2018

AP CollegeBoard

AP Physics C: Electricity and Magnetism

Sample Student Reponses and Scoring Notes

DRAFT

General Notes About 2017 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based and AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8\ \mathrm{m/s^2}$, but use of $10\ \mathrm{m/s^2}$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

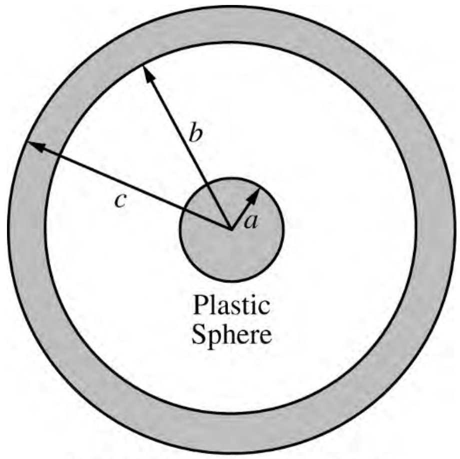
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

      ii. $a < r < b$

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

ii. $a < r < b$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

Justify your answer.

ii. Determine the charge on the outer surface of the conducting shell.

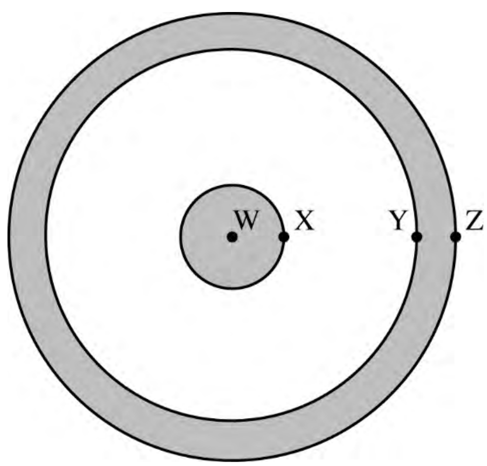
Question 1 continues on the next page.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

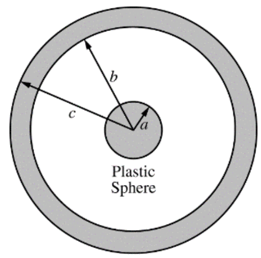
ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

____ W ____ X ____ Y ____ Z



Question 1

15 points total **Distribution of points**



Conducting Spherical Shell

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(a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

i) 3 points

$r < a$

For using the integral to determine the charge enclosed		1 point
$q_{enc} = \int \rho dV = \int \beta r dV$		
For correctly substituting for dV into the integration to determine the charge enclosed		1 point
$q_{enc} = \int \left(4\pi r'^2\right)(\beta r')dr' = 4\beta\pi \int_{r'=0}^{r'=r} r'^3 dr' = 4\beta\pi \left[\frac{r'^4}{4}\right]_{r'=0}^{r'=r}$		
For a correct answer		1 point
$q_{enc} = \beta\pi r^4$		

ii) 1 points

$a < r < b$

For an answer consistent with (a)(i)		1 point
$q_{enc} = \beta\pi a^4$		

Question 1 (continued)

Distribution of points

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i) 2 points

$r < a$

For correctly substituting the area of a Gaussian sphere into Gauss's law		1 point
$\dfrac{q_{enc}}{\varepsilon_0} = \oint \vec{E} \bullet d\vec{A}$		
$\dfrac{q(r)}{\varepsilon_0} = EA_{sphere} = E\left(4\pi r^2\right)$		
For correctly substituting the charge from part (a)(i) into the equation above		1 point
$\dfrac{\beta\pi r^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \dfrac{\beta r^2}{4\varepsilon_0}$		

ii) 1 point

$a < r < b$

For correctly substituting the charge from part (a)(ii) into Gauss's law to calculate the electric field		1 point
$\dfrac{\beta\pi a^4}{\varepsilon_0} = E\left(4\pi r^2\right)$		
$E = \dfrac{\beta a^4}{4\varepsilon_0 r^2}$		

Question 1 (continued)

Distribution of points

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i) 2 points

Determine the charge on the inner surface of the conducting shell.
Justify your answer.

For an answer consistent with part (a)(ii) (must have opposite sign) $q_{inner} = -\beta\pi a^4$		1 point
For a correct justification		1 point
$E_{shell} = 0 = \dfrac{q_{enc}}{\varepsilon_0} \therefore q_{enc} = 0 = q_{sphere} + q_{inner} \therefore q_{inner} = -q_{sphere}$		

ii) 1 point

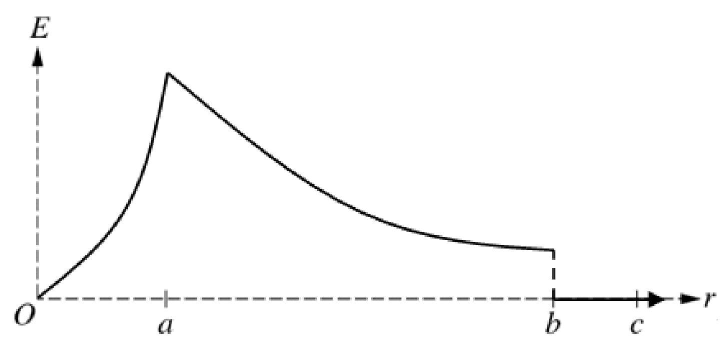
Determine the charge on the outer surface of the conducting shell.

A correct answer		1 point
$q_{outer} = 0$		

(d)

i) 3 points

On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



For concave up graph that begins at the origin and increases to $r = a$		1 point
For a concave up graph that is continuous with the first section and decreases from $r = a$ to non-zero value at $r = b$		1 point
For a graph with a value of $E = 0$ for $r > b$		1 point
Note: Due to scale, discontinuity at point b is not required.		

Question 1 (continued)

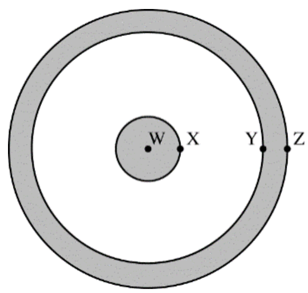
Distribution of points

(d) Con't

ii) 2 points

The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

__1__ W   __2__ X   __3__ Y   __3__ Z



For any ranking that has Y and Z at the same potential		1 point
For ranking the electric potentials: W > X > Y & Z (since Y = Z is the first point)		1 point

PHYSICS C: ELECTRICITY AND MAGNETISM

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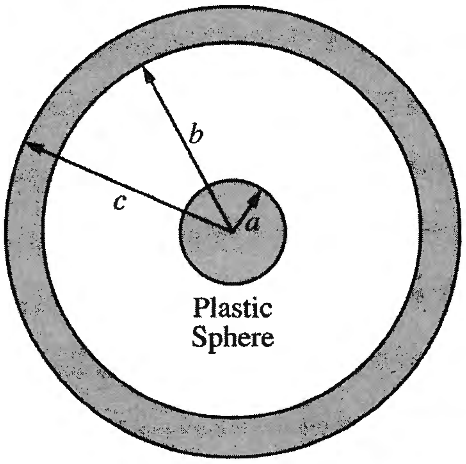


Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

   $\int \rho \, dV = Q_{enc}$

   $V = \frac{4}{3}\pi r^3$

   $dV = 4\pi r^2 dr$

   $\int_0^r \beta r \, 4\pi r^2 \, dr$

   $4\pi\beta \int_0^r r^3 dr$

   $\rightarrow Q_{enc} = 4\pi\beta \left[\frac{r^4}{4}\right]_0^r$

   $\boxed{Q_{enc} = \pi\beta r^4}$

   ii. $a < r < b$

   $\int \rho \, dV = Q_{enc}$

   $\int_0^a \beta r \, 4\pi r^2 \, dr$

   $4\pi\beta \left[\frac{r^4}{4}\right]_0^a$

   $\boxed{Q_{enc} = \pi\beta a^4}$

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$\oint E \cdot dA = \frac{Q_{en}}{\epsilon_0}$

$E(4\pi r^2) = \frac{\pi B r^4}{\epsilon_0}$

$\boxed{E = \frac{Br^2}{4\epsilon_0}}$

ii. $a < r < b$

$\oint E\,dA = \frac{Q_{en}}{\epsilon_0}$

$E(4\pi r^2) = \frac{\pi B a^4}{\epsilon_0}$

$\boxed{E = \frac{Ba^4}{4\epsilon_0 r^2}}$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

$-Q_{shell} = Q_{pk}$

$Q_{net} = 0$

$\boxed{Q_{in} = -\pi B a^4}$

Justify your answer.

因为 the shell is a conductor the inner surface charge will be exact equal to the opposite of the total enclosed charge

ii. Determine the charge on the outer surface of the conducting shell.

- 0C because otherwise an electric field would exist on the outside of the shell

and

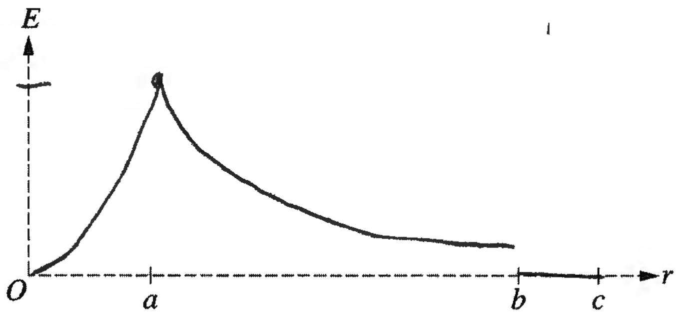
the total charge on the shell would be 0.

Question 1 continues on the next page.

GO ON TO THE NEXT PAGE.

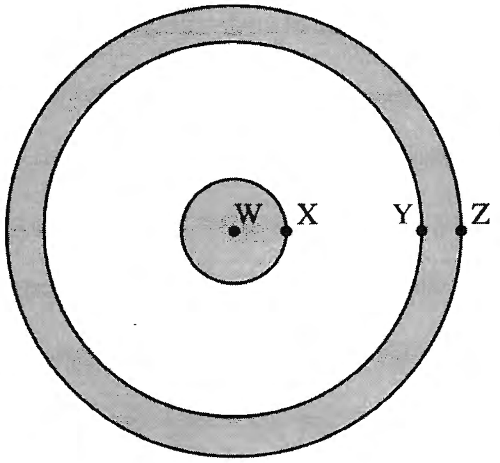
(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

1 W   2 X   3 Y   3 Z



2K

$Z: \frac{KQ}{B} + \frac{KQ}{r}$

$Y = Z$

$X = \frac{KQ}{c} + \frac{KQ}{b}$

$W = K$

GO ON TO THE NEXT PAGE.

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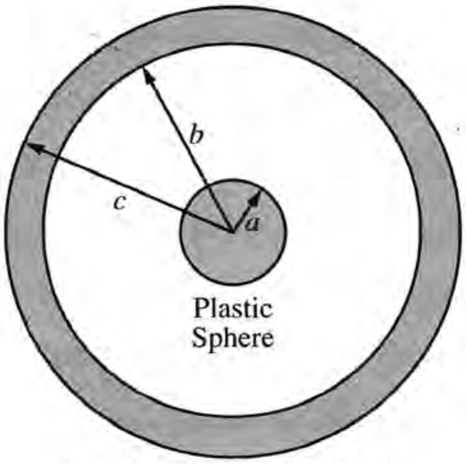

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   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

$$Q = \rho \cdot V = \int_0^r \rho \, dV = \int_0^r \beta r \cdot 4\pi r^2 dr = \boxed{\beta \pi r^4}$$

   ii. $a < r < b$

$$Q = \int_0^a \rho \, dV = \boxed{\beta \pi a^4}$$

GO ON TO THE NEXT PAGE.

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$\oint E \cdot dA = \frac{Q}{\varepsilon_0}$ $E = \frac{Q}{\varepsilon_0 A} = \frac{\beta \pi r^4}{\varepsilon_0 4\pi r^2} = \boxed{\frac{\beta r^2}{4\varepsilon_0}}$

ii. $a < r < b$

$\oint E \cdot dA = \frac{Q}{\varepsilon_0}$ $E = \frac{Q}{\varepsilon_0 A} = \frac{\beta \pi a^4}{\varepsilon_0 4\pi r^2} = \boxed{\frac{\beta a^4}{4\varepsilon_0 r^2}}$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

Justify your answer.

the charge on the inner surface of the conducting shell is same in magnitude of Q of the plastic sphere, $\beta \pi a^4$, but opposite of sign. It is $-\beta \pi a^4$ because the charge inclosed in the shell must be zero, ~~on the~~

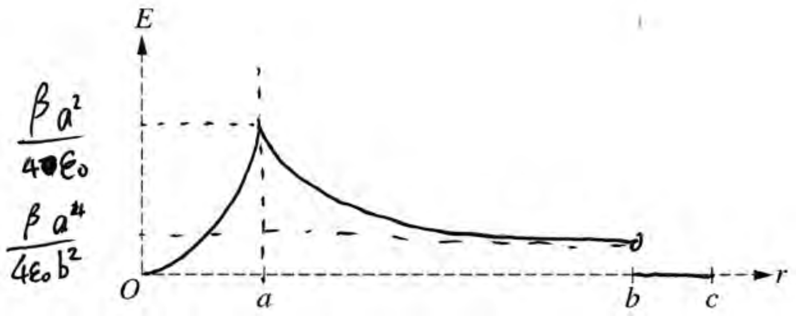
ii. Determine the charge on the outer surface of the conducting shell.

the charge on the outer surface of the conducting shell must be zero. Because the electric field outside the conducting shell is zero, according to Gauss's law $\oint E \cdot dA = \frac{Q_{enclose}}{\varepsilon_0}$ if the electric field, the net charge inclosed must be zero, the inner shell charge canceled with the charge on the plastic sphere. therefore the charge on the outer surface must be zero.

Question 1 continues on the next page.

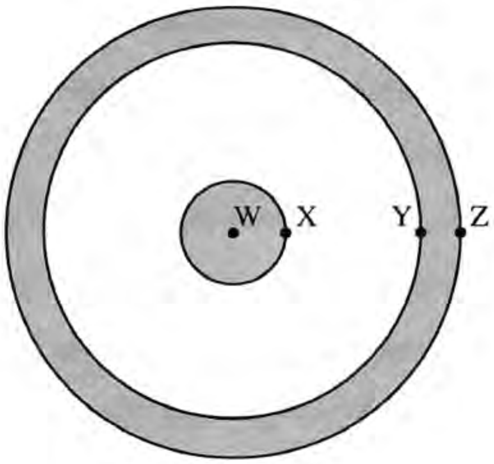
(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

_2_ W   _1_ X   _2_ Y   _2_ Z



PHYSICS C: ELECTRICITY AND MAGNETISM

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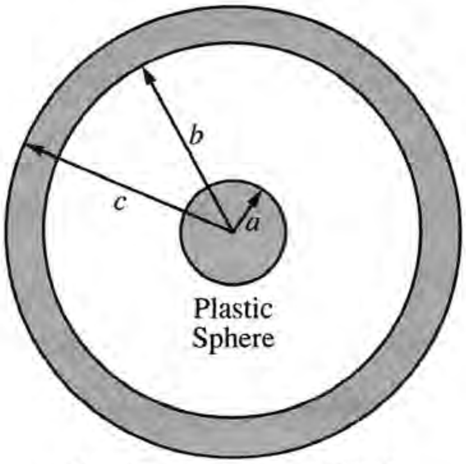

Conducting Spherical Shell

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   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

      $Q_{enc} = \int_0^r \beta r \, 4\pi r^2$

      $\int_0^r \beta r^3 4\pi$

      $Q = \beta\pi r^4$

      ii. $a < r < b$

      $Q_{enc} = \int_0^a \beta 4\pi r^3$

      $= \beta\pi a^4$

GO ON TO THE NEXT PAGE.

