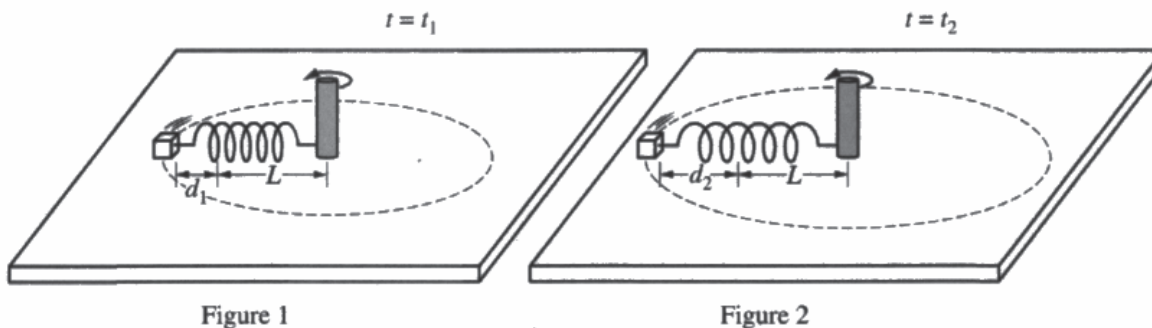
## Question 3

Begin your response to QUESTION 3 on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.



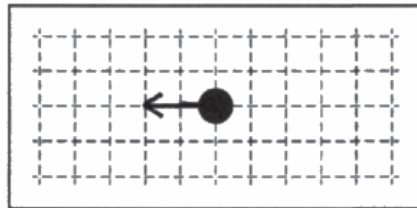
- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

Question 3

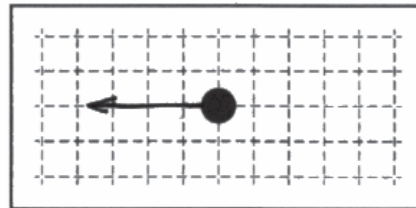
Continue your response to **QUESTION 3** on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



$t = t_1$



$t = t_2$

ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

Since  $d_1$  is smaller than  $d_2$  it means that the spring is not as stretched making it so the the force exerted by the spring on the block is less.

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

\_\_\_\_\_  $v_1 > v_2$     ✓  $v_1 < v_2$     \_\_\_\_\_  $v_1 = v_2$

Justify your answer without using equations.

$v_2$  is greater than  $v_1$  because the block in figure 2 is having to travel a greater distance in the same amount of time as figure 1 meaning that it would have to go faster.

## Question 3

Continue your response to QUESTION 3 on this page.

(b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .

i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$F_{\text{net}} = \frac{2k_0}{(L+d)^2} m_0$$

ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$v = \frac{L+d}{+}$$

(c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

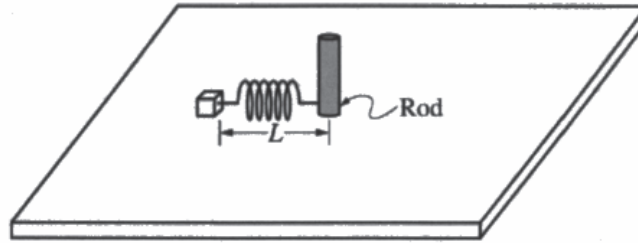
Yes     No

Explain your reasoning.

The equation shows the total radius the block is traveling over time meaning as the distance gets farther  $v$  increases.

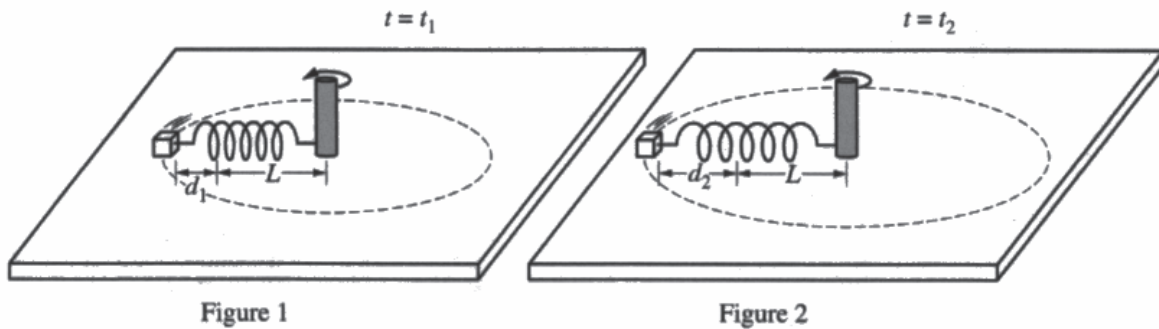
## Question 3

Begin your response to QUESTION 3 on this page.



3. (12 points, suggested time 25 minutes)

A small block of mass  $m_0$  is attached to the end of a spring of spring constant  $k_0$  that is attached to a rod on a horizontal table. The rod is attached to a motor so that the rod can rotate at various speeds about its axis. When the rod is not rotating, the block is at rest and the spring is at its unstretched length  $L$ , as shown. All frictional forces are negligible.



- (a) At time  $t = t_1$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_1$  and the spring is stretched a distance  $d_1$  from the spring's unstretched length, as shown in Figure 1. At time  $t = t_2$ , the rod is spinning such that the block moves in a circular path with a constant tangential speed  $v_2$  and the spring is stretched a distance  $d_2$  from the spring's unstretched length, where  $d_2 > d_1$ , as shown in Figure 2.

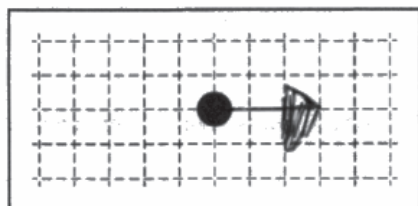


**Question 3**

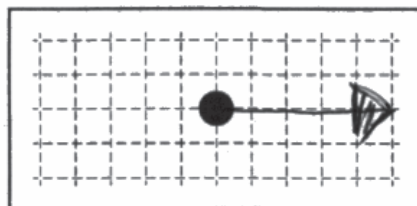
Continue your response to **QUESTION 3** on this page.

i. On the following dots, which represent the block at the locations shown in Figure 1 and Figure 2, draw the force that is exerted on the block by the spring at times  $t = t_1$  and  $t = t_2$ . The spring force must be represented by a distinct arrow starting on, and pointing away from, the dot.

Note: Draw the relative lengths of the vectors to reflect the relative magnitudes of the forces exerted by the spring at both times.



$t = t_1$



$t = t_2$

ii. Referencing  $d_1$  and  $d_2$ , describe your reasoning for drawing the arrows the length that you did in part (a)(i).

$T=t_1$  has a smaller arrow because it is a smaller distance than  $d_2$  therefore making  $T=t_2$  a longer arrow showing

iii. Is the tangential speed  $v_1$  of the block at time  $t = t_1$  greater than, less than, or equal to the tangential speed  $v_2$  of the block at time  $t = t_2$ ?

$v_1 > v_2$       $v_1 < v_2$       $v_1 = v_2$

Justify your answer without using equations.

It is greater because the block is closer to the center of rotation

## Question 3

Continue your response to **QUESTION 3** on this page.

- (b) Consider a scenario where the block travels in a circular path where the spring is stretched a distance  $d$  from its unstretched length  $L$ .
- i. Determine an expression for the magnitude of the net force  $F_{\text{net}}$  exerted on the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$F_{\text{net}} = m_0 \cdot k_0 \cdot d + L$$

- ii. Derive an equation for the tangential speed  $v$  of the block. Express your answer in terms of  $m_0$ ,  $k_0$ ,  $L$ ,  $d$ , and fundamental constants, as appropriate.

$$v = m_0 \cdot d - L$$

- (c) Does your equation for the tangential speed  $v$  of the block from part (b)(ii) agree with your reasoning from part (a)?

Yes     No

Explain your reasoning.

It mentions most criteria used in Part A.

**Question 3****Sample Identifier: P1 Q3 Sample A****Score: 12**

a.i.

- 1 point was earned. The response indicates two arrows drawn to the right.
- 1 point was earned. The response indicates the length of the arrow at  $t_2$  as longer than the arrow at  $t_1$ .

a.ii.

- 1 point was earned. The response states “the spring at  $t_2$  is stretched further and applies a greater force compared to  $t_1$ .” The length relates to the spring force.

a.iii.

- 1 point was earned. The correct selection is checked,  $v_1 < v_2$ , with an attempt to justify the selection.
- 1 point was earned. The response identifies the force as the spring force by stating “The greater force the spring applies to the block,” and indicates the net force as the centripetal force.
- 1 point was earned. The response states “When the tangential velocity of the block is greater, the centripetal acceleration and force will also be greater.”

b.i.

- 1 point was earned. The response correctly identifies the net force expression with the correct variables substituted.

b.ii.

- 1 point was earned. The response attempts to use Newton's second law.
- 1 point was earned. The response correctly substitutes  $L + d$  for the radius.
- 1 point was earned. The correct equation is given.

c.

- 1 point was earned. The response attempts a functional dependence between velocity and distance by stating “If  $v$  increases, so does  $d$ , and vice versa.”
- 1 point was earned. The response correctly relates the equation from part (b) to the reasoning in part (a).



**Sample Identifier: P1 Q3 Sample B**

**Score: 11**

a.i.

- 1 point was earned. The response indicates two arrows drawn to the right.
- 1 point was earned. The response indicates the length of the arrow at  $t_2$  as longer than the arrow at  $t_1$ .

a.ii.

- 1 point was earned. The response states “the further the distance from equilibrium the stronger the force of the spring.” The length is related to the spring force.

a.iii.

- 1 point was earned. The correct selection is checked,  $v_1 < v_2$ , with an attempt to justify the selection.
- 1 point was earned. The response identifies the force as the centripetal force.
- 1 point was earned. The response states “receiving a higher centripetal force at  $t_2$ , then it must be moving at a higher speed.”

b.i.

- 1 point was earned. The response correctly identifies the net force expression with the correct variables substituted.

b.ii.

- 1 point was earned. The response attempts to use Newton's second law to begin the derivation.
- 1 point was earned. The response correctly substitutes  $L + d$  for the radius.
- 0 points were earned. The correct equation is not given.

c.

- 1 point was earned. The response attempts a functional dependence between velocity and distance.
- 1 point was earned. The response correctly relates the equation from part (b) to the reasoning in part (a). The response states “the  $d$  on the top will have a larger impact when it is increased, so the final tangential velocity will be greater if the  $d$  value is increased.”

**Sample Identifier: P1 Q3 Sample C**

**Score: 10**

a.i.

- 1 point was earned. The response indicates two arrows pointing to the right.
- 1 point was earned. The response indicates two arrows where the arrow for  $t_1$  is shorter than the arrow drawn in  $t_2$ .

a.ii.

- 1 point was earned. The response states “ $F_s$  increased so  $t_2$  also increased the stretch length of the spring.”

a.iii.

- 1 point was earned. The response selects the correct choice and attempts a justification.
- 1 point was earned. The response identifies only one force, which is implied to be the spring force from the context of the solution.
- 1 point was earned. The response correctly relates an increase in force to an increase in acceleration and velocity.

b.i.

- 1 point was earned. The response gives a correct expression for the net force.

b.ii.

- 1 point was earned. The response attempts to use Newton's second law to begin the derivation.
- 1 point was earned. The response correctly substitutes  $\frac{v^2}{r}$  for centripetal acceleration.
- 0 points were earned. The correct equation is not given in the response.

c.

- 0 points were earned. The response does not attempt a functional dependence between velocity and stretch length. The response only relates force, acceleration, and velocity.
- 1 point was earned. The response correctly relates the equation in part (b) to support the reasoning in part (a).

**Sample Identifier: P1 Q3 Sample D**

**Score: 9**

a.i.

- 0 points were earned. The response did not indicate two arrows pointing to the right.
- 1 point was earned. The response did draw two arrows where the arrow for  $t_1$  is shorter than the arrow drawn in  $t_2$ .

a.ii.

- 1 point was earned. The response states “the distance is the main factor in determining force of spring.”

a.iii.

- 1 point was earned. The response selects the correct choice and attempts to justify the choice.
- 1 point was earned. The response identifies the spring force as the centripetal force and only refers to that force.
- 1 point was earned. The response states “the velocity has to be greater in  $v_2$  than in  $v_1$  because more force is required.”

b.i.

- 0 points were earned. The response does not give a correct expression for the net force.

b.ii.

- 1 point was earned. The response begins a multistep derivation with Newton's second law.
- 1 point was earned. The response substitutes  $\frac{v^2}{r}$  for the centripetal acceleration.
- 0 points were earned. The equation given in the response is neither correct nor consistent.

c.

- 1 point was earned. The response states “the radius/distance of the spring is the only different factor that determines the tangential speed.”
- 1 point was earned. The response states “Due to the radius being a part of the numerator of the derived equation, the greater the distance (radius) ( $L + d$ ) is, the greater the velocity.”

**Sample Identifier: P1 Q3 Sample E**

**Score: 8**

a.i.

- 1 point was earned. The response draws arrows pointing to the right in both diagrams.
- 1 point was earned. The response indicates the force at  $t_2$  is greater. Extraneous arrows are labeled and correct.

a.ii.

- 1 point was earned. The response states “the more a spring is stretched the greater the force it exerts on an object.”

a.iii.

- 1 point was earned. The response indicates a correct selection and attempts a relevant justification.
- 0 points were earned. The response does not identify the spring force as the net force.
- 0 points were earned. The response does not show a relationship between the spring force and the velocity.

b.i.

- 0 points were earned. The response does not give a correct expression for the net force.

b.ii.

- 1 point was earned. The response attempts to use Newton's second law for a derivation (seen in part (b)(i)).
- 1 point was earned. The response correctly substitutes for the radius.
- 0 points were earned. The correct equation is not given in the response.

c.

- 1 point was earned. The response attempts a functional dependence between velocity and stretch length. The response states “Since  $L + d_2$  is bigger than  $L + d_1$ ,  $V_2$  will be bigger like I said before.”
- 1 point was earned. The response correctly relates the equation in part (b) to support the reasoning in part (a). The response states “My equation shows that multiplying  $(L + d)$  will increase value of  $V$ .”

**Sample Identifier: P1 Q3 Sample F****Score: 7**

a.i.

- 1 point was earned. The response indicates two arrows drawn to the right. Extraneous arrows are labeled and correct.
- 0 points were earned. The response does not indicate the length of the arrow at  $t_2$  as longer than the arrow at  $t_1$ .

a.ii.

- 1 point was earned. The response states “As the distance grows between the block and  $L$  the spring force will get weaker and weaker.” The length is related to the spring force. This is consistent with arrows drawn in part (a)(i).

a.iii.

- 1 point was earned. The correct selection is checked,  $v_1 < v_2$ , with an attempt to justify the selection.
- 0 points were earned. The response does not identify the net force as the spring force.
- 0 points were earned. The response does not correctly relate any relationship between force and velocity.

b.i.

- 0 points were earned. The response does not have the net force expression with the correct variables substituted.

b.ii.

- 1 point was earned. The response attempts to use Newton's second law to begin the equation as seen in part (b)(i).
- 1 point was earned. The response correctly substitutes centripetal acceleration as  $\frac{v^2}{r}$ .
- 0 points were earned. The correct equation is not given.

c.

- 1 point was earned. The response attempts a functional dependence between velocity and distance by stating “an increase in the distance would actually increase the tangential speed of the block.”
- 1 point was earned. The response correctly relates the equation from part (b) to the reasoning in part (a). The response states “As  $L + d$  is in the numerator it means that an increase in the distance would actually increase the tangential speed.”

**Sample Identifier: P1 Q3 Sample G**

**Score: 7**

a.i.

- 0 points were earned. The response does not indicate two arrows pointing to the right.
- 1 point was earned. The response indicates two arrows where the arrow for  $t_1$  is shorter than the arrow drawn in  $t_2$ .

a.ii.

- 1 point was earned. The response states “when the length of the spring increases, the force of the spring also increases.”

a.iii.

- 0 points were earned. The response does not select the correct choice.
- 0 points were earned. The response does not identify the spring force as the net force.
- 0 point were earned. The response does indicate a relationship between the spring force and the velocity.

b.i.

- 0 points were earned. The response does not give a correct expression for the net force.

b.ii.

- 1 point was earned. The response uses Newton's second law to begin the multistep derivation.
- 1 point was earned. The response correctly substitutes  $(L + d)$  for the radius.
- 1 point was earned. The response is consistent with the expression for spring force in (b)(i) and correctly substitutes into the derived equation.

c.

- 1 point was earned. The response states “making the velocity greater as the distance increases.”
- 1 point was earned. The response correctly relates the equation in part (b) to support the reasoning in part (a). The response states “The distance of the block from the center is in the numerator of the equation, making the velocity greater as the distance increases.”



**Sample Identifier: P1 Q3 Sample H**

**Score: 5**

a.i.

- 1 point was earned. The response indicates two arrows pointing to the right.
- 1 point was earned. The response indicates the rightward arrow at  $t_2$  is greater than  $t_1$ . Extraneous forces are labeled and correct.

a.ii.

- 1 point was earned. The response states “I drew the arrow for tension at  $t = t_1$  smaller than  $t = t_2$  because  $d_1$  is smaller.” The force (tension) is related to the stretch.

a.iii.

- 1 point was earned. The correct response is indicated, and an attempt at a relevant justification is made.
- 0 points were earned. The response does not identify the spring force as the net force.
- 0 points were earned. The response does not show a relationship between the spring force and the velocity.

b.i.

- 0 points were earned. The response does not give a correct expression for the net force.

b.ii.

- 0 points were earned. The response does not attempt to use Newton's second law.
- 1 point was earned. The response correctly substitutes  $L + d$  for the radius.
- 0 points were earned. The correct equation is not given in the response.

c.

- 0 points were earned. The response does not attempt a functional dependence between velocity and stretch length.
- 0 points were earned. The response does not attempt to relate the equation in part (b) to support the reasoning in part (a).

**Sample Identifier: P1 Q3 Sample I**

**Score: 5**

a.i.

- 0 points were earned. The response does not indicate two arrows pointing to the right.
- 1 point was earned. The response indicates two arrows where the arrow for  $t_1$  is shorter than the arrow drawn in  $t_2$ .

a.ii.

- 1 point was earned. The response states “the spring is not as stretched making it so the the force exerted by the spring on the block is less.”

a.iii.

- 1 point was earned. The response indicates the correct selection, with an attempt at a relevant justification.
- 0 points were earned. The response does not identify the spring force as the net force.
- 0 point were earned. The response does not show a relationship between the spring force and the velocity.

b.i.

- 0 points were earned. The response does not give a correct expression for the net force.

b.ii.

- 0 points were earned. The response does not attempt to use Newton's second law.
- 0 points were earned. The response does not have any correct substitutions for the spring force, centripetal acceleration, or the radius.
- 0 points were earned. The correct equation is not given in the response.

c.

- 1 point was earned. The response does attempt a functional dependence between velocity and stretch length. The response states “as the distance gets farther  $v$  increases”
- 1 point was earned. The response correctly relates the equation in part (b) to support the reasoning in part (a).

**Sample Identifier: P1 Q3 Sample J**

**Score: 3**

a.i.

- 1 point was earned. The response indicates two arrows pointing to the right.
- 1 point was earned. The response indicates two arrows where the arrow for  $t_1$  is shorter than the arrow drawn in  $t_2$ .

a.ii.

- 1 point was earned. The response states “ $T = t_1$  has a smaller arrow because  $d_1$  is a smaller distance.” The response indicates that the force (the arrow) is related to the stretch length ( $d_1$ ).

a.iii.

- 0 points were earned. The response does not select the correct choice.
- 0 points were earned. The response does not identify the spring force as the net force.
- 0 point were earned. The response does not show a relationship between the spring force and the velocity.

b.i.

- 0 points were earned. The response does not give a correct expression for the net force.

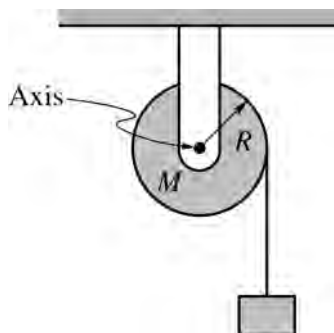
b.ii.