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$\int E da = \frac{Q_{enc}}{\varepsilon_0}$

$E 4\pi r^2 = \frac{\beta \pi r^4}{\varepsilon_0}$

$E = \frac{\beta r^2}{4\varepsilon_0}$

Q_{enc} from part a

ii. $a < r < b$

$\int E da = \frac{Q_{enc}}{\varepsilon_0}$

$E 4\pi r^2 = \frac{\beta \pi a^4}{\varepsilon_0}$

$E = \frac{\beta a^4}{4\varepsilon_0}$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

$-\beta a^4 \pi$

Justify your answer.

It has to have the same charge as the sphere within it so that it is able to make the enclosed charge equal to 0C, and the electric field outside 0.

ii. Determine the charge on the outer surface of the conducting shell.

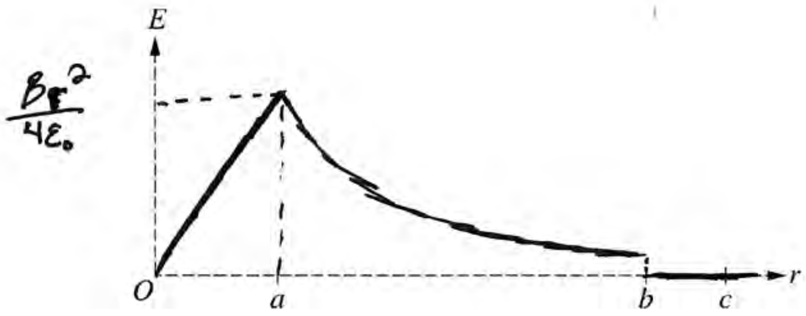
0C

Question 1 continues on the next page.

GO ON TO THE NEXT PAGE.

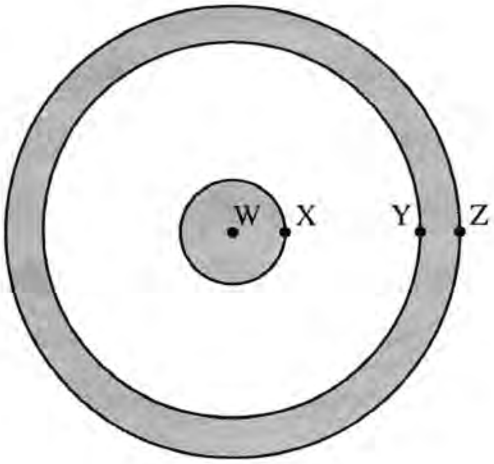
(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

1 W   2 X   3 Y   3 Z



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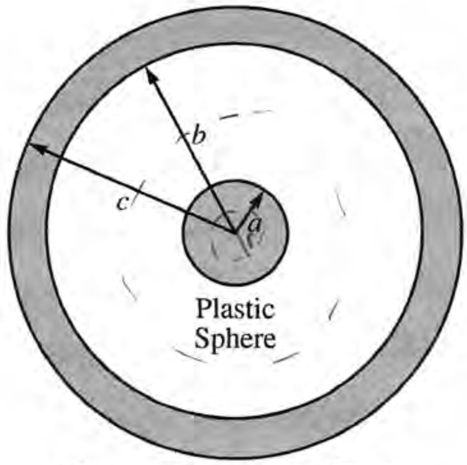

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   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$

$dq = \rho \, dV$

$dq = \rho(4\pi r^2 dr)$

$q_{in} = \pi \beta r^8$

$q_{in} = \int_0^r \beta r (4\pi r^2) dr = 4\pi\beta \int_0^r r^3 dr = 4\pi\beta \frac{r^4}{4}\Big|_0^r = \pi\beta r^4$

      ii. $a < r < b$

$\cancel{q_{in} = \int_a^b \beta r(4\pi r^2) dr = 4\pi\beta \int_a^b r^3 dr = \frac{4\pi\beta}{4} r^4 \Big|_a^b = \pi\beta(b^4 - a^4)}$

$\cancel{q_{in} = \pi\beta(b^4 - a^4)}$

$\boxed{q_{in} = \beta a}$

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$E(4\pi r) = \frac{Q_{in}}{\epsilon_0}$

$E(4\pi r) = \frac{\pi \beta r^4}{\epsilon_0}$

$E = \frac{\cancel{\pi} \beta r^{\cancel{4}\,3}}{4\cancel{\pi r}\epsilon_0} = \frac{\beta r^3}{4\epsilon_0}$

$E = \frac{\beta r^3}{4\epsilon_0}$

ii. $a < r < b$

$E(4\pi r) = \frac{\beta a}{\epsilon_0}$

$E = \frac{\beta a}{4\pi\epsilon_0 r}$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

Negative

Justify your answer.

The charge per unit volume of the center plastic sphere is +, so it repels the electrons towards the inner surface of the shell

• Must be opposite sign of outer surface's charge in order for $E = 0$.

ii. Determine the charge on the outer surface of the conducting shell.

Positive

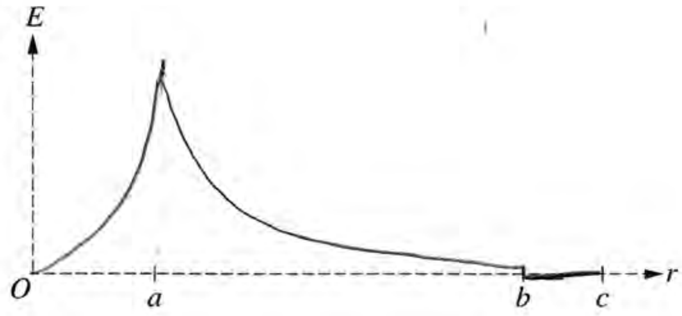
Question 1 continues on the next page.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.

$E = \frac{\beta r^3}{4\epsilon_0}$

$E = \frac{\beta a}{4\pi\epsilon_0 r}$



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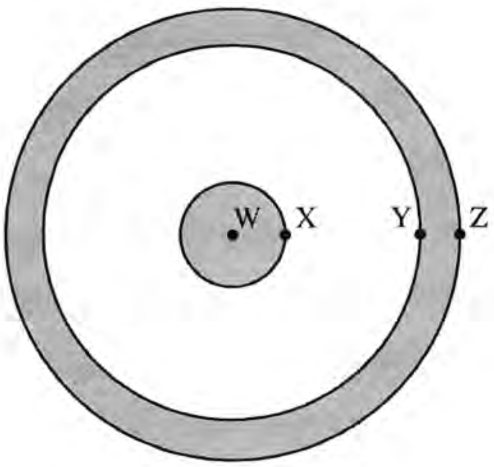
_1_ W   _2_ X   _3_ Y   _3_ Z



E =

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

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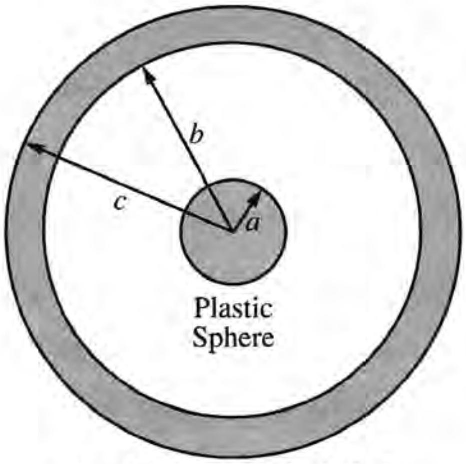

Conducting Spherical Shell

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   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

      $Q_{enc} = \beta r$

      ii. $a < r < b$

      $Q_{enc} = \beta a$

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$

$E(4\pi r^2) = \frac{\beta r}{\varepsilon}$

$\boxed{E = \frac{\beta}{4\pi \varepsilon r}}$

ii. $a < r < b$

$E(4\pi r^2) = \frac{\beta a}{\varepsilon_0}$

$\boxed{E = \frac{\beta a}{4\pi \varepsilon_0 r^2}}$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

$-\beta a$

Justify your answer.

Since the outside of the shell's electric field is zero, the shell must have an opposite charge thats equal in magnitude to the plastic shell in order to cancell out the electric fields

ii. Determine the charge on the outer surface of the conducting shell.

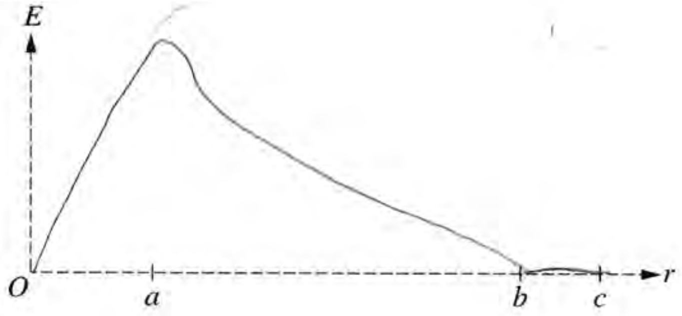
$\boxed{0\ C}$

Question 1 continues on the next page.

GO ON TO THE NEXT PAGE.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



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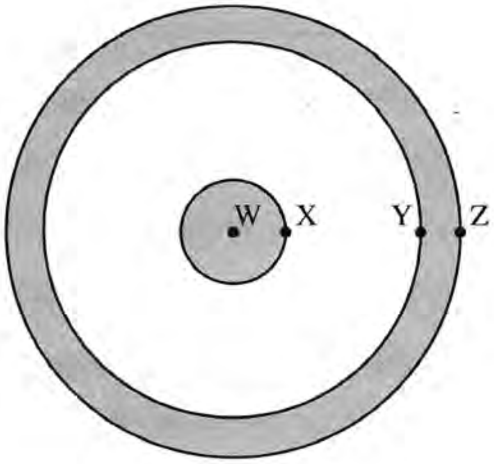
1 W   1 X   2 Y   2 Z



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PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

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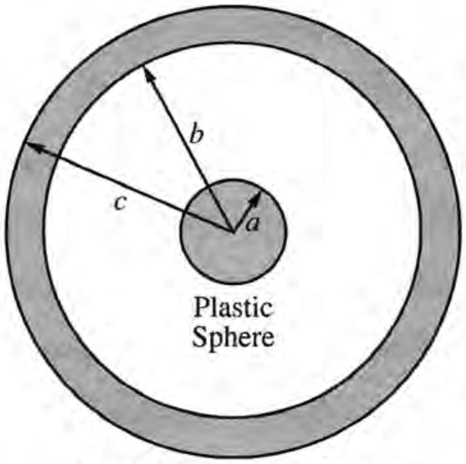

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   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

$\frac{q}{V} = \rho(r) = \beta r$

$q = V\beta r$

$\boxed{q = \frac{4}{3}\pi r^4 \beta}$

$\cancel{\Phi_E = \oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}}$ $\cancel{E = \frac{4\pi r^4 \beta}{3\varepsilon_0 4\pi r^2}}$

$\cancel{E\oint dA = \frac{4\pi r^4 \beta}{3\varepsilon_0}}$ $\cancel{E = \frac{\beta r^2}{3\varepsilon_0}}$

$\cancel{EA = \frac{4\pi r^4 \beta}{3\varepsilon_0}}$

      ii. $a < r < b$

$\frac{q}{V} = \rho(r) = \beta r$

$q = \beta a V$

$\boxed{q = \frac{4}{3}\pi a^4 \beta}$

$\cancel{\Phi_E = \oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}}$

$\cancel{E 4\pi r^2 = \frac{\frac{4}{3}\pi a^4 \beta}{\varepsilon_0}}$

$\cancel{E = \frac{\beta a^4}{3\varepsilon_0 r^2}}$

GO ON TO THE NEXT PAGE.

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$\phi_E = \oint E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$

$EA = \frac{4\pi r^4 \beta}{3\varepsilon_0}$

$E 4\pi r^2 = \frac{4\pi r^4 \beta}{3\varepsilon_0}$

$\rightarrow E = \frac{r^2 \beta}{3\varepsilon_0}$

ii. $a < r < b$

$\phi_E = \oint E\, dA = \frac{q_{enc}}{\varepsilon_0}$

$EA = \frac{4}{3}\pi a^4 \beta / \varepsilon_0$

$E 4\pi r^2 = \frac{4\pi a^4 \beta}{3\varepsilon_0}$

$\rightarrow E = \frac{a^4 \beta}{3\varepsilon_0 r^2}$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

$-\frac{4}{3}\pi a^4 \beta$

Justify your answer.

The charge on the inner sphere polarizes the shell, inducing an equal but opposite charge on the inside of the shell

ii. Determine the charge on the outer surface of the conducting shell.

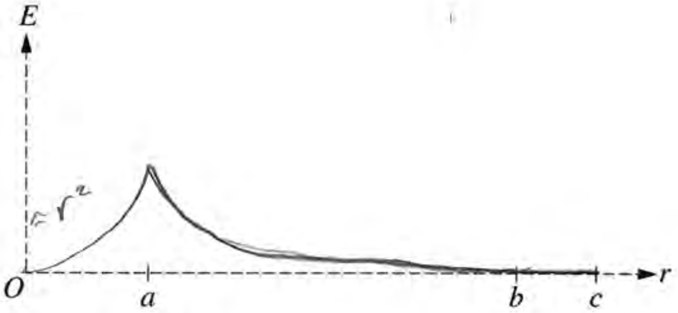
$\frac{4}{3}\pi a^4 \beta$

Question 1 continues on the next page.

GO ON TO THE NEXT PAGE.

(d)

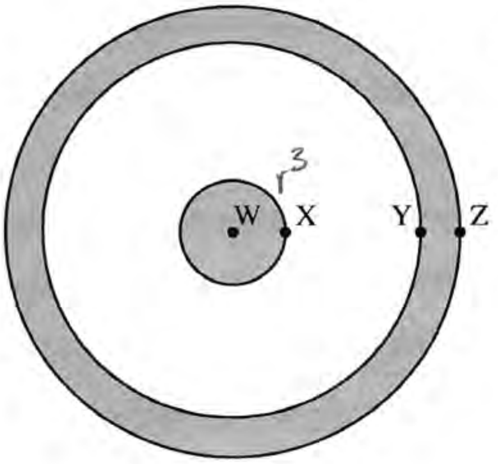
i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



ii. The figure below shows the sphere and shell with four points labeled W, X, Y, and Z. Point W is at the center of the sphere, point X is on the surface of the sphere, and points Y and Z are on the inner and outer surface of the shell, respectively. Rank the points according to the electric potential at that point, with 1 indicating the largest electric potential. If two points have the same electric potential, give them the same numerical ranking.