- 0 points were earned. The response does not attempt to use Newton's second law.
- 0 points were earned. The response does not have correct substitutions for the spring force, centripetal acceleration, or the radius.
- 0 points were earned. The correct equation is not given in the response.

c.

- 0 points were earned. The response does not attempt a functional dependence between velocity and stretch length.
- 0 points were earned. The response does not correctly relate the equation in part (b) to support the reasoning in part (a).

Begin your response to **QUESTION 4** on this page.



4. (7 points, suggested time 13 minutes)

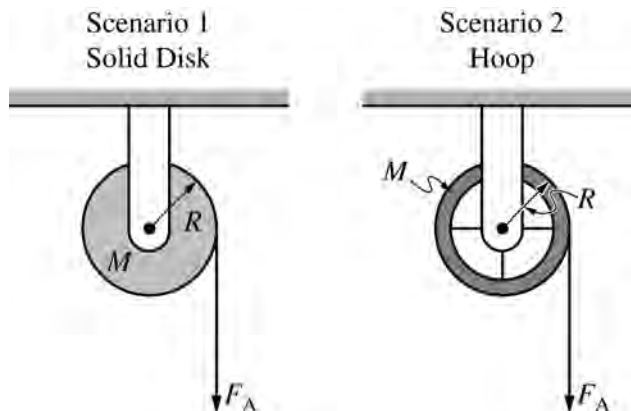
A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass  $M$  and radius  $R$ . The rotational inertia of the pulley is described by the equation  $I = \frac{1}{2}MR^2$ . The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is  $F_T$ .

(a) Determine an expression for the magnitude of the angular acceleration  $\alpha_D$  of the disk as the block travels downward. Express your answer in terms of  $M$ ,  $R$ ,  $F_T$ , and physical constants as appropriate.

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 4** on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass  $M$  and radius  $R$  as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force  $F_A$  for the same time interval  $\Delta t$ , causing the pulleys to rotate without the string slipping. After time interval  $\Delta t$ , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

- (b) Consider scenarios 1 and 2 at the end of time interval  $\Delta t$ . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

**GO ON TO THE NEXT PAGE.**

**Question 4: Short Answer Paragraph Argument****7 points**

- 
- (a) For a correct expression for the angular acceleration of the pulley in terms of the **1 point**

appropriate quantities:  $\alpha_D = \frac{2F_T}{MR}$

---

**Example Response**

$$\alpha_D = \frac{RF_T}{\frac{1}{2}MR^2} \quad \text{OR} \quad \alpha_D = \frac{2F_T}{MR}$$

---

**Total for part (a) 1 point**

---



(b)	For indicating that the torque, $\tau$ , is the same for both pulleys	<b>1 point</b>
	For indicating that the impulse, $\tau\Delta t$ , (or change in momentum $\Delta L$ ) is the same for both pulleys because $\tau$ and $\Delta t$ are the same	<b>1 point</b>
	For indicating that the rotational inertia, $I$ , of the disk and hoop are different	<b>1 point</b>
	For providing reasoning that because the rotational inertia, $I$ , are different for the disk and hoop, the kinematic quantities ( $\Delta\theta$ , $\omega$ , $\alpha$ ) are also different for the disk and hoop	<b>1 point</b>
	For <b>one</b> of the following:	<b>1 point</b>
	<ul style="list-style-type: none"> <li>• Relating <math>I</math> and <math>\omega</math> to reason that <math>\Delta K</math> is greater for the disk</li> <li>• Indicating that because <math>\Delta\theta</math> is greater for the disk the work done on the disk is greater</li> </ul>	
	For a logical, relevant, and internally consistent argument that follows the guidelines described in the published requirements for the paragraph-length response	<b>1 point</b>

### Example Response

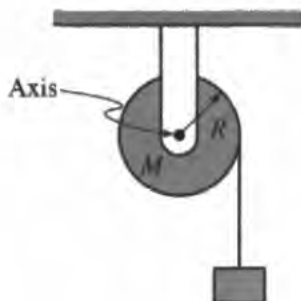
*The rotational inertia,  $I$ , of the hoop is larger than the rotational inertia of the disk because the hoop's mass is all on the outside instead of distributed throughout like the disk. Equal forces are applied to both pulleys at the same distance, which means that the torques exerted on the pulleys will also be equal. Since the same torque is applied to both pulleys for the same time period, the change in angular momentum will be the same for the disk and hoop. The magnitude of the angular velocity for the hoop will be smaller than that of the disk since angular velocity is inversely proportional to the rotational inertia ( $\omega = \frac{L}{I}$ ). Since kinetic energy is proportional to rotational inertia and the square of angular velocity ( $K_R = \frac{1}{2}I\omega^2$ ), the difference in angular velocity more greatly affects the rotational kinetic energy. That means the disk will have a greater rotational kinetic energy than the hoop.*

**Total for part (b) 6 points**

**Total for question 4 7 points**

Question 4

Begin your response to QUESTION 4 on this page.



4. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass  $M$  and radius  $R$ . The rotational inertia of the pulley is described by the equation  $I = \frac{1}{2}MR^2$ . The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is  $F_T$ .

(a) Determine an expression for the magnitude of the angular acceleration  $\alpha_D$  of the disk as the block travels downward. Express your answer in terms of  $M$ ,  $R$ ,  $F_T$ , and physical constants as appropriate.

$$\alpha = \frac{T_{net}}{I}$$

$$\alpha = \frac{r \sin \theta}{\frac{1}{2}MR^2}$$

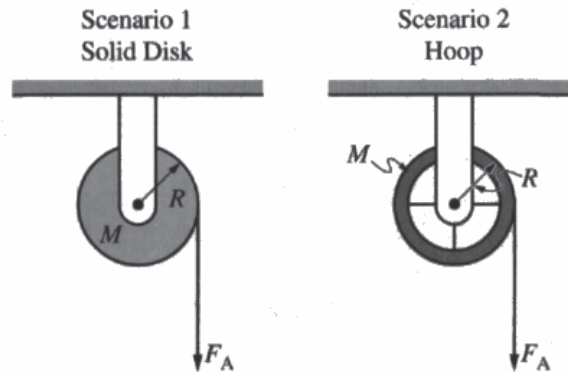
$$\alpha = \frac{2F_T \sin 90}{MR}$$

$$\alpha = \frac{2F_T}{MR}$$



## Question 4

Continue your response to QUESTION 4 on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass  $M$  and radius  $R$  as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

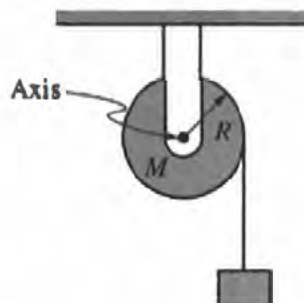
In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force  $F_A$  for the same time interval  $\Delta t$ , causing the pulleys to rotate without the string slipping. After time interval  $\Delta t$ , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

- (b) Consider scenarios 1 and 2 at the end of time interval  $\Delta t$ . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

The change in ang. momentum is the same because the  $T_{net}$  and  $t$  are the same and  $T_{net} t = \text{change in ang. momentum}$ .  $K_R$  is greater for the disk because the hoop has a greater  $I$ , so a lower  $\omega$ . And  $\omega$  rotates more in the equation for  $K_R$  because  $\omega$  is squared.

## Question 4

Begin your response to **QUESTION 4** on this page.



4. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass  $M$  and radius  $R$ . The rotational inertia of the pulley is described by the equation  $I = \frac{1}{2}MR^2$ . The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is  $F_T$ .

(a) Determine an expression for the magnitude of the angular acceleration  $\alpha_D$  of the disk as the block travels downward. Express your answer in terms of  $M$ ,  $R$ ,  $F_T$ , and physical constants as appropriate.

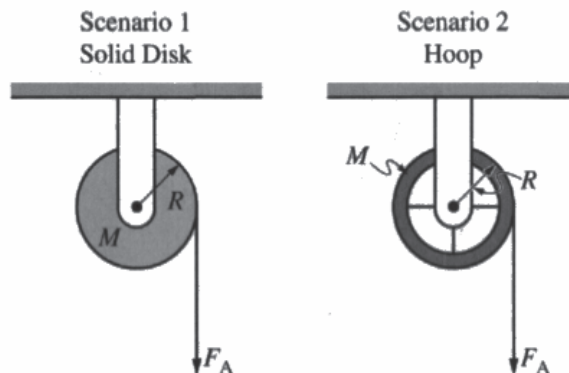
$$\alpha_D = \frac{R F_T}{\frac{1}{2} M R^2}$$

$$\alpha_D = \frac{F_T}{\frac{1}{2} M R}$$



Question 4

Continue your response to QUESTION 4 on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass  $M$  and radius  $R$  as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

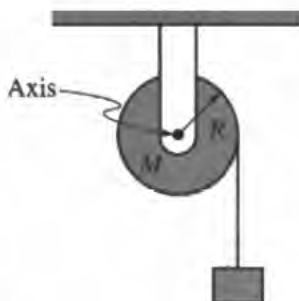
In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force  $F_A$  for the same time interval  $\Delta t$ , causing the pulleys to rotate without the string slipping. After time interval  $\Delta t$ , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

- (b) Consider scenarios 1 and 2 at the end of time interval  $\Delta t$ . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

Because <sup>change in</sup> angular momentum is determined by the equation  $\Delta L = \tau \Delta t$  and torque and change in time are constant in both scenarios. However inertia is not because in a disk  $I = \frac{1}{2} MR^2$  but a hoop  $I = MR^2$  and since both have equal torques ~~in~~ scenario 2's angular acceleration is going to be smaller gives it's higher inertia due to the equation  $\alpha = \frac{\tau_{net}}{I}$  and so scenario 1 would have a greater angular velocity ~~at~~ at the end of time interval  $\Delta t$  thus giving it higher <sup>rotational</sup> kinetic energy with the equation  $K = \frac{1}{2} I \omega^2$

## Question 4

Begin your response to **QUESTION 4** on this page.



4. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass  $M$  and radius  $R$ . The rotational inertia of the pulley is described by the equation  $I = \frac{1}{2}MR^2$ . The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

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(a) Determine an expression for the magnitude of the angular acceleration  $\alpha_D$  of the disk as the block travels downward. Express your answer in terms of  $M$ ,  $R$ ,  $F_T$ , and physical constants as appropriate.

$$\alpha_D = \frac{R F_T \sin \theta}{\frac{1}{2} M R^2}$$

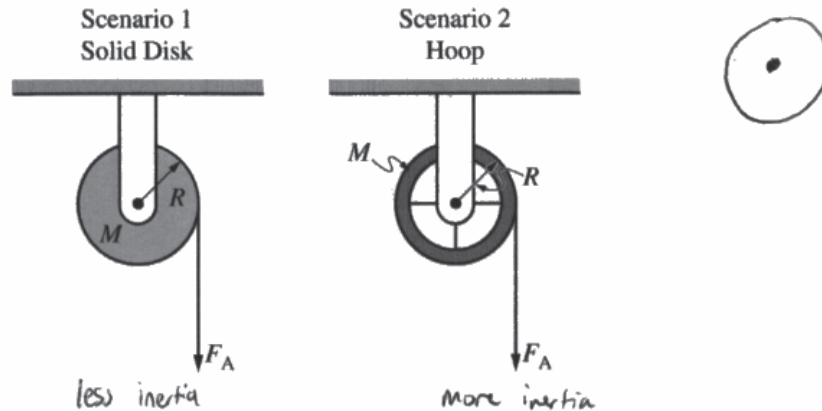
$$= \frac{2 F_T \sin \theta}{M R}$$





Question 4

Continue your response to QUESTION 4 on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass  $M$  and radius  $R$  as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

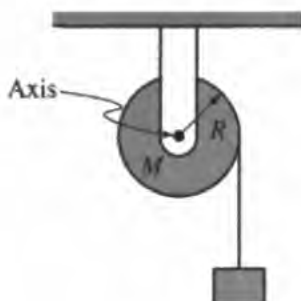
In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force  $F_A$  for the same time interval  $\Delta t$ , causing the pulleys to rotate without the string slipping. After time interval  $\Delta t$ , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

(b) Consider scenarios 1 and 2 at the end of time interval  $\Delta t$ . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

Key  
Answers ↓  
Considering that both pulleys with the hoop & disk have the same mass  $M$  & radius  $R$ , there's a reason why they have the same change in angular momentum but the disk has a greater change in  $K_{rot}$ . The reason for the same change in angular momentum is that although the disk has less inertia than the hoop (because the mass is closer to the center of mass than the hoop), the pulley is going to have a faster  $\omega$  for the disk than the hoop simply because it has less inertia & it will accelerate faster. And since the  $\omega$  is less for the hoop, it has an equal momentum to the disk because of its greater inertia. The reason why the  $K_{rot}$  for the disk is greater than the hoop is because  $K_{rot} = \frac{1}{2} I \omega^2$ , meaning that a difference between the two's angular speeds will cause a significant difference in  $K_{rot}$ . But in the situation where we were comparing the ~~rotational~~ angular momentums, where  $L = I \omega$ , nothing was squared so it would lead to them being equal if no significant difference in momentums between the pulley with the disk & the one with the hoop.

## Question 4

Begin your response to **QUESTION 4** on this page.



4. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass  $M$  and radius  $R$ . The rotational inertia of the pulley is described by the equation  $I = \frac{1}{2}MR^2$ . The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is  $F_T$ .

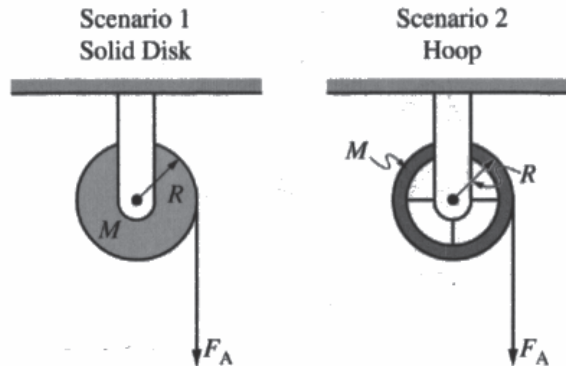
(a) Determine an expression for the magnitude of the angular acceleration  $\alpha_D$  of the disk as the block travels downward. Express your answer in terms of  $M$ ,  $R$ ,  $F_T$ , and physical constants as appropriate.

$$a = \frac{T_{\text{net}}}{I}$$

$$a = \frac{F_T}{\frac{1}{2}MR^2}$$

Question 4

Continue your response to QUESTION 4 on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass  $M$  and radius  $R$  as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force  $F_A$  for the same time interval  $\Delta t$ , causing the pulleys to rotate without the string slipping. After time interval  $\Delta t$ , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

- (b) Consider scenarios 1 and 2 at the end of time interval  $\Delta t$ . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

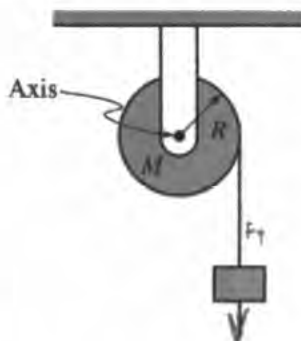
$AL = T \cdot \Delta t$   
 $\Delta L = F \cdot R \cdot T$

~~is same~~

Both pulleys have the same torque acting in the same direction. This is because the force of  $F_A$  is equal, and the radius  $R$  from the pivot is equal. Since they have the same torque for the same amount of time, they have the same change in angular momentum. However, rotational kinetic energy is determined on mass. Although everything else stays constant, the hoop has less mass than the disk. Therefore, the disk will have more rotational kinetic energy.

## Question 4

Begin your response to QUESTION 4 on this page.



4. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass  $M$  and radius  $R$ . The rotational inertia of the pulley is described by the equation  $I = \frac{1}{2}MR^2$ . The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

When the block is released from rest and as the block travels toward the ground, the magnitude of the tension exerted on the block by the string is  $F_T$ .

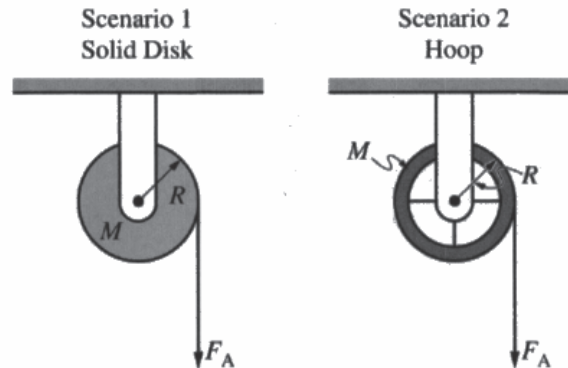
(a) Determine an expression for the magnitude of the angular acceleration  $\alpha_D$  of the disk as the block travels downward. Express your answer in terms of  $M$ ,  $R$ ,  $F_T$ , and physical constants as appropriate.

$$\frac{RF_T}{\frac{1}{2}MR^2} = \alpha_D = \frac{F_T}{\frac{1}{2}MR}$$



## Question 4

Continue your response to QUESTION 4 on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass  $M$  and radius  $R$  as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

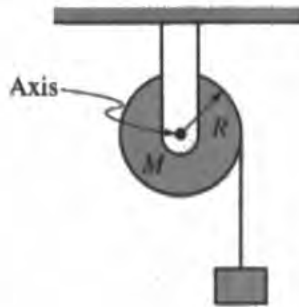
In both scenarios, the pulleys begin at rest. Then both strings are pulled with the same constant force  $F_A$  for the same time interval  $\Delta t$ , causing the pulleys to rotate without the string slipping. After time interval  $\Delta t$ , the change in angular momentum of the disk is equal to the change in angular momentum of the hoop, but the change in rotational kinetic energy for the disk is greater than that of the hoop.

- (b) Consider scenarios 1 and 2 at the end of time interval  $\Delta t$ . In a clear, coherent paragraph-length response that may also contain equations and drawings, explain why the change in angular momentum of both pulleys is the same but the change in rotational kinetic energy is greater for the disk.

The change in angular momentum is the same because they both have the same values for  $M$ ,  $R$ , and  $F_A$ . The change in rotational kinetic energy is greater for the disk because it has a lower rotational inertia than the hoop.

## Question 4

Begin your response to **QUESTION 4** on this page.



4. (7 points, suggested time 13 minutes)

A block of unknown mass is attached to a long, lightweight string that is wrapped several turns around a pulley mounted on a horizontal axis through its center, as shown. The pulley is a uniform solid disk of mass  $M$  and radius  $R$ . The rotational inertia of the pulley is described by the equation  $I = \frac{1}{2}MR^2$ . The pulley can rotate about its center with negligible friction. The string does not slip on the pulley as the block falls.

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(a) Determine an expression for the magnitude of the angular acceleration  $\alpha_D$  of the disk as the block travels downward. Express your answer in terms of  $M$ ,  $R$ ,  $F_T$ , and physical constants as appropriate.

$$I = \frac{1}{2}MR^2 \quad \alpha_D \quad \sum \tau = F_T R$$

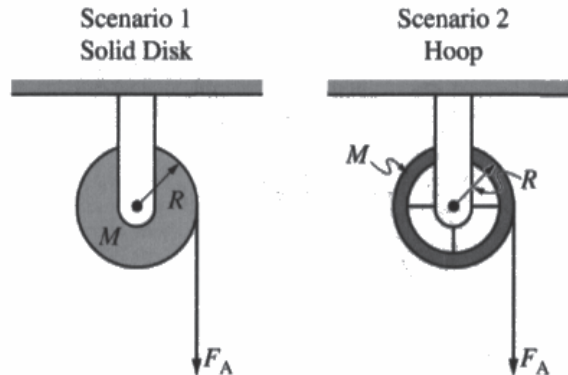
$$\alpha_D = \frac{\sum \tau}{I} = \frac{F_T R}{\frac{1}{2}MR^2} = \frac{2F_T R}{MR^2} = \frac{2F_T}{MR}$$

$$\boxed{\alpha_D = \frac{2F_T}{MR}}$$



## Question 4

Continue your response to QUESTION 4 on this page.