<u>3</u> W <u>2</u> X <u>1</u> Y <u>4</u> Z

$V = \int E\,dr$
$= E \int dr$
$= Er$



$W < X < Y$

GO ON TO THE NEXT PAGE.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II
Time—45 minutes
3 Questions

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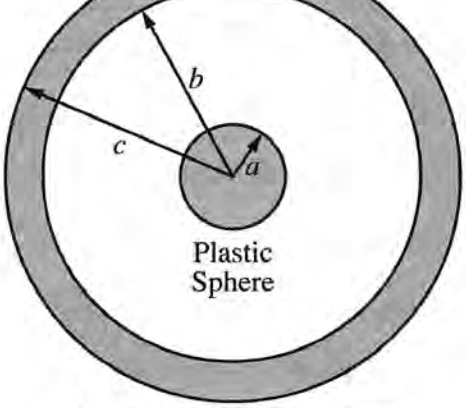


Conducting Spherical Shell

1. A solid plastic sphere of radius a and a conducting spherical shell of inner radius b and outer radius c are shown in the figure above. The shell has an unknown charge. The solid plastic sphere has a charge per unit volume given by $\rho(r) = \beta r$, where β is a positive constant and r is the distance from the center of the sphere. Express your answers to parts (a), (b), and (c) in terms of β, r, a, and physical constants, as appropriate.

   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

   $$Q = \rho V = \rho(r)\,\frac{4}{3}\pi r^3 = \beta r\,\frac{4}{3}\pi r^3 = \frac{4\pi}{3}\beta r^4$$

   ii. $a < r < b$

   $$Q = \rho V = \rho(a)\,\frac{4}{3}\pi a^3 = \beta a\,\frac{4}{3}\pi a^3 = \frac{4\pi}{3}\beta a^4$$

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0} = E \oint dA = E(\cancel{4}\cancel{\pi} r^2) = \frac{\cancel{4}\cancel{\pi}}{3\varepsilon_0} \beta r^4$$

$$E = \frac{\beta r^{4_2}}{3\varepsilon_0 \cancel{r}} = \boxed{\frac{\beta r^2}{3\varepsilon_0}}$$

ii. $a < r < b$

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0} = E \oint dA = E(\cancel{4}\cancel{\pi} a^2) = \frac{\cancel{4}\cancel{\pi}}{3\varepsilon_0} \beta a^4$$

$$E = \frac{\beta a^{4_2}}{3\varepsilon_0 \cancel{a}} = \boxed{\frac{\beta a^2}{3\varepsilon_0}}$$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

Justify your answer.

ii. Determine the charge on the outer surface of the conducting shell.

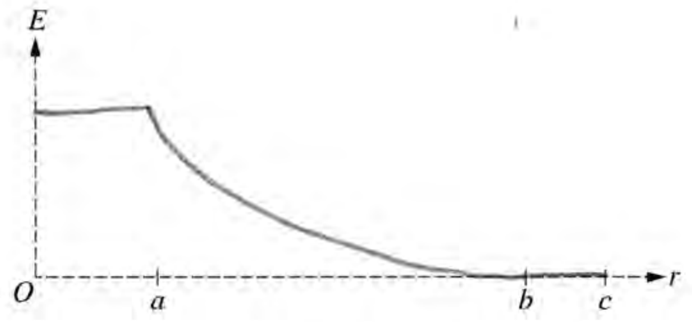
$$\boxed{Q = 0\ C}$$

Question 1 continues on the next page.

GO ON TO THE NEXT PAGE.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



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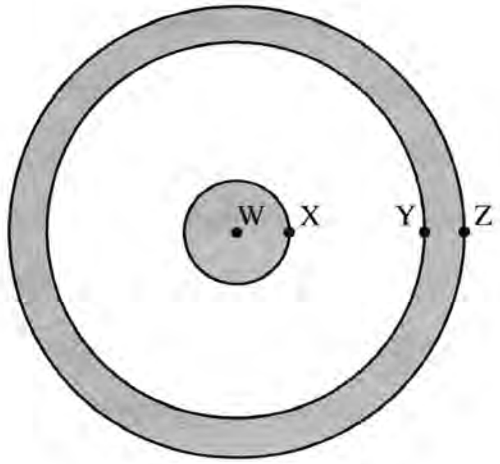
3 W   1 X   2 Y   3 Z



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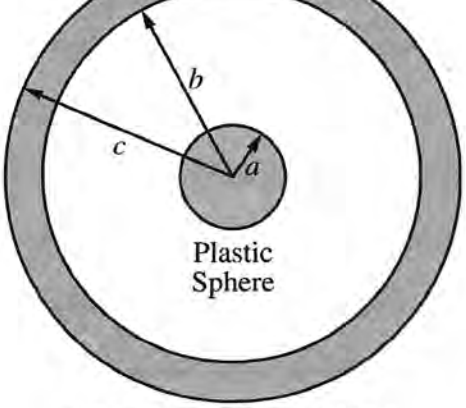

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   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

   $\frac{q_{enc}}{\varepsilon_0} = \int E\cdot dA$ $\quad V = \frac{4}{3}\pi r^3$

   $A = 4\pi r^2$

   $q_{enc} = E 4\pi r^2 \varepsilon_0$

   ii. $a < r < b$

   $\frac{q_{enc}}{\varepsilon_0} = \int E\cdot dA$ $\quad 4\pi b^2 - 4\pi a^2$

   $4\pi(b^2 - a^2)$

   $q = 4\pi(b^2 - a^2)E\cdot\varepsilon_0$

GO ON TO THE NEXT PAGE.

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$$\boxed{E = \frac{kq}{r^2} = \frac{q}{4\pi r^2 \varepsilon_0}}$$

$dA = 4\pi r^2$

ii. $a < r < b$

$$\frac{q_{enc}}{\varepsilon_0} = \int E\, dA \quad 4\pi(b-a)^2$$

$$E = \frac{q}{\varepsilon 4\pi (b-a)^2}$$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

βb

Justify your answer.

on the inner sphere, the charge is equal to the charge/unit volume on point b.

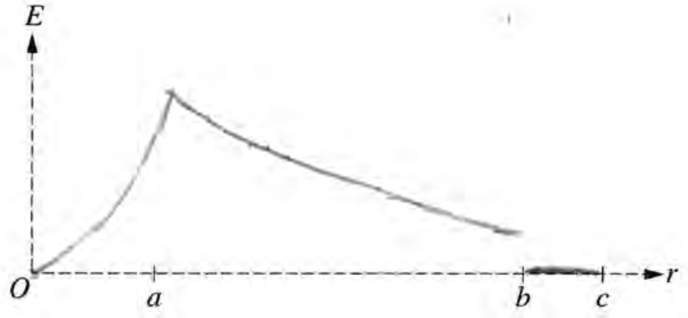
ii. Determine the charge on the outer surface of the conducting shell.

$-\beta b \rightarrow$ there must be a net of zero

Question 1 continues on the next page.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



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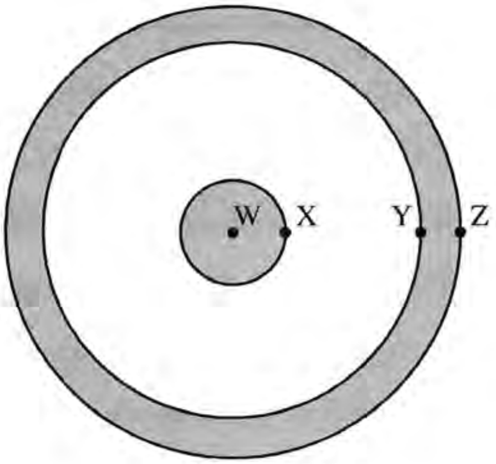
1 W 2 X 3 Y 3 Z



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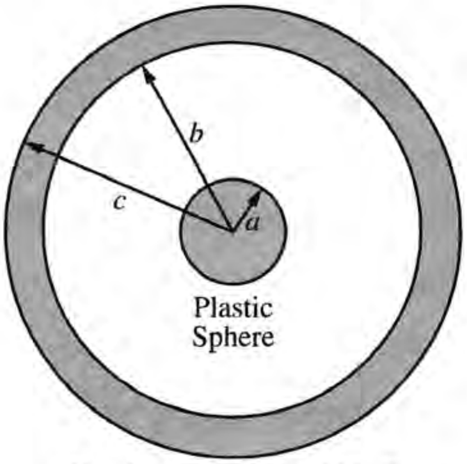

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   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

   i. $r < a$

   let C represent the charge enclosed by the Gaussian sphere

   $$\boxed{C = \frac{4\pi r^4 \beta}{3}}$$

   ii. $a < r < b$

   $$\boxed{C = \frac{4\pi r^4 \beta}{3}}$$

GO ON TO THE NEXT PAGE.

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

$\oint E \cdot d\vec{A}$ $\quad E = -\frac{dV}{dx}$

$E = \frac{dV}{dx}\left(\frac{4\pi r^4 \beta}{3}\right)$ $\quad E = \frac{4\pi\beta}{3}\left(\frac{dV}{dr}\left(r^4\right)\right)$

$$\boxed{E = \frac{16\pi\beta r^3}{3}}$$

ii. $a < r < b$

$\oint E \cdot d\vec{A}$ $\quad E = -\frac{dV}{dx}$

$E = \frac{dV}{dx}\left(\frac{4\pi r^4 \beta}{3}\right)$ $\quad E = \frac{4\pi\beta}{3}\left(\frac{dV}{dr}\left(r^4\right)\right)$ $\quad \boxed{E = \frac{16\pi\beta r^3}{3}}$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

The charge will be 0.

Justify your answer.

As there is no electric field past the outside of the conducting shell, there will be no charge on the outside of the shell and thus, no charge will accumulate on the inside of the charge.

ii. Determine the charge on the outer surface of the conducting shell.

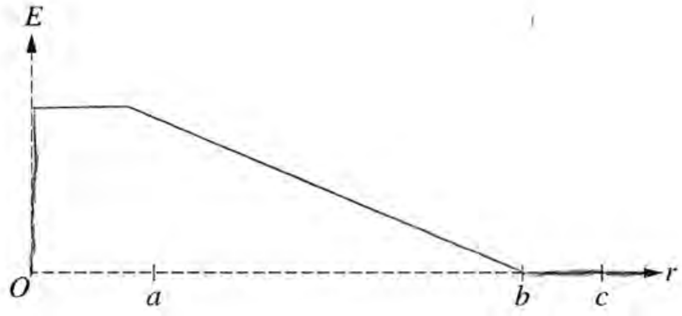
The electric charge will be 0 on the outer surface as there is no electric field beyond the outside of the conducting shell.

Question 1 continues on the next page.

GO ON TO THE NEXT PAGE.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



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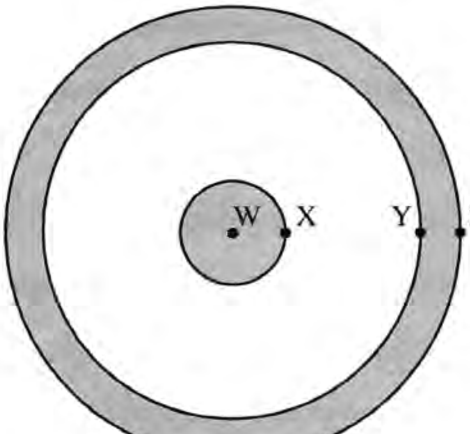
_1_ W   _1_ X   _4_ Y   _4_ Z



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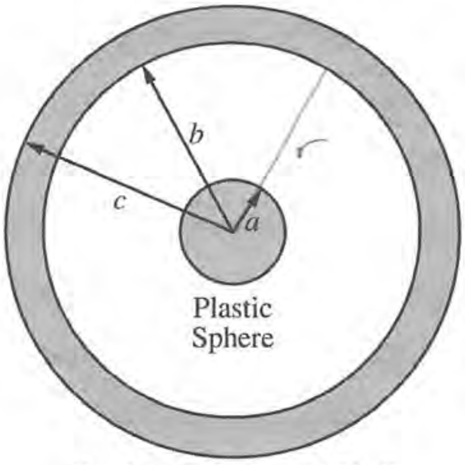

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   (a) Consider a Gaussian sphere of radius r concentric with the plastic sphere. Derive an expression for the charge enclosed by the Gaussian sphere for the following regions.

      i. $r < a$

      $$\int E \cdot dA = \frac{Q_{enc}}{\varepsilon_0} = \frac{\beta a}{\varepsilon_0}$$

      $$E(2\pi r) = \frac{\beta a}{\varepsilon_0} \rightarrow r = \frac{\beta a}{2\pi \varepsilon_0 E}$$

      ii. $a < r < b$

(b) Use Gauss's law to derive an expression for the magnitude of the electric field in the following regions.

i. $r < a$

ii. $a < r < b$

(c) At any point outside of the conducting shell, it is observed that the magnitude of the electric field is zero.

i. Determine the charge on the inner surface of the conducting shell.

Justify your answer.

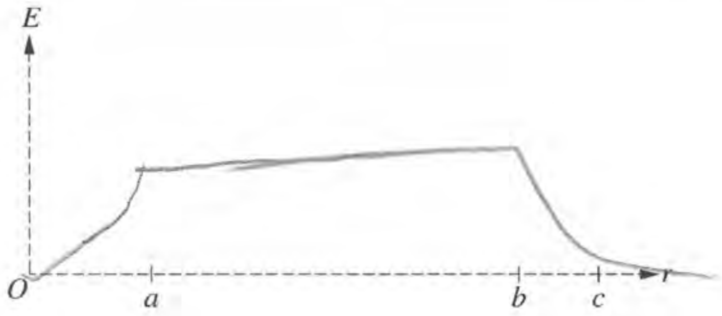
ii. Determine the charge on the outer surface of the conducting shell.

Question 1 continues on the next page.

GO ON TO THE NEXT PAGE.

(d)

i. On the axes below, sketch the electric field E as a function of distance r from the center of the sphere. Sketch the graph for the range $r = 0$ at the center of the sphere to $r = c$ at the outside of the conducting shell.