Scenarios 1 and 2 show two different pulleys. In Scenario 1, the pulley is the same solid disk referenced in part (a). In Scenario 2, the pulley is a hoop that has the same mass  $M$  and radius  $R$  as the disk. Each pulley has a lightweight string wrapped around it several turns and is mounted on a horizontal axle, as shown. Each pulley is free to rotate about its center with negligible friction.

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The change in angular momentum of both pulleys is the same (solid disk & hoop), but the change in rotational KE is greater for disk b/c  $\Delta L = I\omega$  which means that rotational Inertia  $\times$  angular speed is the same  $\therefore$  isn't affected b/c it is same in moment but when talking about rot. KE which has the  $K = \frac{1}{2} I\omega^2 \rightarrow$  since change is:  $\Delta K = \frac{1}{2} I\omega^2$  when it gets more specific of it cause of KE, there is a  $\frac{1}{2}$  multiplied by the rotational Inertia multiplied by angular speed squared, which more specific  $\therefore$  since scenario 2 has a hoop it will slow down the rotational Kinetic Energy

## Question 4

### Sample Identifier: P1 Q4 Sample A

Score: 7

a.

- 1 point was earned. The response correctly determines an expression for  $\alpha$  in terms of appropriate variables.

b.

- 1 point was earned. The response correctly indicates that the torque was the same for both pulleys.
- 1 point was earned. The response correctly connects torque and time to the same change in angular momentum.
- 1 point was earned. The response indicates that the hoop and the disk have different rotational inertias.
- 1 point was earned. The response relates rotational inertia to an angular kinematic quantity change.
- 1 point was earned. The response correctly justifies why the change in the kinetic energy of the disk is greater than the change in kinetic energy of the hoop. The response includes an explanation that changes in angular speed affect the kinetic energy more than changes in rotational inertia.
- 1 point was earned. The response includes a logical, relevant, and internally consistent argument.

**Sample Identifier: P1 Q4 Sample B**

**Score: 6**

a.

- 1 point was earned. The response correctly determines an expression for  $\alpha$  in terms of appropriate variables.

b.

- 1 point was earned. The response correctly indicates that the torque was the same for both pulleys.
- 1 point was earned. The response correctly connects torque and time to the same change in angular momentum.
- 1 point was earned. The response indicates that the hoop and the disk have different rotational inertias.
- 1 point was earned. The response relates rotational inertia to an angular kinematic quantity change.
- 0 points were earned. The response states that the change in the kinetic energy of the disk is greater than the change in kinetic energy of the hoop but does not include an explanation that changes in angular speed affect the kinetic energy more than changes in rotational inertia.
- 1 point was earned. The response includes a logical, relevant, and internally consistent argument.

**Sample Identifier: P1 Q4 Sample C**

**Score: 4**

a.

- 0 points were earned. While the response includes an expression for  $\alpha$  the response also includes an unknown quantity  $\theta$ .

b.

- 0 points were earned. The response does not include any comparison of torque for the two pulleys.
- 0 points was earned. The response does not connect either torque and time or impulse to the change in angular momentum.
- 1 point was earned. The response indicates that the hoop and the disk have different rotational inertias.
- 1 point was earned. The response relates rotational inertia to an angular kinematic quantity change.
- 1 point was earned. The response correctly justifies why the change in the kinetic energy of the disk is greater than the change in kinetic energy of the hoop. The response includes an explanation that changes in angular speed affect the kinetic energy more than changes in rotational inertia.
- 1 point was earned. The response includes a logical, relevant, and internally consistent argument.

**Sample Identifier: P1 Q4 Sample D**

**Score: 3**

a.

- 0 points were earned. The response incorrectly determines an expression for  $\alpha$ .

b.

- 1 point was earned. The response correctly indicates that the torque was the same for both pulleys.
- 1 point was earned. The response correctly connects torque and time to the same change in angular momentum.
- 0 points were earned. The response indicates that the hoop and the disk have different masses not different rotational inertias.
- 0 points were earned. The response does not relate rotational inertia to an angular kinematic quantity change.
- 0 points were earned. The response incorrectly justifies why the change in the kinetic energy of the disk is greater than the change in kinetic energy of the hoop.
- 1 point was earned. The response includes a logical, relevant, and internally consistent argument.

**Sample Identifier: P1 Q4 Sample E**

**Score: 2**

a.

- 1 point was earned. The response correctly determines an expression for  $\alpha$  in terms of appropriate variables.

b.

- 0 points were earned. The response did not include any comparison of torque for the two pulleys.
- 0 points were earned. The response states given information that the change of angular momentum of the two scenarios are the same but did not connect it to torque and time or impulse.
- 1 point was earned. The response indicates that the hoop and the disk have different rotational inertias.
- 0 points were earned. The response does not relate rotational inertia to an angular kinematic quantity change.
- 0 points were earned. The response mentions there is a change of kinetic energy but incorrectly justifies why the change in the kinetic energy of the disk is greater than the change in kinetic energy of the hoop.
- 0 points were earned. The argument is not complete and does not have logically consistent statements.



**Sample Identifier: P1 Q4 Sample F**

**Score: 1**

- a.
- 0 points were earned. The response incorrectly determines an expression for  $\alpha$ .
- b.
- 0 point were earned. The response does not indicate that the torque was the same for both pulleys. There is no mention of torque in the response.
  - 0 points were earned. The response does not connect the same torque and time (angular impulse) to the same change in angular momentum.
  - 0 points were earned. The response does not indicate that the hoop and the disk have different rotational inertias. While the response does mention that rotational inertia multiplied by the angular speed is equal for both disk and hoop, the response does not specify that rotational inertia and angular speed are different for both the disk and hoop.
  - 0 points were earned. The response does not relate rotational inertia to an angular kinematic quantity change. While the response does mention that rotational inertia multiplied by the angular speed is equal for both disk and hoop, the response does not specify that an angular kinematic quantity changes due to a change in rotational inertia.
  - 0 points were earned. The response does not justify why the change in the kinetic energy of the disk is greater than the change in kinetic energy of the hoop. While the response gives the equation for rotational kinetic energy, the response does not explain why the changes in kinetic energy are not the same.
  - 1 point was earned. Although incorrect, the response includes a relevant, and internally consistent argument.

Begin your response to **QUESTION 5** on this page.

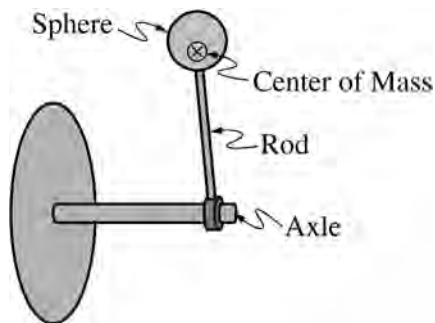


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a  $\otimes$ , as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

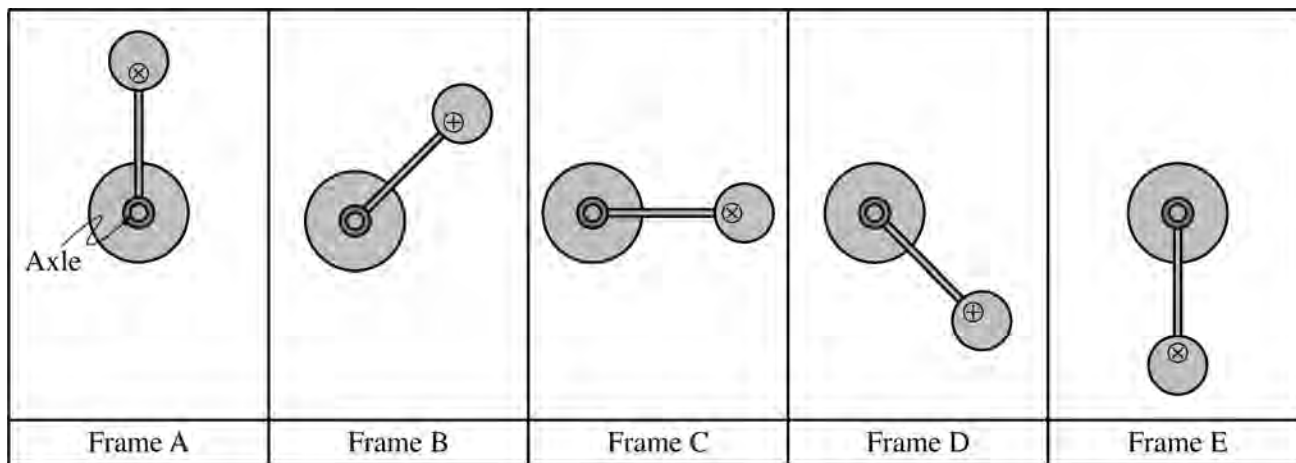


Figure 2

**GO ON TO THE NEXT PAGE.**

Continue your response to **QUESTION 5** on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

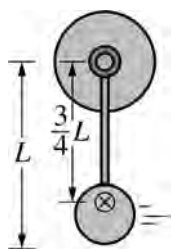


Figure 3

(b) The rod-sphere system has mass  $M$  and length  $L$ , and the center of mass is located a distance  $\frac{3}{4}L$  from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of  $M$ ,  $L$ , and fundamental constants, as appropriate.