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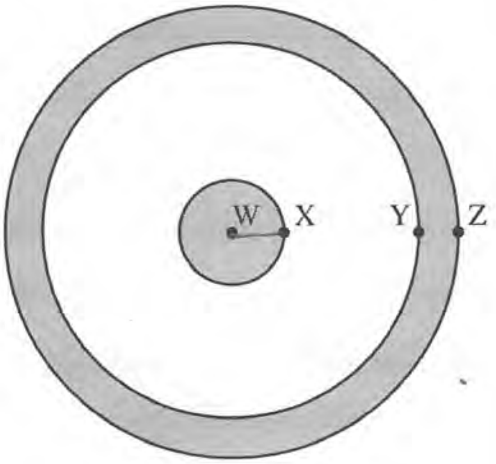
4 W   3 X   1 Y   1 Z

$\frac{kQ_1Q_2}{r}$

(9.0×10^9)



Question 1

Sample Identifier: E Q1 A
Score: 14

- a(i) Full credit ... 3 pts
- a(ii) Full credit ... 1 pt
- b(i) Full credit ... 2 pts
- b(ii) Full credit ... 1 pt
- c(i) 1 pt for charge consistent with a(ii) but does not identify $E_{shell} = 0$
- c(ii) Full credit ... 1 pt
- d(i) Full credit ... 3 pts
- d(ii) Full credit ... 2 pts

Sample Identifier: E Q1 B
Score: 13

- a(i) Full credit ... 3 pts
- a(ii) Full credit ... 1 pt
- b(i) Full credit ... 2 pts
- b(ii) Full credit ... 1 pt
- c(i) 1 pt for charge consistent with a(ii) but does not identify $E_{shell} = 0$
- c(ii) Full credit ... 1 pt
- d(i) Full credit ... 3 pts
- d(ii) 1 pt for points Y & Z equal, but W is not greater than Y & Z

Sample Identifier: E Q1 C
Score: 11

- a(i) 1 pt for correct answer, but integral is missing dr, and no clear indiction that $dV = 4\pi r^2 dr$
- a(ii) Full credit ... 1 pt
- b(i) Full credit ... 2 pts
- b(ii) Full credit ... 1 pt
- c(i) 1 pt for charge consistent with a(ii) but does not identify $E_{shell} = 0$
- c(ii) Full credit ... 1 pt
- d(i) 1 pt for $E = 0$ for $r > b$, 1 pt for correct shape for $a < r < b$, but graph should not be linear for $r < a$
- d(ii) Full credit ... 2 pts

Sample Identifier: E Q1 D
Score: 10

- a(i) Full credit ... 3 pts
- a(ii) Answer is not consistent with a(i) ... No credit
- b(i) 1 pt for correct use of Gauss's law but area is incorrect
- b(ii) Correctly substituted charge from a(ii) ... 1 pt
- c(i) Answer not consistent with a(ii) and no clear indication of $q_{enc} = 0$ ... No credit
- c(ii) Incorrect answer ... No credit
- d(i) Full credit ... 3 pts
- d(ii) Full credit ... 2 pts

Sample Identifier: E Q1 E
Score: 8

- a(i) No integral to determine charge, no indication that $dV = 4\pi r^2 dr$, and incorrect answer ... No credit
- a(ii) Answer consistent with a(i) ... 1 pt
- b(i) Correct area and answer consistent with a(i) ... 2 pts
- b(ii) Answer consistent with a(ii) ... 1 pt
- c(i) 1 pt for charge consistent with a(ii) but does not identify $E_{shell} = 0$
- c(ii) Full credit ... 1 pt
- d(i) 1 pt for $E = 0$ for $r > b$, but incorrect shape for $r < a$, and graph should not approach zero as r approaches b
- d(ii) 1 pt for points Y & Z equal, but W is not greater than Y & Z

Sample Identifier: E Q1 F
Score: 7

- a(i) No integral to determine charge, no indication that $dV = 4\pi r^2 dr$, and incorrect answer ... No credit
- a(ii) Answer consistent with a(i) ... 1 pt
- b(i) Correct area and answer consistent with a(i) ... 2 pts
- b(ii) Answer consistent with a(ii) ... 1 pt
- c(i) 1 pt for charge consistent with a(ii) but does not identify $E_{shell} = 0$
- c(ii) Incorrect answer ... No credit
- d(i) 1 pt for correct shape for $r < a$, 1 pt for $E = 0$ for $r > b$, but graph should not approach zero as r approaches b
- d(ii) $Y \neq Z$ and W is not greater than X ... No credit

Sample Identifier: E Q1 G
Score: 6

- a(i) No integral to determine charge, no indication that $dV = 4\pi r^2 dr$, and incorrect answer ... No credit
- a(ii) Answer consistent with a(i) ... 1 pt
- b(i) Correct area and answer consistent with a(i) ... 2 pts
- b(ii) Answer consistent with a(ii) ... 1 pt
- c(i) Blank ... No credit
- c(ii) Full credit ... 1 pt
- d(i) 1 pt for $E = 0$ for $r > b$, but incorrect shape for $r < a$, and graph should not approach zero as r approaches b
- d(ii) $Y \neq Z$ and W is not greater than X ... No credit

Sample Identifier: E Q1 H
Score: 5

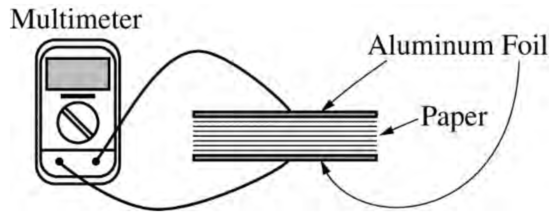
- a(i) No integral to determine charge, no indication that $dV = 4\pi r^2 dr$, and incorrect answer ... No credit
- a(ii) Answer not consistent with a(i) ... No credit
- b(i) No use of Gauss's law and no use of charge from a(i) ... No credit
- b(ii) Student substituted charge from a(ii) into dA in Gauss's Law (instead of for q_{enc}) ... No credit
- c(i) Charge is inconsistent with a(ii) and does not identify $E_{shell} = 0$ ... No credit
- c(ii) $q \neq 0$ ... No credit
- d(i) Full credit ... 3 pts
- d(ii) Full credit ... 2 pts

Sample Identifier: E Q1 I
Score: 3

- a(i) No integral to determine charge, no indication that $dV = 4\pi r^2 dr$, and incorrect answer ... No credit
- a(ii) a should be substuted for r ... No credit
- b(i) No use of Gauss's law and charge from a(i) not properly substituted into Gauss's Law ... No credit
- b(ii) Charge is incorrect and not properly substuted into Gauss's Law ... No credit
- c(i) Charge is inconsistent with a(ii) and electric field IN THE SHELL equal to zero is not indicated ... No credit
- c(ii) Full credit ... 1 pt
- d(i) 1 pt for $E = 0$ for $r > b$, but incorrect shape for $r < a$, and graph should not approach zero as r approaches b
- d(ii) d(ii) 1 pt for points Y & Z equal, but W is not greater than X

Sample Identifier: E Q1 J
Score: 2

- a(i) No integral to determine charge, no dV with $4\pi r^2 dr$, and incorrect answer ... No credit
- a(ii) Blank ... No credit
- b(i) Blank ... No credit
- b(ii) Blank ... No credit
- c(i) Blank ... No credit
- c(ii) Blank ... No credit
- d(i) 1 pt for correct shape for $r < a$, but graph should not approach zero as r approaches b, and $E \neq 0$ for $r > b$
- d(ii) d(ii) 1 pt for points Y & Z equal, but W is not greater than X



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

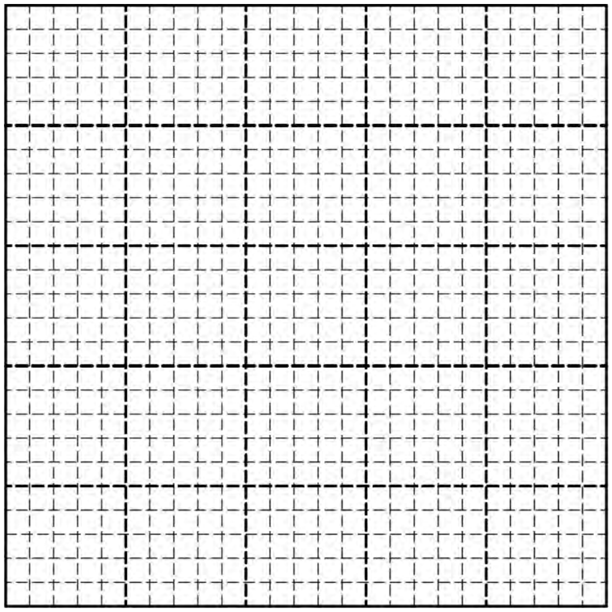
(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

Question 2 continues on the next page.

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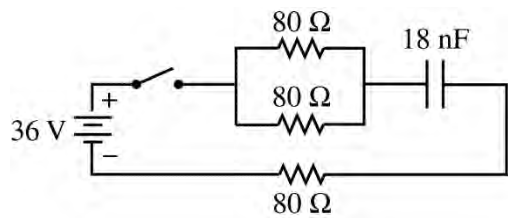


The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

(e) Determine the time constant for this circuit.

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

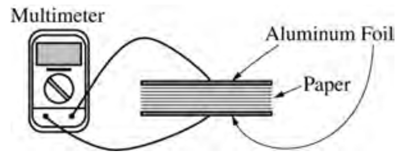
ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Question 2

15 points total **Distribution of points**



An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) 1 point

Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: ____________

Horizontal axis: ____________

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

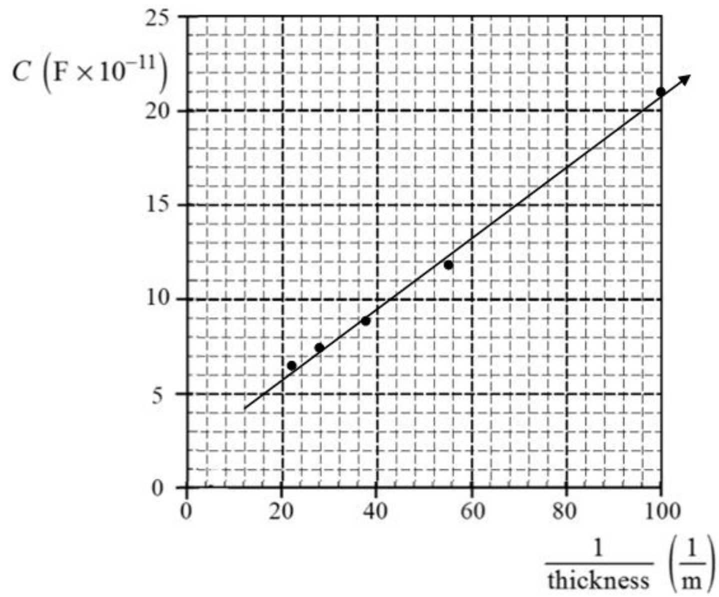
For indicating variables that will create a straight line whose slope can be used to determine the dielectric constant of the paper		1 point
Example: Vertical axis: C *Horizontal axis:* $\dfrac{1}{d}$		
Note: Student earns full credit if axes are reversed or if they use another acceptable combination		

Question 2 (continued)

Distribution of points

(b) 4 points

Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



For a correct scale that uses more than half the grid		1 point
For correctly labeling the axes including units		1 point
For correctly plotting the data		1 point
For drawing a straight line consistent with the plotted data		1 point

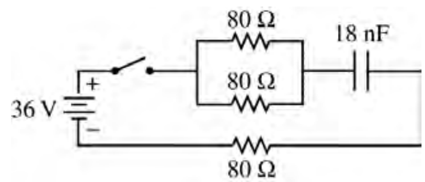
(c) 3 points

Using the straight line, calculate a dielectric constant for the paper.

For correctly calculating the slope from the best-fit line and not the data points unless the points fall on the best-fit line		1 point
$\text{slope} = \dfrac{\Delta y}{\Delta x} = \dfrac{(17-8)\left(\mathrm{F}\times10^{-11}\right)}{(80-32)(1/\mathrm{m})} = 0.19\ \mathrm{F{\bullet}m}$ (Linear regression $= 0.187\ \mathrm{F{\bullet}m}$)		
For correctly relating the slope to the dielectric constant		1 point
$\text{slope} = \kappa\varepsilon_0 A$		
$\kappa = \dfrac{\text{slope}}{\varepsilon_0 A} = \dfrac{(0.19\ \mathrm{F{\bullet}m})}{\left(8.85\times10^{-12}\ \mathrm{C^2/N{\bullet}m^2}\right)\left(0.060\ \mathrm{m^2}\right)}$		
For a correct answer		1 point
$\kappa = 3.58$ (Linear regression $= 3.52$)		

Question 2 (continued)

Distribution of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical $80\ \Omega$ resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors		1 point
$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80\ \Omega} + \frac{1}{80\ \Omega} = \frac{2}{80\ \Omega} = \frac{1}{40\ \Omega}$		
$R_P = 40\ \Omega$		
For using Ohm's law with the potential difference across the capacitor equal to zero		1 point
$I = \frac{V}{R} = \frac{V}{R_P + R_3} = \frac{(36\ \text{V})}{R_P + R_3}$		
For substitution of values for resistance including the value for combined resistance above		1 point
$I = \frac{V}{R_P + R_3} = \frac{(36\ \text{V})}{(40\ \Omega + 80\ \Omega)} = 0.30\ \text{A}$		

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above		1 point
$\tau = R_{eq}C = (120\ \Omega)(18\ \text{nF})$		
For an answer with units consistent with part (d)		1 point
$t = 2.16\ \mu\text{s}$		

Question 2 (continued)

Distribution of points

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i) 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select "Larger than measured"		
For an appropriate explanation		1 point
Example: The battery is not ideal and has internal resistance. So the actual resistance for the circuit is larger than the measured resistance.		

ii) 1 points

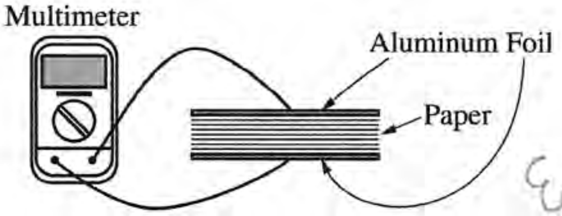
Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select "Larger than measured"		
For an appropriate explanation		1 point
Example: Some of the sheets of paper may be thinner than expected.. So the actual capacitance for the circuit is larger than the measured capacitance.		

E Q2 A p1



$\varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2} \cdot \frac{m^2}{m}$

2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ m^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

$C = \frac{K\varepsilon_0 A}{d}$

Sheets of Paper	d (m)	C (F)	$\frac{\varepsilon_0 A}{d}$ $\left(\frac{C^2}{Nm}\right)$	
500	0.045	6.5×10^{-11}	1.18×10^{-11}	
400	0.036	7.4×10^{-11}	1.475×10^{-11}	
300	0.027	8.9×10^{-11}	1.97×10^{-11}	
200	0.018	11.9×10^{-11}	2.95×10^{-11}	
100	0.010	21.0×10^{-11}	5.31×10^{-11}	

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: C

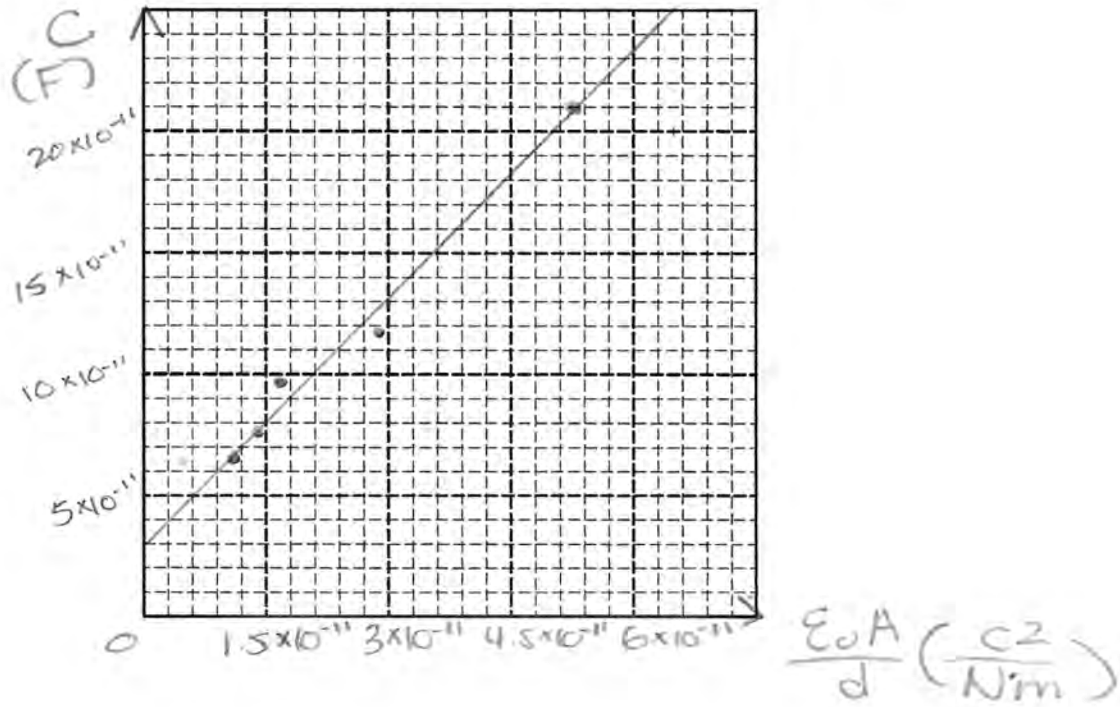
Horizontal axis: $\frac{\varepsilon_0 A}{d}$

$C \uparrow \quad \rightarrow \frac{\varepsilon_0 A}{d}$

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$F = K \underbrace{\frac{\frac{C^2}{Nm^2}(m^2)}{m}}_{\frac{C^2}{Nm}}$$

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

$(1.5\times10^{-11}, 8\times10^{-11})$

$(3\times10^{-11}, 13\times10^{-11})$

$$k = \frac{13\times10^{-11}\,F - 8\times10^{-11}\,F}{3\times10^{-11}\,\frac{C^2}{Nm} - 1.5\times10^{-11}\,\frac{C^2}{Nm}}$$

$$= 3.33\,\frac{FNm}{C^2}$$

Question 2 continues on the next page.