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

**GO ON TO THE NEXT PAGE.**

**Question 5: Short Answer****7 points**

- 
- (a)(i) For indicating “Frame C” with correct reasoning about the magnitude of the torque being the greatest **1 point**

Accept **one** of the following:

- This is the instant when the lever arm is greatest
- This is when the angle between radius vector and weight force vector is most perpendicular

---

For correctly relating torque and angular acceleration:  $\alpha \propto \tau$  **1 point**

**Example Response**

*The angular acceleration is greatest in Frame C because angular acceleration is proportional to torque, and in Frame C the gravitational force vector is directed perpendicular to the rod (lever arm) which means this is where the torque will be the greatest.*

- 
- (a)(ii) For indicating “Frame E” with correct reasoning **1 point**

Accept **one** of the following:

- Work or energy (e.g., this is when the maximum work has been done on the system by gravity)
- Angular momentum (e.g., the torque due to gravity is clockwise the entire time, causing the rod to gain angular momentum)
- Kinematics (e.g., the rod speeds up the entire time)

**Example Response**

*The rotational kinetic energy is greatest in Frame E because this is where the rod-sphere system has the greatest rotational speed since the torque has been in the same direction as the motion the entire time.*

---

**Total for part (a) 3 points**

---

---

**(b)(i)** For a multistep derivation that begins with conservation of energy **1 point**

$$E_i = E_f \quad \text{OR} \quad \Delta E = 0 \quad \text{OR} \quad U_{gi} + K_i = U_{gf} + K_f$$

---

For indicating the change in height is equal to  $\frac{3}{2}L$  **1 point**

$$\Delta y = \frac{3}{2}L$$

---

For an answer consistent with the height change indicated previously in the response **1 point**

$$K_f = \frac{3}{2}MgL$$

**Scoring Note:** A correct answer of  $K_f = \frac{3}{2}MgL$  with no supporting work can earn only this point.

---

**Example Response**

$$E_i = E_f$$

$$U_{gi} + K_i = U_{gf} + K_f$$

$$\Delta K = U_{gi} - U_{gf}$$

$$\Delta K = Mg\Delta y$$

$$\Delta y = \frac{3L}{4} + \frac{3L}{4} = \frac{3}{2}L$$

$$\Delta K = \frac{3}{2}MgL$$

---

**(b)(ii)** For indicating that the gravitational force is the external force that does work on the rod-sphere system **1 point**

---

**Example Response**

*The rod and sphere gain kinetic energy due to the positive work done by the gravitational force, which is an external force for the rod-sphere system.*

---

**Total for part (b) 4 points**

---

**Total for question 5 7 points**

Question 5

Begin your response to QUESTION 5 on this page.

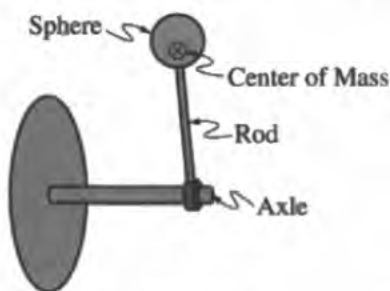


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a  $\otimes$ , as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

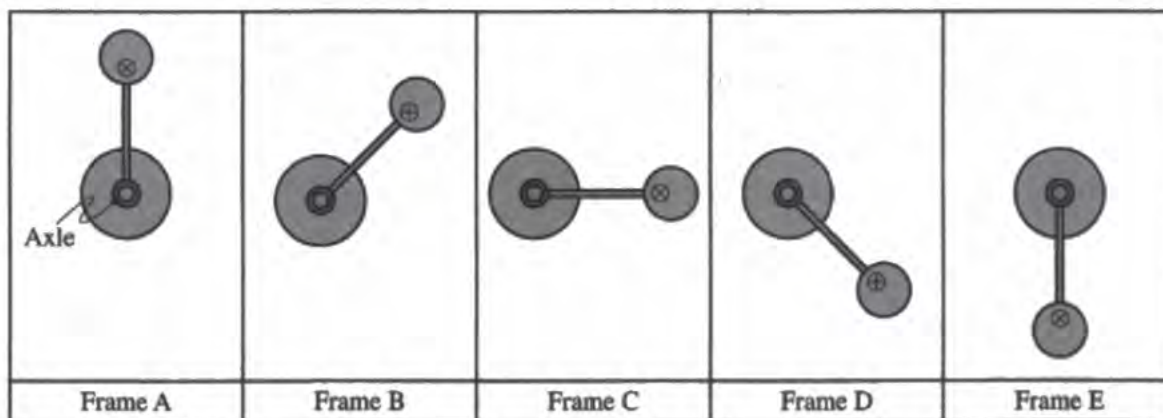


Figure 2



Question 5

Continue your response to QUESTION 5 on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

Frame C.  $\tau = I\alpha$ , and  $I$  is constant in all frames.  
 The torque is the greatest in Frame C as the force of grav.  $\vec{F}$  is perpendicular to the radius.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

Frame E, angular velocity, is the greatest as positive torque has been applied for the maximum amount of time, and  
 $KE_{rot} = \frac{1}{2} I \omega^2$

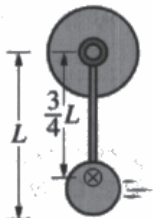


Figure 3

$\Delta KE = W = F_g d$   
 $\Delta KE = Mg \pi \frac{3}{4} L$   $d = 2\pi \left(\frac{3}{4}\right) L$

(b) The rod-sphere system has mass  $M$  and length  $L$ , and the center of mass is located a distance  $\frac{3}{4}L$  from the

axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of  $M$ ,  $L$ , and fundamental constants, as appropriate.

$\Delta KE = KE_f - KE_i = KE_f$   $\Delta KE = KE_{final} - KE_{initial}$   
 $U_g = KE_f - KE_i = \Delta KE$   
 $U_g = mgh$   $\Delta KE = \frac{3}{4} MgL$

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

If Earth is not included in the system, work is done on the system by gravity, thereby gaining kinetic energy.

**Question 5**

Begin your response to **QUESTION 5** on this page.

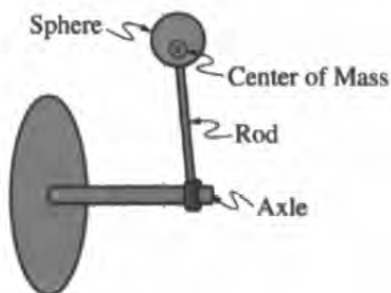


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a  $\otimes$ , as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

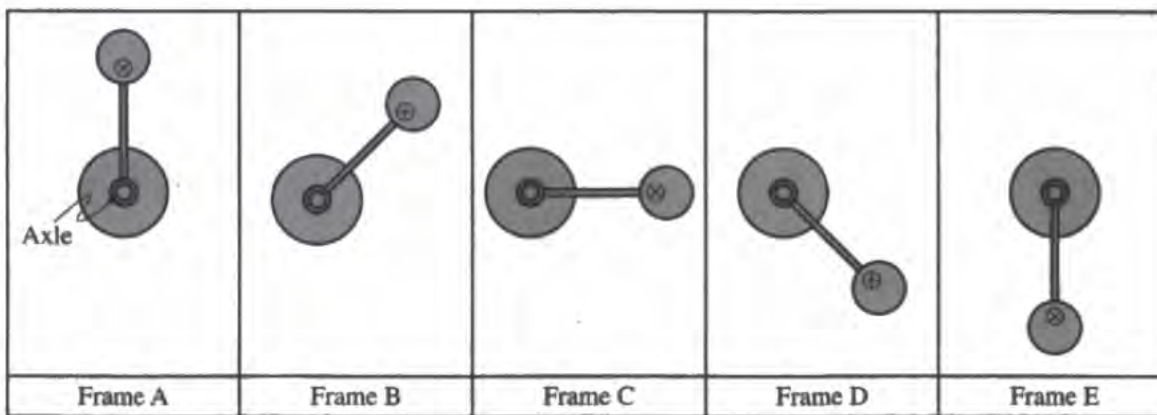


Figure 2

Question 5

Continue your response to QUESTION 5 on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

Frame C. Angular acceleration can be gotten from ~~torque~~ torque, and since torque is equal to  $Frsin\theta$ , and force and radius are constant, only the angle the force is applied changes. Since the max sin value is at  $90^\circ$ , frame C has the largest torque, since it is applied at a  $90^\circ$  angle.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

Since rot. KE. is equal to  $\frac{1}{2}I\omega^2$ , and  $I$  is constant in this scenario, the point of maximum  $\omega$  gives the greatest rot. KE.

This means frame E has the largest, since the entire time the object accelerates downwards and does not lose speed due to any outside force, E allows the most time for the rod to accelerate, giving it the greatest  $\omega$  and rot. KE.

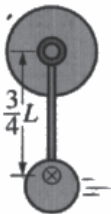


Figure 3

(b) The rod-sphere system has mass  $M$  and length  $L$ , and the center of mass is located a distance  $\frac{3}{4}L$  from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of  $M$ ,  $L$ , and fundamental constants, as appropriate.

$mgh = \frac{1}{2}mv^2$      $h = 2L$   
 ~~$2MgL$~~

kin energy at the bottom = ~~from~~  $\Delta GPE$  (from  $L$  to  $-L$ )

$\Delta KE = KE_2 - KE_1 = 2MgL - 0 = 2MgL$

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

The rod & sphere gain kinetic energy, even if Earth is not included because Earth becomes an outside force. This means the Earth still accelerated the rod & sphere, but there is just a change in the system's total M.E.

**Question 5**

Begin your response to **QUESTION 5** on this page.

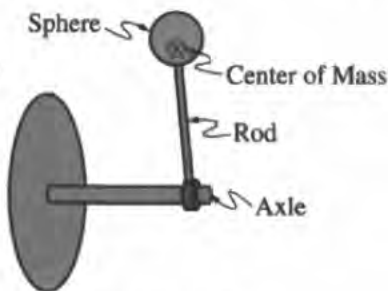


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a  $\otimes$ , as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

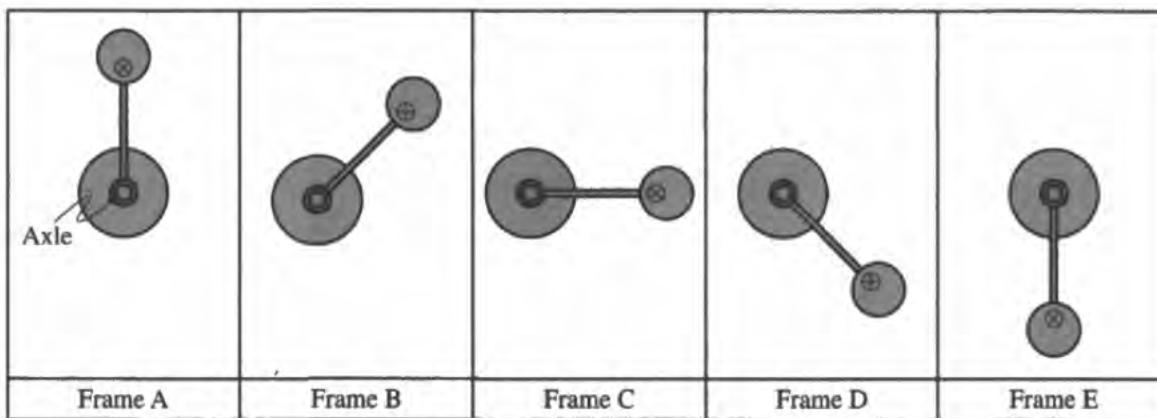


Figure 2





Question 5

Continue your response to QUESTION 5 on this page.