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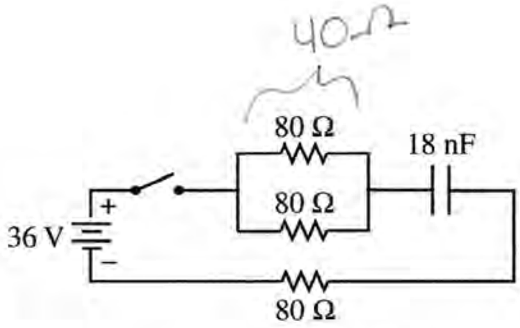


The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$R_{eq} = 40\Omega + 80\Omega = 120\Omega$

$I = \frac{V}{R} = \frac{36V}{120\Omega} = .3A$

(e) Determine the time constant for this circuit.

$\tau = RC$

$= 120\Omega(18 \times 10^{-9} F)$

$= 2.16 \times 10^{-6} \Omega \cdot F$

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ✓ Smaller than measured

Explain experimentally what could account for this.

Resistance in the wires could account for this

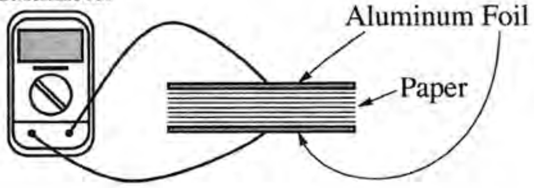
ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

✓ Larger than measured ____ Smaller than measured

$C = \frac{\varepsilon_0 A}{d}$

Explain experimentally what could account for this.

The student could have measured the distance wrong when calculating capacitance.



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ m^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)	$1/d$ (m^{-1})	$C \times 10^{12}$ (pF)
500	0.045	6.5×10^{-11}	22.22	65
400	0.036	7.4×10^{-11}	27.778	74
300	0.027	8.9×10^{-11}	37.04	89
200	0.018	11.9×10^{-11}	55.56	119
100	0.010	21.0×10^{-11}	100	210

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: C

Horizontal axis: d^{-1}

$C = \frac{K \varepsilon_0 A}{d}$ $\quad \frac{C}{d^{-1}} = K A \varepsilon_0$

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

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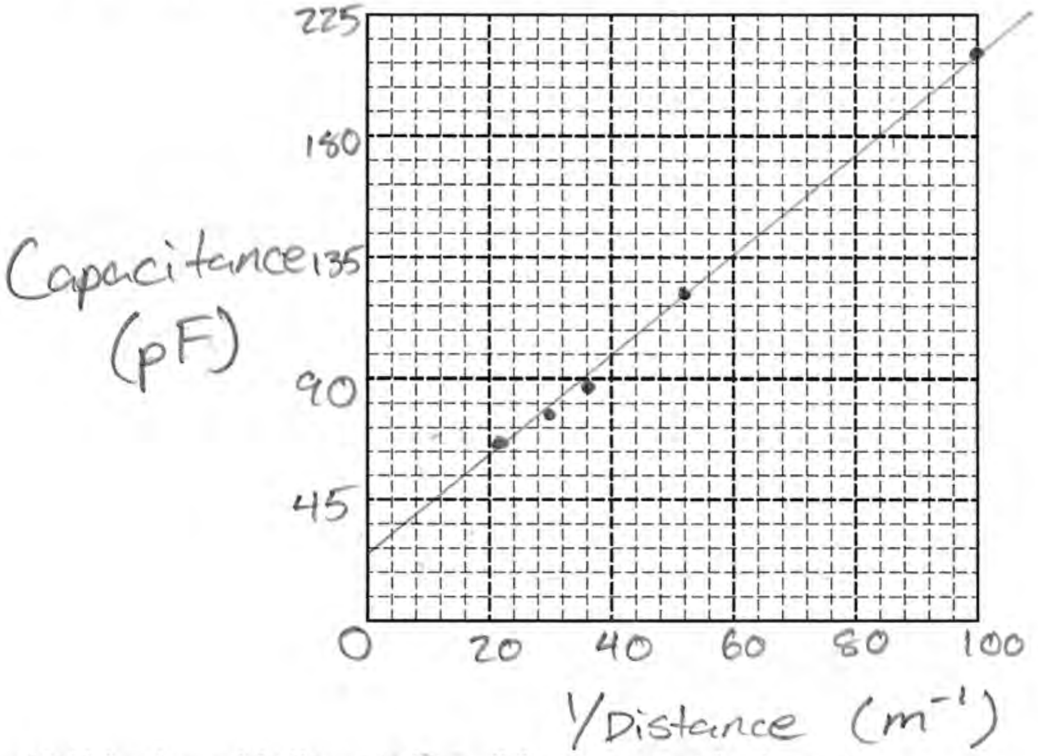
(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{C \times 10^{12}}{d^{-1}} = \frac{180-45}{84-12}$$

$$\text{slope} = 1.875 \times 10^{-12}$$

$$\frac{(\text{slope})}{A\,\varepsilon_0} = K$$

$$\boxed{K \approx 3.53}$$

Question 2 continues on the next page.

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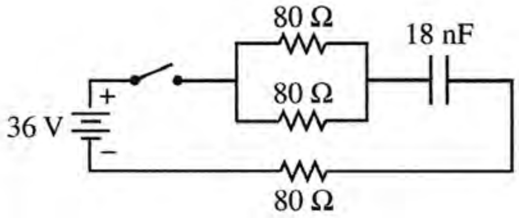


The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$R_{eq} = \frac{1}{\frac{1}{80}+\frac{1}{80}} + 80 = 120\Omega$

$V = IR$ $\boxed{I = 0.3A}$

(e) Determine the time constant for this circuit.

$CR_{eq} = \tau$

$\boxed{\tau = 2.16\times10^{-6}\ s}$

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

__✓__ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

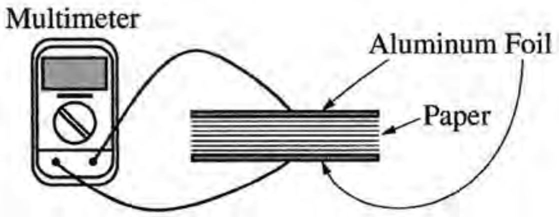
a larger resistance results in a larger time constant

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

__✓__ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

If the resistance is correct then the capacitance must be larger



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)	$\varepsilon_0 A/d$ ($C^2/N\cdot m$)	
500	0.045	6.5×10^{-11}	1.18×10^{-11}	
400	0.036	7.4×10^{-11}	1.475×10^{-11}	
300	0.027	8.9×10^{-11}	1.967×10^{-11}	
200	0.018	11.9×10^{-11}	2.95×10^{-11}	
100	0.010	21.0×10^{-11}	5.31×10^{-11}	

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: C

Horizontal axis: $\varepsilon_0 A/d = \frac{5.31\times10^{-12}}{d}$

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$C = \frac{K\varepsilon_0 A}{d}$ $y = mx$ $y = C$ $x = \frac{\varepsilon_0 A}{d} = \frac{C^2}{N\cdot m}$ $= \frac{8.85\times10^{-12}\cdot .06}{d}$ $\frac{5.31\times10^{-13}}{d}$

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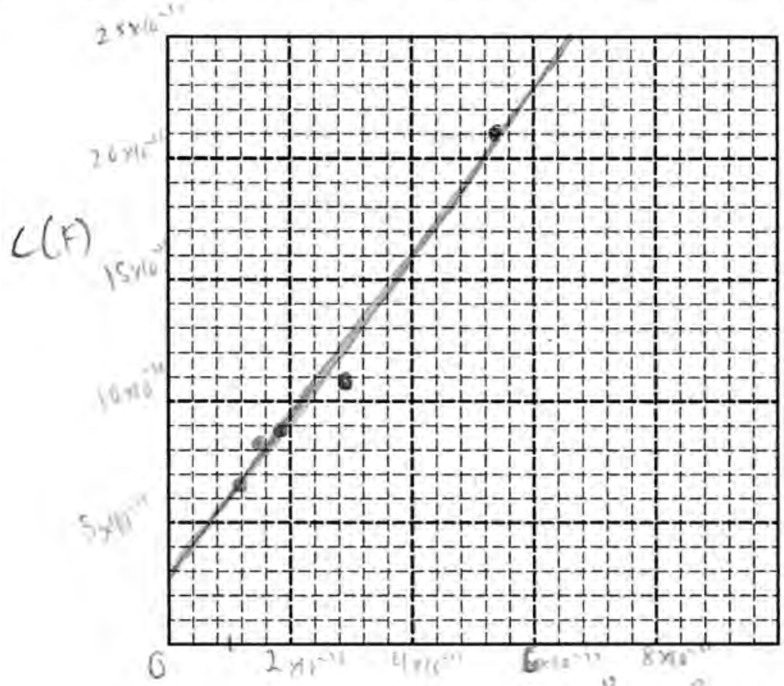
(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



$$\frac{\varepsilon_0 A}{d} = \frac{5.31\times10^{-13}}{d} \left(\frac{C^2}{Nm}\right)$$

(c) Using the straight line, calculate a dielectric constant for the paper.

Using linear regression via calculator, $y = 3.5168x + 2.0796\times10^{-11}$

thus, $m = 3.5168$

$K_{paper} = 3.52 \frac{FNm}{C^2}$

Question 2 continues on the next page.

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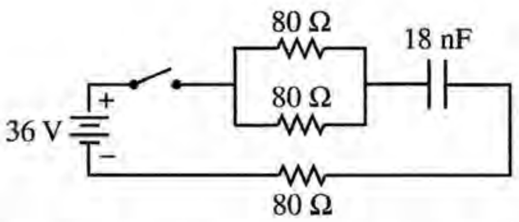


The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$V=IR \quad I=\frac{V}{R} \quad R_c=0 \quad R_{eq}=(80^{-1}+80^{-1})^{-1}+80=120\Omega$

$R_{eq}=120\Omega$

$I=\frac{36V}{120\Omega}=.3A$

$\boxed{I=.3A}$

(e) Determine the time constant for this circuit.

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

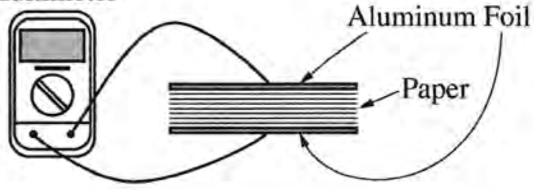
____ Larger than measured   _X_ Smaller than measured

Explain experimentally what could account for this.

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

_X_ Larger than measured   ____ Smaller than measured

Explain experimentally what could account for this.



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)	$\varepsilon_0 A / d$	
500	0.045	6.5×10^{-11}	1.18×10^{-11}	
400	0.036	7.4×10^{-11}	1.475×10^{-11}	
300	0.027	8.9×10^{-11}	1.967×10^{-11}	
200	0.018	11.9×10^{-11}	2.95×10^{-11}	
100	0.010	21.0×10^{-11}	5.31×10^{-11}	

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: C

$C = \frac{K \varepsilon_0 A}{d}$

Horizontal axis: $\varepsilon_0 A / d$

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

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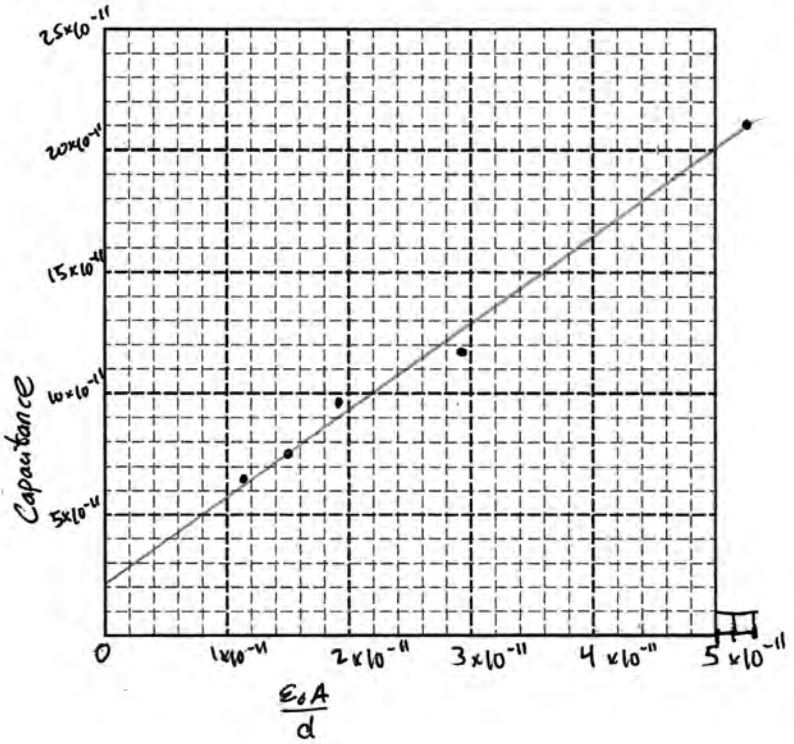
(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

$$\frac{18\times10^{-11} - 3\times10^{-11}}{4.4\times10^{-11} - 0.2\times10^{-11}} = \frac{1.5\times10^{-10}}{4.2\times10^{-11}} \boxed{= 3.57}$$

Question 2 continues on the next page.