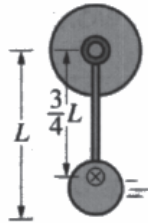
(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

The angular acceleration is greatest in frame C because the torque caused by gravity is greatest there.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

The rotational kinetic energy is greatest at frame E because the rod-sphere system is moving with the greatest angular velocity there.



This is because the torque has been applied for the longest at frame E.

Figure 3

(b) The rod-sphere system has mass  $M$  and length  $L$ , and the center of mass is located a distance  $\frac{3}{4}L$  from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of  $M$ ,  $L$ , and fundamental constants, as appropriate.

$$mgh = \frac{1}{2} I \omega^2 \quad h = 2 \cdot \frac{3}{4} L = \frac{3}{2} L$$

$$mgh = \Delta K \quad \Delta K = M \cdot g \cdot \left(\frac{3}{2} L\right)$$

$$\Delta K = \frac{3}{2} MgL$$

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

The rod and sphere gain kinetic energy because the angular velocity at frame E is greater than at frame A, so the rotational kinetic energy increases since  $K_r = \frac{1}{2} I \omega^2$

**Question 5**

Begin your response to **QUESTION 5** on this page.

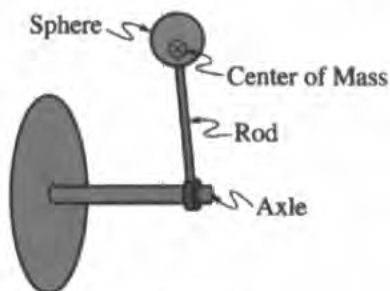


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a  $\otimes$ , as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

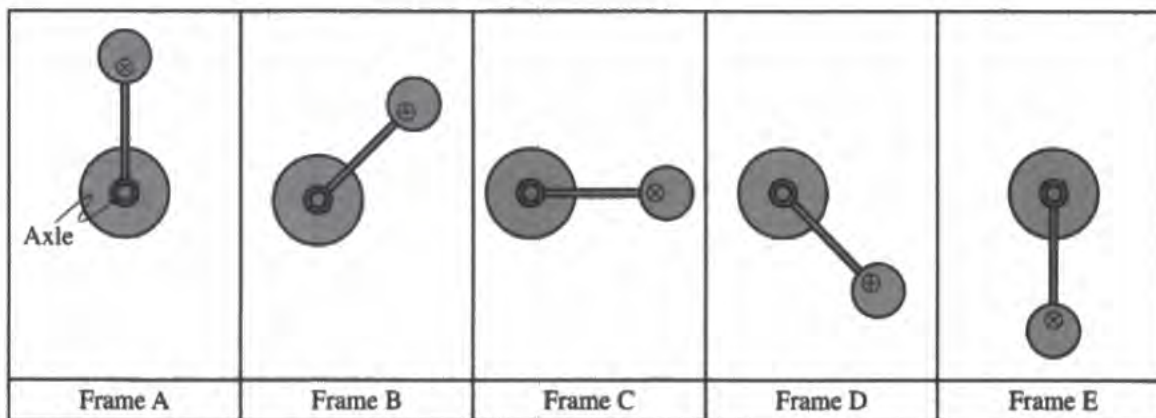


Figure 2





## Question 5

Continue your response to QUESTION 5 on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

In frame C because torque is greatest with  $F_g$  perpendicular to the rod.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

Frame E because it has been accelerating for the longest at this point.

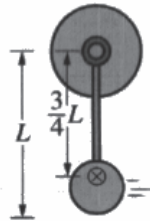


Figure 3

(b) The rod-sphere system has mass  $M$  and length  $L$ , and the center of mass is located a distance  $\frac{3}{4}L$  from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of  $M$ ,  $L$ , and fundamental constants, as appropriate.

$$M \cdot g \cdot \frac{3}{4} L \cdot 2$$

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

They decrease in height lowering  $U$  increasing  $K$

**Question 5**

Begin your response to **QUESTION 5** on this page.

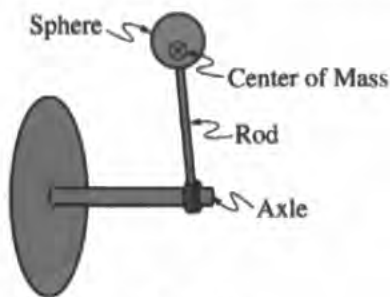


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a  $\otimes$ , as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

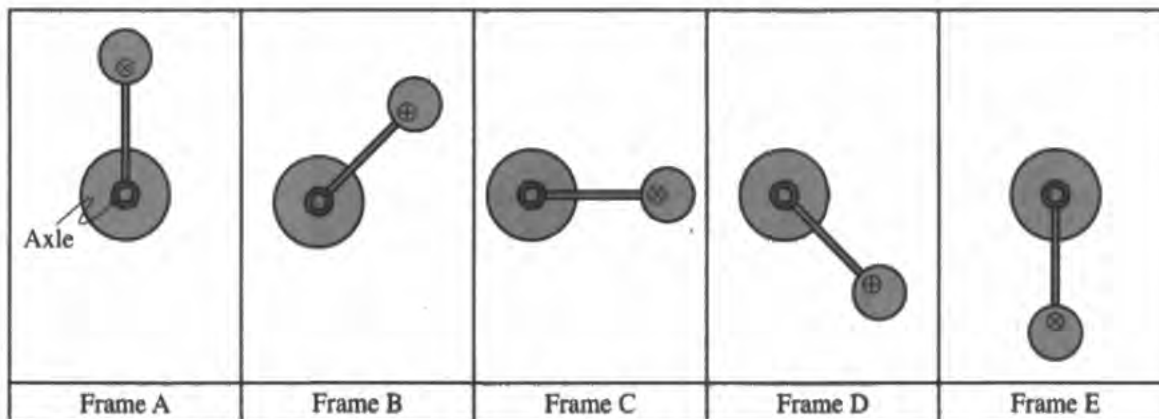


Figure 2



Question 5

Continue your response to QUESTION 5 on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

In Frame C the angular acceleration is the greatest. The equation for angular acceleration is  $\frac{\tau_{net}}{I}$ . Because  $I$  remains constant, the only changing value is  $\tau_{net}$ .  $\tau_{net}$  is the perpendicular force multiplied by the radius in which that force is applied. The only force is from gravity, and at C  $F_g$  is perpendicular to the rod.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

Frame E. This is because the  $\omega$  at Frame e is the greatest, since the system has been accelerating for the longest time. The equation for rotational kinetic energy is  $KE = \frac{1}{2} I \omega^2$ .  $I$  remains constant, and  $\omega$  is greatest at Frame E.

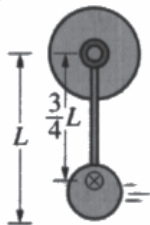


Figure 3

(b) The rod-sphere system has mass  $M$  and length  $L$ , and the center of mass is located a distance  $\frac{3}{4}L$  from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of  $M$ ,  $L$ , and fundamental constants, as appropriate.

$$KE_{total} = \frac{1}{2} M v^2 + \frac{1}{2} I \omega^2 \rightarrow KE_{total} = \frac{1}{2} M v^2 + \frac{1}{2} (M) \left(\frac{3}{4}L\right)^2$$

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

The system still has potential energy, and as the system falls, potential energy is converted to kinetic.

**Question 5**

Begin your response to **QUESTION 5** on this page.

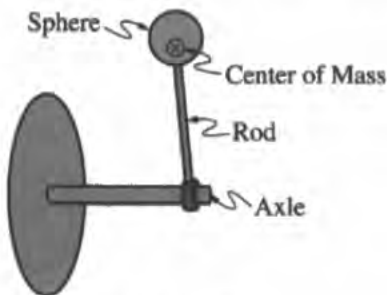


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a  $\otimes$ , as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

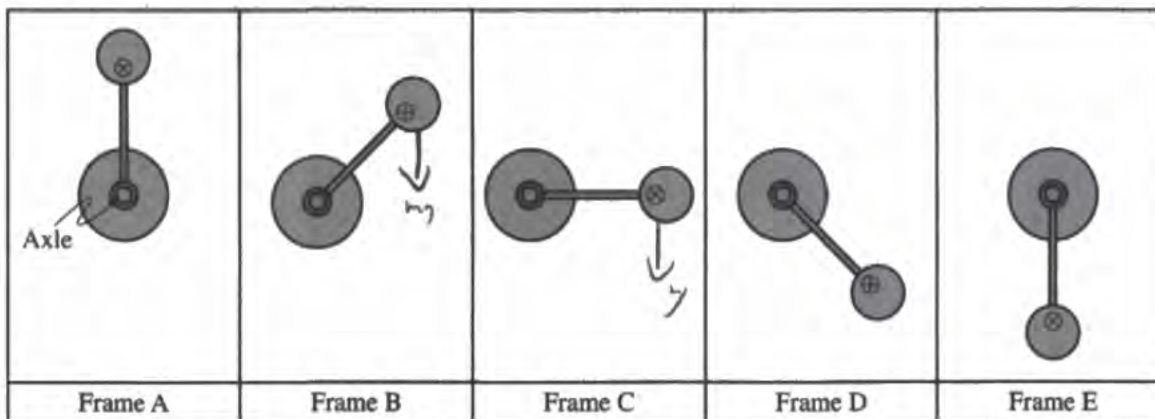


Figure 2



Question 5

Continue your response to QUESTION 5 on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

$\alpha = \frac{\tau_{\text{net}}}{I}$  ( $I$  is the same in all.)

$\tau = rF \sin \theta$ ,  $L = \text{constant torque}$   
 $r$  is const and  $F$ .