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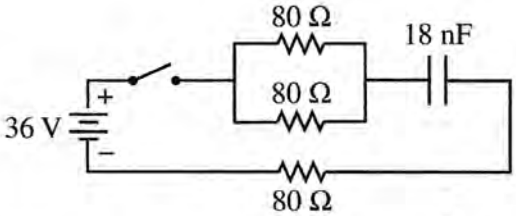


The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$R_{EQ} = 80 + \left(\frac{1}{\frac{1}{80}+\frac{1}{80}}\right) = 80 + 40 = 120\,\Omega$

$V = IR$

$36 = I \cdot 120 \quad \boxed{I = 0.3A}$

(e) Determine the time constant for this circuit.

$\tau = \frac{R}{C} = \frac{120}{3.57} = 33.613$

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

__✓__ Larger than measured ____ Smaller than measured

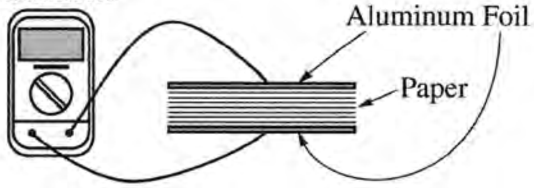
Explain experimentally what could account for this.

Resistence in the wires would add more resistence

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured __✓__ Smaller than measured

Explain experimentally what could account for this.



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ m^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
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300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

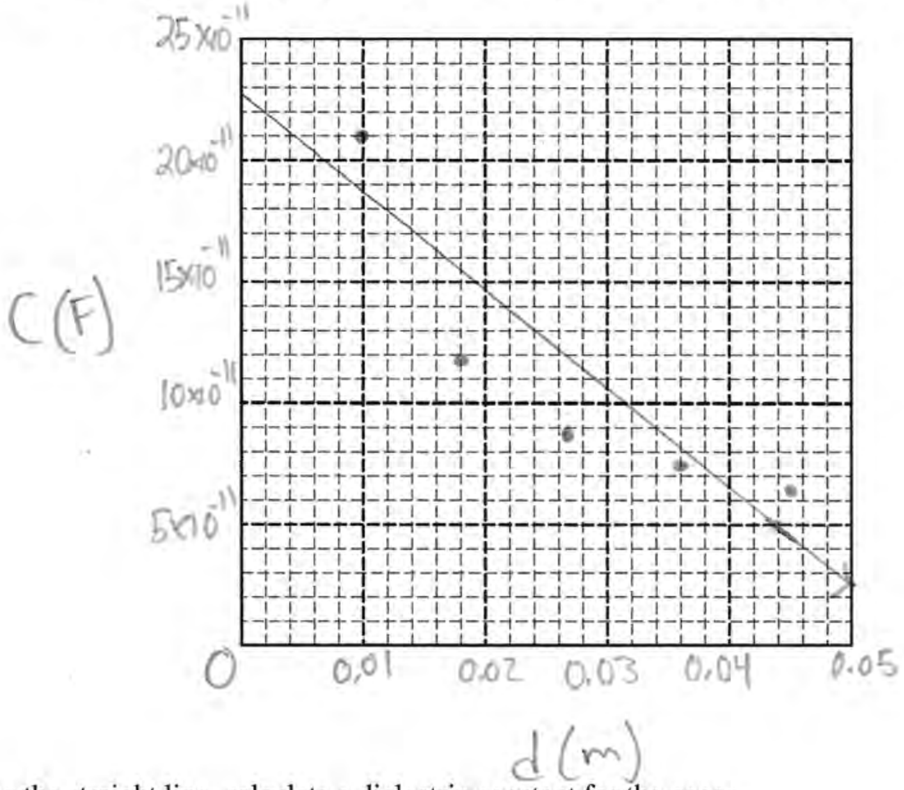
Vertical axis: C (F)

Horizontal axis: d (m)

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$C = \frac{K \varepsilon_0 A}{d}$

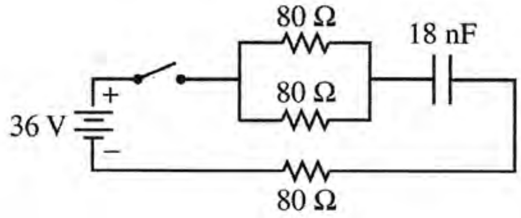
(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

$$K = \frac{(5\times10^{-11}) - (18\times10^{-11})}{0.044 - 0.012} = \boxed{-4.06\times10^{-9}}$$

Question 2 continues on the next page.



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$$\frac{1}{\frac{1}{80}+\frac{1}{80}} = 40+80 = 120\,\Omega$$

$$V_T = I_T R_T \qquad \frac{36}{120} = \boxed{I = 0.3\text{A}}$$

(e) Determine the time constant for this circuit.

$$\tau = RC$$

$$\tau = (120)(18\times10^{-9}) = \boxed{2.16\times10^{-6}}$$

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

_X_ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

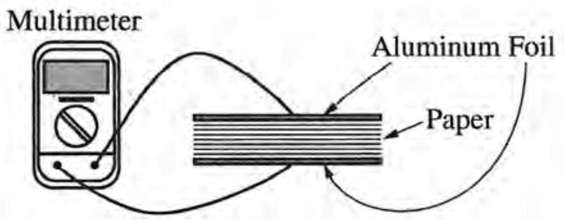
The cross-sectional Area of the wire could've been greater than expected causing the actual resistance to increase

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured _X_ Smaller than measured

Explain experimentally what could account for this.

their measurements for capacitance could've been measured inaccurately



2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060 \text{ m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: d(m)

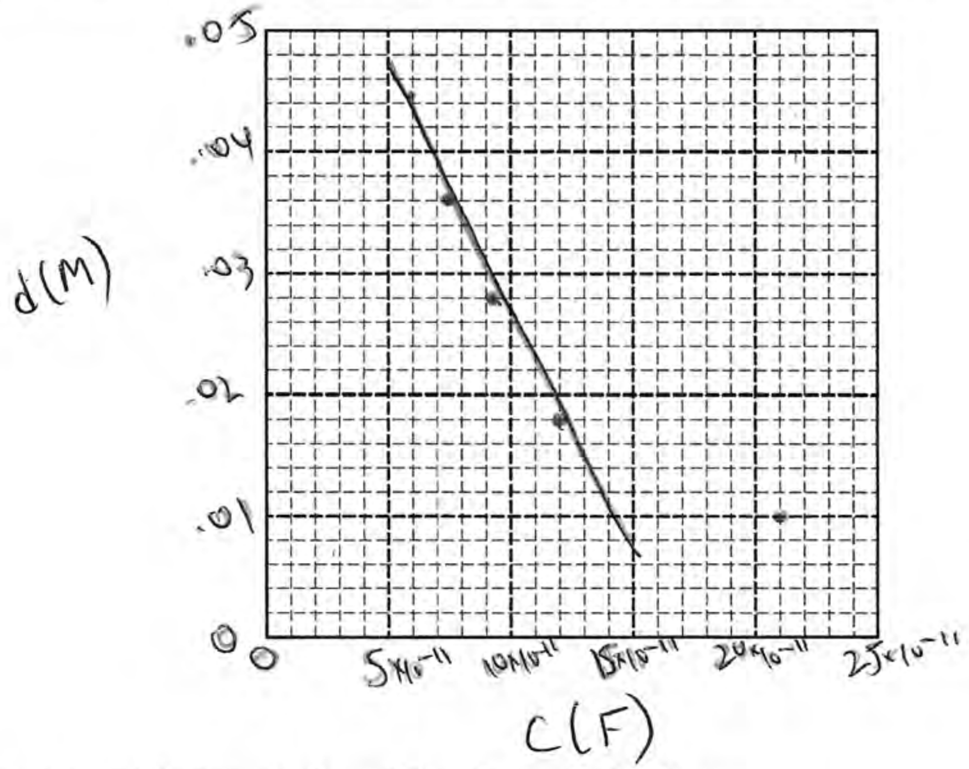
Horizontal axis: C (F)

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$\cancel{Cd = \frac{\kappa \varepsilon_0 A}{\varepsilon_0 A}}$$

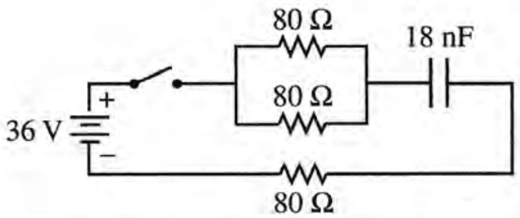
(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

$$\left(\frac{.045 - .018}{6.5\times10^{-11} - 11.9\times10^{-11}}\right)^{-1} = \boxed{-2\times10^{-9}\ F/m}$$

Question 2 continues on the next page.



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$V = IR$

$36V = I(120\,\Omega)$

$\boxed{.3A = I}$

(e) Determine the time constant for this circuit.

$V = TC$

$36V = T(18 \times 10^{-9}\,F)$

$\boxed{T = 2 \times 10^{9}\,s}$

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

_✓_ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

The wire adds resistance which is unaccounted for

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured _✓_ Smaller than measured

Explain experimentally what could account for this.

The capacitor might be old and not functioning correctly

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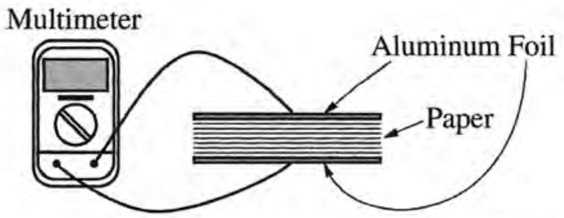


$C = \frac{k \varepsilon_0 A}{d}$

$C = \frac{k \varepsilon_0 (.06)}{d}$ $k = \frac{Cd}{.06 \varepsilon_0}$

2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060 \text{ m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)	ln C (F^{-1})	
500	0.045	6.5×10^{-11}	1.87	
400	0.036	7.4×10^{-11}	2.00	
300	0.027	8.9×10^{-11}	2.19	
200	0.018	11.9×10^{-11}	~~B~~ 2.48	
100	0.010	21.0×10^{-11}	3.04	

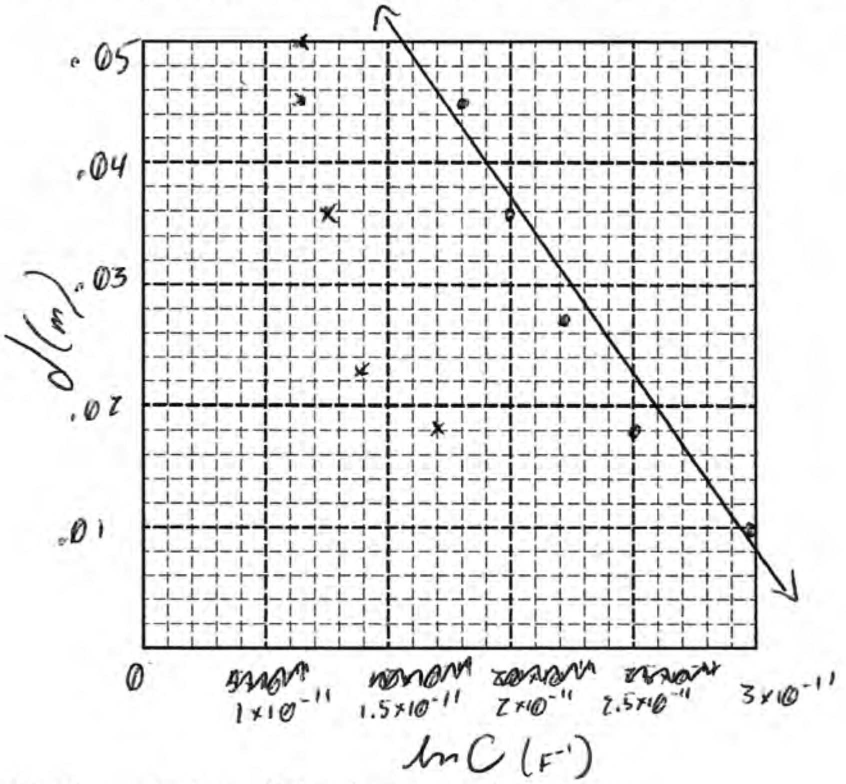
(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: d

Horizontal axis: ln C

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

$$C = \frac{k \varepsilon_0 A}{d}$$

$$\int AC\, dx = Cd \qquad k = \frac{Cd}{.06\varepsilon_0}$$

where x = distance

$$m = \frac{.045 - .010}{1.57 - 3.04} = -3$$

Question 2 continues on the next page.

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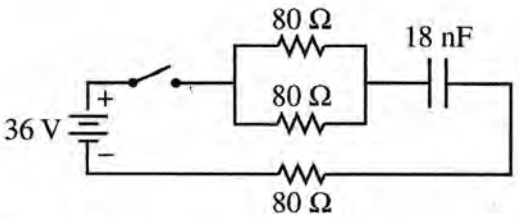

The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$R_T = 80\Omega + \left(\frac{1}{80} + \frac{1}{80}\right)^{-1} = 80 + 40 = 120\Omega$

$V = IR \quad 36 = 120I \quad \boxed{I = 3.33\ A}$

(e) Determine the time constant for this circuit.

$\tau =$

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

✓ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

if the students disregard the internal resistance of the built cap., such as the high resistance of Al. foil, the current calculated would be falsely high due to a higher than calculated resistance, making the cap. charge unexpectedly slow

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

✓ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

if the students determined the C value by discharging it, innefficiencies in the discharge circuit such as internal resistance of wire used or insufficiently accurate meters could skew the results to suggest a falsely low capacitance

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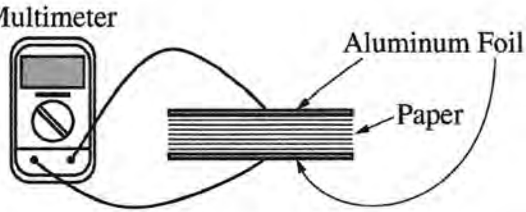


E Q2 H p1

2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
400	0.036	7.4×10^{-11}		
300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

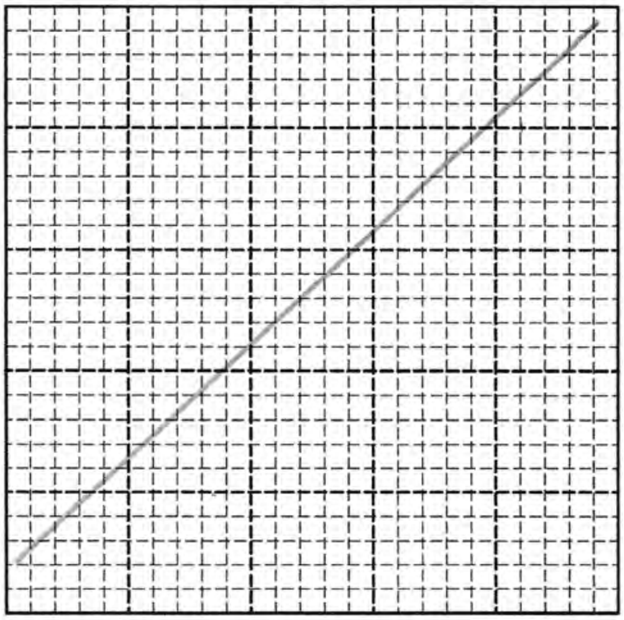
Vertical axis: Sheets of Paper

Horizontal axis: capacitance

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

$$C = \frac{k_0 \varepsilon_0 A}{d}$$

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

$\kappa \neq 1$

Question 2 continues on the next page.

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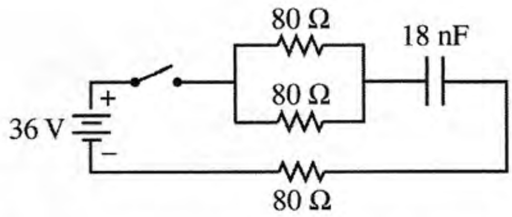

The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$R_{eq} = 80 + \left(\frac{1}{80} + \frac{1}{80}\right)^{-1} = 120\Omega$

$I(0) = \frac{\mathcal{E}}{R} e^{0} = \frac{\mathcal{E}}{R} = \frac{36}{R_{eq}} = \frac{36}{120} = 0.3\ \text{A}$

(e) Determine the time constant for this circuit.

$\tau = RC = R_{eq}\,(18\ \mu F)$

$R_{eq} = 80 + \left(\frac{1}{80} + \frac{1}{80}\right)^{-1} = 120\ \Omega$

$\boxed{\tau = 120 \cdot 18 \cdot 10^{-6} = 2160 \times 10^{-6}\ s}$

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ✓ Smaller than measured

Explain experimentally what could account for this.

Since the time is longer than expected one must conclud the time constan is larger than it is, since student A assumed the capacitance was correct He must think He measured the resistance to High

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ✓ Smaller than measured

Explain experimentally what could account for this.

Since the time is longer than expected one must think the time constant is to large, since student B assumes the resistance is correct He must assume He measured the capacitance as to large

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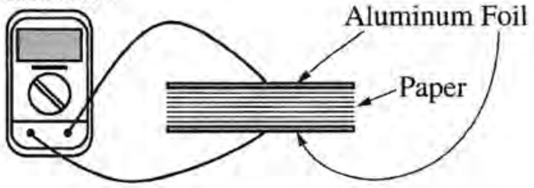

2. An experiment is designed to measure the dielectric constant of paper that has an area $A = 0.060\ \text{m}^2$. Using aluminum foil, two parallel plates are created with the same area as the paper. Five hundred sheets of paper are placed between the aluminum foil plates to create a parallel plate capacitor, as shown in the figure above. Using a multimeter, the capacitance C of the capacitor is measured. The number of sheets and the total thickness d of the stack of paper are recorded. The experiment is repeated, reducing the number of sheets of paper each time. The data are recorded in the table below.

Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
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300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: __________

Horizontal axis: Sheets of paper

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

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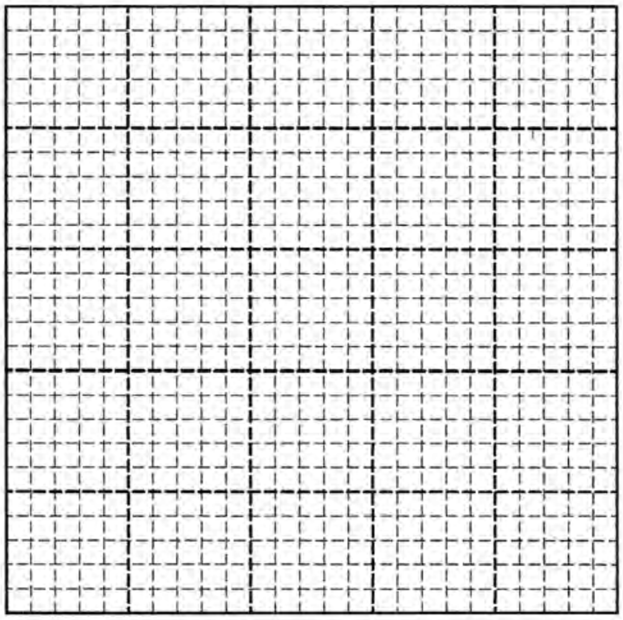
(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

Question 2 continues on the next page.

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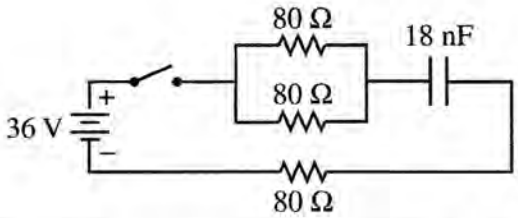

The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$V = IR$

$\frac{1}{80} + \frac{1}{80} = \frac{2}{80}$ $\frac{80}{2} = 40$

$36 = I(120)$

$40 + 80 = 120$

$I = \boxed{0.3}$

(e) Determine the time constant for this circuit.

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

__✓__ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Less current due to the increased resistance would make it take longer for the capacitor to accumulate a charge.

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

__✓__ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

A larger capacitor would take longer to get charged up

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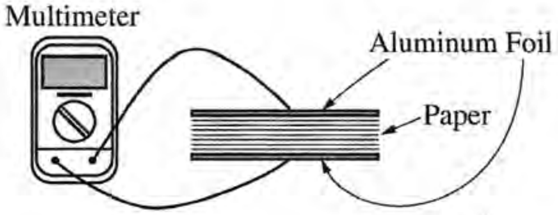

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Sheets of Paper	d (m)	C (F)		
500	0.045	6.5×10^{-11}		
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300	0.027	8.9×10^{-11}		
200	0.018	11.9×10^{-11}		
100	0.010	21.0×10^{-11}		

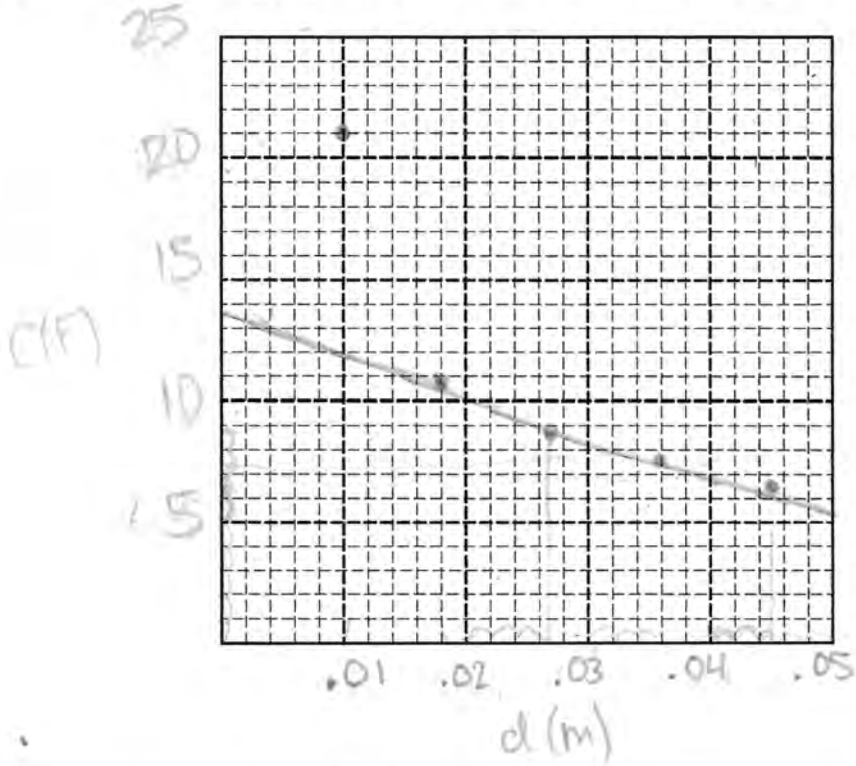
(a) Indicate below which quantities should be graphed to yield a straight line whose slope could be used to calculate a numerical value for the dielectric constant of the paper.

Vertical axis: C(F)

Horizontal axis: d(m)

Use the remaining columns in the table above, as needed, to record any quantities that you indicated that are not given. Label each column you use and include units.

(b) Plot the data points for the quantities indicated in part (a) on the graph below. Clearly scale and label all axes, including units if appropriate. Draw a straight line that best represents the data.



(c) Using the straight line, calculate a dielectric constant for the paper.

Question 2 continues on the next page.

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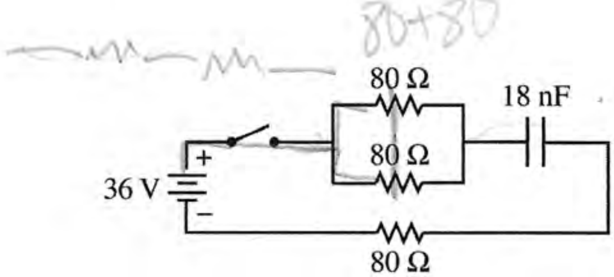
80+80



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) Calculate the current in the battery immediately after the switch is closed.

$36 - I160 + 18 - 80I = 0$

$36 - 240I + 18 = 0$

$54 = 240I$ $I = 4.44\text{ A}$

(e) Determine the time constant for this circuit.

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

_✓_ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

If you have more resistance than current, not a lot of current will get through the resistor

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured _✓_ Smaller than measured

Explain experimentally what could account for this.

If the resistance is correct, that means you don't have a lot of charge in your capacitor, which causes voltage to take longer than expected

GO ON TO THE NEXT PAGE.

Question 2

Sample Identifier: E Q2 A
Score: 14

- a) Full credit ... 1 pt
- b) Full credit ... 4 pts
- c) Full credit ... 3 pts
- d) Full credit ... 3 pts
- e) Full credit ... 2 pts
- f(i) Explanation is ok but incorrect selection ... No credit
- f(ii) Full credit ... 1 pt

Sample Identifier: E Q2 B
Score: 13

- a) Full credit ... 1 pt
- b) Full credit ... 4 pts
- c) Full credit ... 3 pts
- d) Full credit ... 3 pts
- e) Full credit ... 2 pts
- f(i) No connection to experiment ... No credit
- f(ii) No connection to experiment ... No credit

Sample Identifier: E Q2 C
Score: 11

- a) Full credit ... 1 pt
- b) Full credit ... 4 pts
- c) Correct slope calculation through linear regression ok ... 3 pts
- d) Full credit ... 3 pts
- e) Full credit ... 2 pts
- f(i) Blank ... No credit
- f(ii) No explanation ... No credit

Sample Identifier: E Q2 D
Score: 10

- a) Full credit ... 1 pt
- b) 1 pt for appropriate scale, 1 pt for data points, 1 point for appropriate trend line, but no units on axes
- c) 1 pt for slope calculation, 1 pt for correct answer, but did not show relation between k and slope (so did not indicate what the answer means) ... 3 pts
- d) Full credit ... 3 pts
- e) Incorrect equation and no units on answer ... No credit
- f(i) Full credit ... 1 pt
- f(ii) Incorrect selection and no connection to experiment ... No credit

Sample Identifier: E Q2 E
Score: 9

- a) Variables do not yeild liner graph … No credit
- b) Graph ok and consistent with choice of variables … 4 pts
- c) 1 pt for calculation of slope, but slope does not equal k and incorrect answer
- d) Full credit … 3 pts
- e) Correct equation and no units on answer … 1 pt
- f(i) Explantion would result in smaller resistance … No credit
- f(ii) No connection to experiment … No credit

Sample Identifier: E Q2 F
Score: 7

- a) Variables do not yeild liner graph … No credit
- b) 1 pt for appropriate scale, 1 pt for axes labeled with units, 1 pt for data points consistent with part (a), but best fit line ignored data point
- c) Slope calculated from data points and not best fit line, slope not related to k, and incorrect answer … No credit
- d) Full credit … 3 pts
- e) Incorrect equation and incorrect answer … No credit
- f(i) Full credit … 1 pt
- f(ii) Incorrect explanation … No credit

Sample Identifier: E Q2 G
Score: 6

- a) Variables do not yeild liner graph … No credit
- b) 1 pt for best fit line, but horizontal axis has incorrect units, scale does not use half of graph in x-direction, and data points not correctly plotted
- c) Slope calculated incorrectly (incorrect values for ln C), slope incorrectly related to k, and wrong answer … No credit
- d) Full credit … 3 pts
- e) No equation or answer … No credit
- f(i) Full credit … 1 pt
- f(ii) Full credit … 1 pt

Sample Identifier: E Q2 H
Score: 4

- a) Variables do not yeild liner graph ... No credit
- b) No scale, no labeling of axis, no data points, so trend line ignored ... No credit
- c) No calculation of slope, no relation to k, and wrong answer ... No credit
- d) Full credit ... 3 pts
- e) 1 pt for correct equation but incorrect answer
- f(i) No connection to experiment ... No credit
- f(ii) No connection to experiment ... No credit

Sample Identifier: E Q2 I
Score: 3

- a) Only one variable listed ... No credit
- b) Blank ... No credit
- c) Blank ... No credit
- d) Full credit ... 3 pts
- e) Blank ... No credit
- f(i) No connection to experiment ... No credit
- f(ii) No connection to experiment ... No credit

Sample Identifier: E Q2 J
Score: 2

- a) Variables do not yeild liner graph ... No credit
- b) 1 pt for appropriate scale, 1 point for labeled axes, but data points are not correct and trend line no appropriate for plotted data
- c) Blank ... No credit
- d) Full credit ... 3 pts
- e) Blank ... No credit
- f(i) No connection to experiment ... No credit
- f(ii) No connection to experiment ... No credit

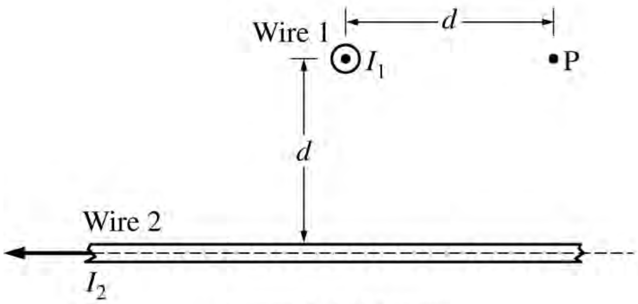


Figure 1. Side view

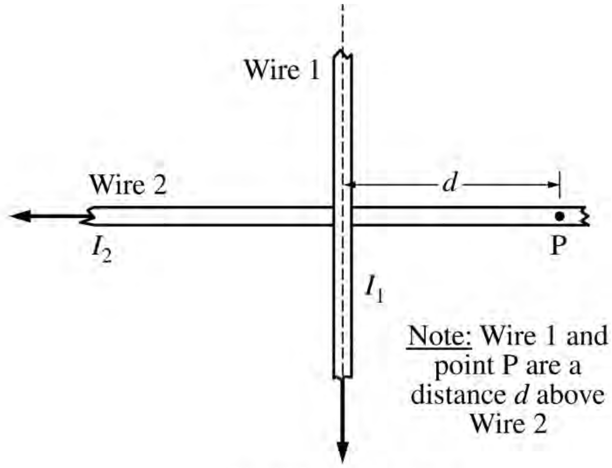


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

(b) Derive an expression for the magnitude of the net magnetic field at point P.