Frame C, bc  $F_g$  is perpendicular to the lever arm.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

$K_{\text{rot}} = \frac{1}{2} I \omega^2$

$L = L_e$   
 $I_{\text{cm}} = I_{\text{cm}}$

They are all equal bc the period is the same in all frames and angular momentum is conserved.

$I$  is const.  $T = \frac{2\pi}{\omega}$   $T_p = 2\pi \sqrt{\frac{L}{g}}$

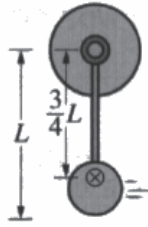


Figure 3

(b) The rod-sphere system has mass  $M$  and length  $L$ , and the center of mass is located a distance  $\frac{3}{4}L$  from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of  $M$ ,  $L$ , and fundamental constants, as appropriate.

$K_{\text{rot}} = \frac{1}{2} I \omega^2$

$I = \frac{1}{2} M \left(\frac{3}{4}L\right)^2$   
 $I = \frac{3}{8} ML^2$

$\omega = \frac{2\pi}{T}$   $T = 2\pi \sqrt{\frac{3/4 L}{g}}$

$\omega = \frac{1}{\sqrt{3/4 L/g}}$

$K_{\text{rot}} = \frac{3}{16} ML \left( \frac{1}{\sqrt{3/4 L/g}} \right)^2$

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

There is still angular acceleration, even without  $F_g$ .

$\alpha = \frac{\Sigma \tau}{I}$

$\tau = r F \sin \theta$   
 $I = m r^2$

**Question 5**

Begin your response to **QUESTION 5** on this page.

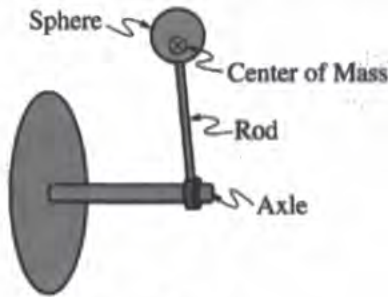


Figure 1

5. (7 points, suggested time 13 minutes)

A rod with a sphere attached to the end is connected to a horizontal mounted axle and carefully balanced so that it rests in a position vertically upward from the axle. The center of mass of the rod-sphere system is indicated with a  $\otimes$ , as shown in Figure 1. The sphere is lightly tapped, and the rod-sphere system rotates clockwise with negligible friction about the axle due to the gravitational force.

A student takes a video of the rod rotating from the vertically upward position to the vertically downward position. Figure 2 shows five frames (still shots) that the student selected from the video.

Note: these frames are not equally spaced apart in time.

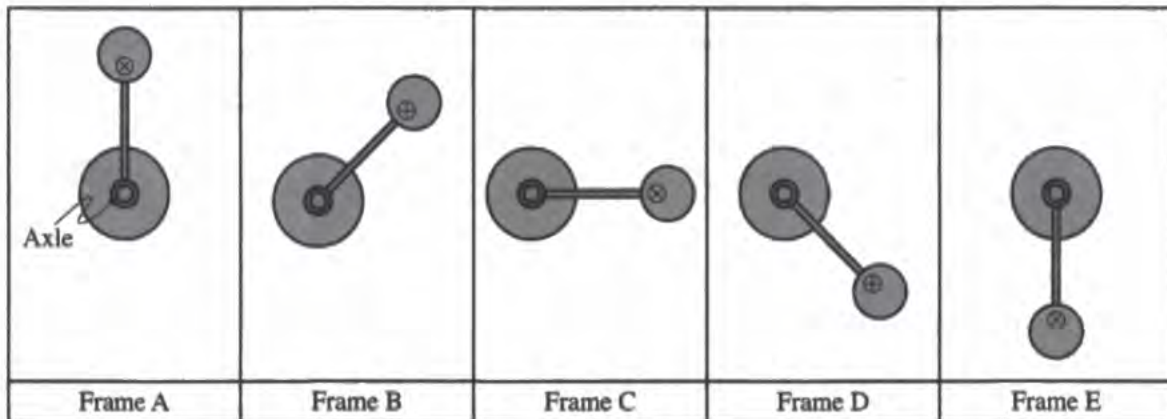


Figure 2



Question 5

Continue your response to QUESTION 5 on this page.

(a) Use the frames of the video shown in Figure 2 to answer the following questions.

i. In which frame is the angular acceleration of the rod-sphere system the greatest? Justify your answer.

Frame A, it has gravity pulling on it with more force than the other frames. It still has the same force applied.

ii. In which frame is the rotational kinetic energy of the rod-sphere system the greatest? Briefly justify your answer.

Frame E. This is the point with least potential energy because it has the least height, so it must have the most kinetic energy, given mechanical energy must be constant.

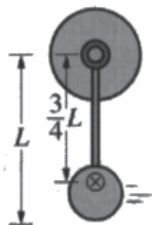


Figure 3

(b) The rod-sphere system has mass  $M$  and length  $L$ , and the center of mass is located a distance  $\frac{3}{4}L$  from the axle, shown in Figure 3.

i. Derive an expression for the change in kinetic energy of the rod-sphere-Earth system from the moment shown in Frame A to the moment shown in Frame E. Express your answer in terms of  $M$ ,  $L$ , and fundamental constants, as appropriate.

$$\Delta KE = 2 \left( \frac{mg}{3L^2} \right)$$

$$KE = \frac{1}{2} I \omega^2$$

ii. Briefly explain why the rod and sphere gain kinetic energy, even if Earth is not included in the system.

The rod & sphere still gain kinetic energy because of their rotational velocity increasing near the bottom.

## Question 5

### Sample Identifier: P1 Q5 Sample A

Score: 7

a.i.

- 1 point was earned. The response states Frame C and indicates that the force of gravity is perpendicular to the radius.
- 1 point was earned. The response uses the equation for Newton's second law to relate torque and angular acceleration.

a.ii.

- 1 point was earned. The response states Frame E and correctly relates the position to the greatest kinetic energy.

b.i.

- 1 point was earned. The response begins with a concept of conservation of energy and has multiple steps.
- 1 point was earned. The response correctly substitutes the value  $\frac{6}{4}L$  for the height shown in the equation.
- 1 point was earned. The response has a correct answer.

b.ii.

- 1 point was earned. The response indicates that the gravitational force is now an external force on the system and does work.

**Sample Identifier: P1 Q5 Sample B**

**Score: 6**

a.i.

- 1 point was earned. The response states Frame C and indicates that the force of gravity is perpendicular to the rod.
- 1 point was earned. The response indicates a direct relationship between angular acceleration and torque.

a.ii.

- 1 point was earned. The response states Frame E and correctly relates the position to the greatest kinetic energy.

b.i.

- 1 point was earned. The response begins with a concept of conservation of energy and has multiple steps.
- 0 point was earned. The response incorrectly substitutes the value  $2L$  for the height shown in the equation.
- 1 point was earned. The response has a correct answer consistent with the value of height.

b.ii.

- 1 point was earned. The response indicates that the gravitational force is now an external force on the system and does work.

**Sample Identifier: P1 Q5 Sample C**

**Score: 5**

a.i.

- 0 points were earned. The response does not indicate a relationship between torque and the position of the rod-sphere in Frame C.
- 1 point was earned. The response indicates a proportional relationship between torque and angular acceleration.

a.ii.

- 1 point was earned. The response states Frame E and correctly relates the position to the greatest kinetic energy.

b.i.

- 1 point was earned. The response begins with a concept of conservation of energy and has multiple steps.
- 1 point was earned. The response correctly substitutes the value  $\frac{3}{2}L$  for the height shown in the equation.
- 1 point was earned. The response has a correct answer.

b.ii.

- 0 points were earned. The response does not indicate that the gravitational force is now an external force on the system.

**Sample Identifier: P1 Q5 Sample D**

**Score: 4**

a.i.

- 1 point was earned. The response states Frame C and indicates that the force of gravity is perpendicular to the radius.
- 1 point was earned. The response implies that the angular acceleration is directly proportional to the torque.

a.ii.

- 1 point was earned. The response states Frame E and correctly relates the position to the greatest kinetic energy.

b.i.

- 0 points were earned. The response is not a multi-step derivation and does not begin with a concept of conservation of energy.
- 0 points were earned. The response has no explicit indication of a height or change in height.
- 1 point was earned. The response does have the correct answer.

b.ii.

- 0 points were earned. The response does not indicate that the gravitational force is now an external force on the system.

**Sample Identifier: P1 Q5 Sample E**

**Score: 3**

a.i.

- 1 point was earned. The response states Frame C and indicates that the force of gravity is perpendicular to the radius.
- 1 point was earned. The response uses the equation for Newton's second law to relate torque and angular acceleration.

a.ii.

- 1 point was earned. The response states Frame E and correctly relates the position to the greatest kinetic energy.

b.i.

- 0 points were earned. The response does not begin with a concept of conservation of energy.
- 0 points were earned. The response has no indication of a height or change in height.
- 0 points were earned. The response does not have a correct answer.

b.ii.

- 0 points were earned. The response does not indicate that the gravitational force is now an external force on the system.



**Sample Identifier: P1 Q5 Sample F**

**Score: 2**

a.i.

- 1 point was earned. The response states Frame C and indicates that the force of gravity is perpendicular to the lever arm.
- 1 point was earned. The response uses the equation for Newton's second law to relate torque and angular acceleration.

a.ii.

- 0 points were earned. The response does indicate the correct frame.

b.i.

- 0 points were earned. The response does not begin with a concept of conservation of energy.
- 0 points were earned. The response has no indication of a height or change in height.
- 0 points were earned. The response does not have a correct answer.

b.ii.

- 0 points were earned. The response does not indicate that the gravitational force is now an external force on the system.

**Sample Identifier: P1 Q5 Sample G**

**Score: 1**

a.i.

- 0 points were earned. The response does not state Frame C.
- 0 points were earned. The response does not indicate a correct relationship between torque and angular acceleration.

a.ii.

- 1 point was earned. The response states Frame E and correctly relates the position to the greatest kinetic energy.

b.i.

- 0 points were earned. The response is not a multi-step derivation and does not begin with a concept of conservation of energy.
- 0 points were earned. The response has no indication of a height or change in height.
- 0 points were earned. The response does not have a correct answer.

b.ii.

- 0 points were earned. The response does not indicate that the gravitational force is now an external force on the system.