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

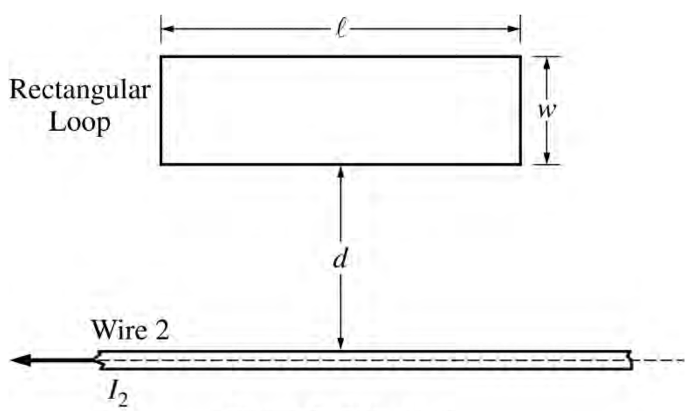


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

Question 3

15 points total **Distribution of points**

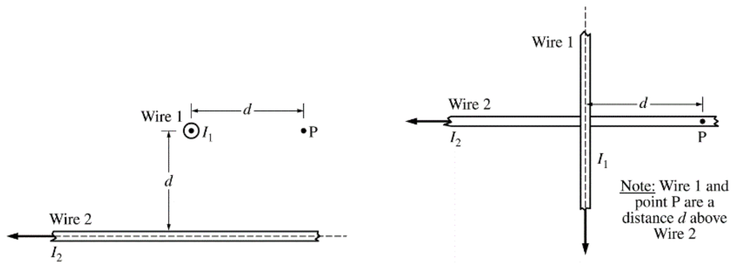


Figure 1. Side view Figure 2. Top view

The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance *d* above wire 2. Point P is a horizontal distance *d* from wire 1 and a distance *d* directly above wire 2. Express your answers to parts (a) and (b) in terms of I, *d*, and physical constants, as appropriate.

(a) 2 points

Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

For attempting to use Ampere's law to calculate the magnetic field at point *P*		1 point
$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$		
$B(2\pi d) = \mu_0 I$		
For a correct answer		1 point
$B = \dfrac{\mu_0 I}{2\pi d}$		

(b) 2 points

Derive an expression for the magnitude of the net magnetic field at point P.

For indicating $B_2 = 2B_1$		1 point
$B_1 = \dfrac{\mu_0 I}{2\pi d}$ & $B_2 = \dfrac{\mu_0 (2I)}{2\pi d} = \dfrac{\mu_0 I}{\pi d}$		
For an indication that the magnitude of the magnetic field is the vector sum of B_1 and B_2		1 point
$B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{B^2 + B^2} = \sqrt{\left(\dfrac{\mu_0 I}{2\pi d}\right)^2 + \left(\dfrac{\mu_0 (2I)}{2\pi d}\right)^2} = \dfrac{\sqrt{5}\mu_0 I}{2\pi d}$		

Question 3 (continued)

Distribution of points

(c) 2 points

Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

For correctly relating the angle to the individual magnetic fields		1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right)$		
For correctly substituting B_1 and B_2 into the equation		1 point
$\theta = \tan^{-1}\left(\frac{B_1}{B_2}\right) = \tan^{-1}\left(\frac{\left(\frac{\mu_0 I}{2\pi d}\right)}{\left(\frac{\mu_0(2I)}{2\pi d}\right)}\right) == \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$		

(d) 2 points

Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

For stating there is no translational motion since the magnetic forces on the wire cancel.		1 point
For stating there is a net torque which causes rotation.		1 point
Example: In Figure 2, the top portion of wire 1 will be in a magnetic field into the page from wire 2 and thus will experience a force to the right. The bottom portion of wire 1 will be in a magnetic field out of the page from wire 2 and thus will experience a force to the left. So the net force will be zero but there will be a net clockwise torque so the wire will rotate clockwise.		

Question 3 (continued)

Distribution of points

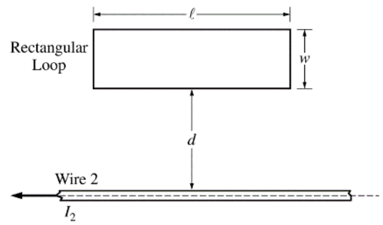


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1-kt)$, where k is a positive constant with units of s^{-1}.

(e) 1 point

Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

__X__ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$ ____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

For selecting $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$		1 point

Question 3 (continued)

Distribution of points

(f) 3 points

Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell(1-kt)}{\pi}\ln\left(\frac{d+w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

For attempting to take the time derivative of the magnetic flux to calculate the emf		1 point
$\varepsilon = \left\lvert -\frac{d\Phi}{dt}\right\rvert = \frac{\mu_0 I_0 \ell}{\pi}\ln\left(\frac{d+w}{d}\right)\left\lvert\frac{d}{dt}[(1-kt)]\right\rvert$		
$\varepsilon = \frac{\mu_0 I_0 k\ell}{\pi}\ln\left(\frac{d+w}{d}\right)$		
For dividing the emf by the resistance to calculate the current		1 point
$I = \frac{\varepsilon}{R} = \frac{\frac{\mu_0 I_0 k\ell}{\pi}\ln\left(\frac{d+w}{d}\right)}{R}$		
For a correct answer		1 point
$I = \frac{\mu_0 I_0 k\ell}{\pi R}\ln\left(\frac{d+w}{d}\right)$		

(g) 3 points

What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise   ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

For selecting "Clockwise" with an attempt at a relevant justification		1 point
For indicating that the flux inside the loop will decrease		1 point
For using Lenz's law to relate the decrease in the flux to the clockwise current		1 point
Example: Since the current in the wire is decreasing, the flux in the loop will decrease. According to Lenz's law, the induced current should create a magnetic field to oppose this decrease. Thus the induced magnetic field must be into the page and the current in the loop must be clockwise.		

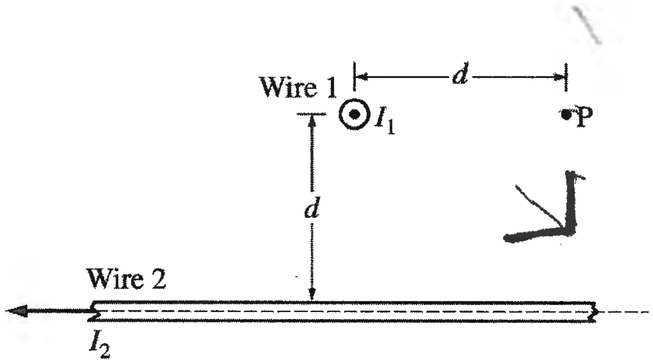


Figure 1. Side view

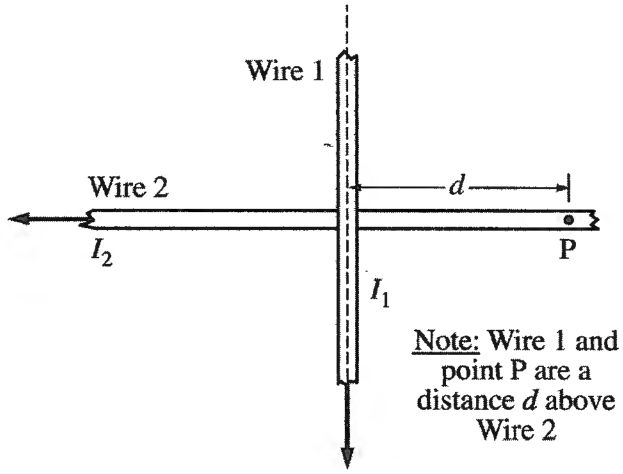


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

Imagine Amperian loop centered at I_1 of radius d.

$\oint B \cdot dl = \mu_0 I$,

$\oint B \cdot dl = B \oint dl = B(2\pi d)$

$B = \frac{\mu_0 I}{2\pi d}$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

$B_{wire\,1} = \frac{\mu_0 I}{2\pi d}$ up (Fig 1), $B_{wire\,2} = \frac{\mu_0 (2I)}{2\pi d}$ in page (Fig 1)

$B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 I}{2\pi d}\right)^2 + \left(\frac{\mu_0 I}{\pi d}\right)^2} = \sqrt{\frac{5\mu_0^2 I^2}{4\pi^2 d^2}} = \frac{\sqrt{5}\mu_0 I}{2\pi d}$,

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

Summing vectors:

B_1, B_2 =

B_1, B_2, $\tan\theta = \frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{2\pi d}}{\frac{\mu_0 I}{\pi d}} = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$.

GO ON TO THE NEXT PAGE.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

✓ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

(Figure 2) The B field on the upper part of wire 1 will be into the page, and the lower part will be out of the page, by the right hand rule. Since $F = I\ell \times B$, the force on the upper part is east and the lower part is west, causing clockwise rotation.

N E W S

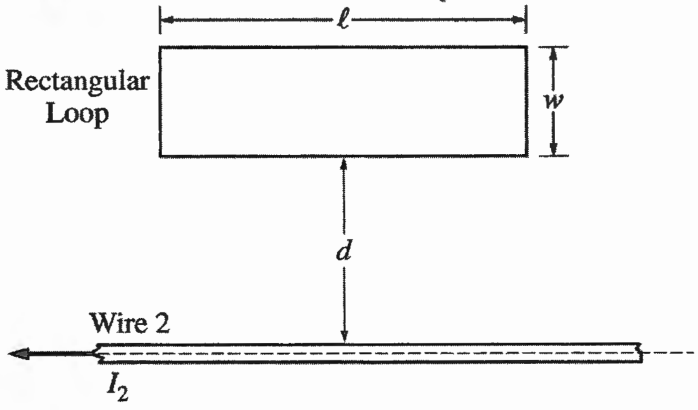


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1-kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

✓ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

Faraday's Law: $\mathcal{E} = -\frac{d\Phi}{dt} = \frac{\mu_0 I_0 \ell k}{\pi} \ln\left(\frac{d+w}{d}\right)$

$$I = \frac{\mathcal{E}}{R} = \frac{\mu_0 I_0 \ell k \ln\left(\frac{d+w}{d}\right)}{\pi R}$$

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

✓ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

The B field from I_2 is going in the page by the right hand rule, and is decreasing. By Lenz's Law, the current will flow to oppose the change, and therefore flow to induce a B into the page. This is achieved with a clockwise current, by the right hand rule.

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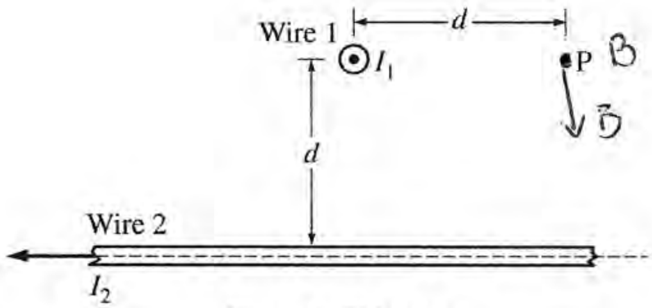


Figure 1. Side view

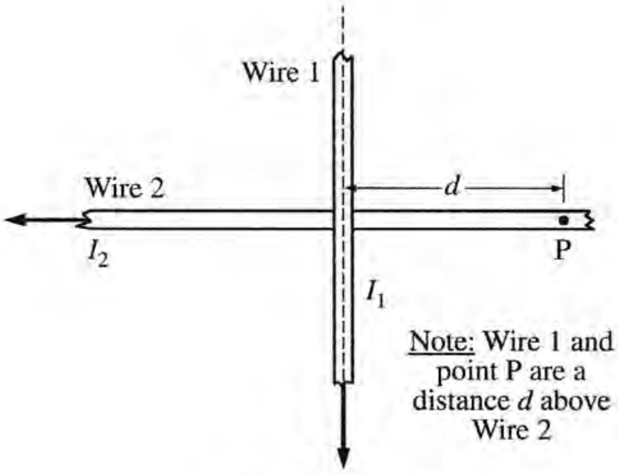


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

Ampere's law

$B = \frac{\mu_0 I}{2\pi r}$ $\quad \vec{B} = \frac{\mu_0 I_1}{2\pi d} = \frac{\mu_0 I}{2\pi d}$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

$\vec{B}_2 = \frac{\mu_0 I_2}{2\pi d}$ $\quad \vec{B}_1 = \frac{\mu_0 I_1}{2\pi d}$

$\vec{B}_{net} = \sqrt{\vec{B}_1^2 + \vec{B}_2^2} = \sqrt{\frac{\mu_0^2 I_2^2}{4\pi^2 d^2} + \frac{\mu_0 I_1^2}{4\pi^2 d^2}} = \sqrt{\frac{\mu_0^2 (I_2^2 + I_1^2)}{4\pi^2 d^2}} = \frac{\mu_0}{2\pi d}\sqrt{I_2^2 + I_1^2}$

$\Uparrow$

$\vec{B}_{net} = \frac{\mu_0}{2\pi d}\sqrt{4I^2 + I^2} = \frac{\mu_0 I}{2\pi d}\sqrt{5}$

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

~~tan(θ)=~~

$\tan\theta = \frac{\vec{B}_1}{\vec{B}_2} = \frac{\frac{\mu_0 I}{2\pi d}}{\frac{\mu_0 2I}{2\pi d}} = \frac{1}{2}$ $\quad \tan^{-1}\left(\frac{1}{2}\right) = \theta$

$\theta = 26.56^\circ$

GO ON TO THE NEXT PAGE.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

_✓_ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer. According to the right hand rule, the magnetic field created by wire 2 will be into the page, which means $\vec{B}_2$ is parallel to wire 1. Since $\vec{F} = \vec{I}\ell\vec{B}$, the force will be zero as B & I are parallel.

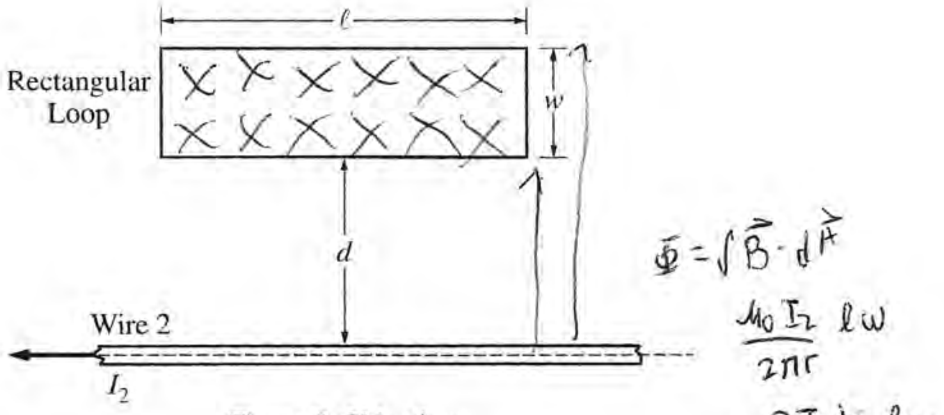


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\cancel{\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr}$

_✓_ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\cancel{\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr}$

Question 3 continues on the next page.

GO ON TO THE NEXT PAGE.

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

$I = \frac{\mathcal{E}}{R} \qquad \mathcal{E} = -\frac{d\Phi}{dt} = \boxed{\frac{\mu_0 I_0 \ell k}{\pi} \ln\left(\frac{d+w}{d}\right)}$

$I = \frac{\mu_0 I_0 \ell k}{\pi R} \ln\left(\frac{d+w}{d}\right)$

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

_✓_ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

According to the equation above, $\Phi = \frac{\mu_0 I_0 \ell (1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$, as time goes on, Φ decreases. According to the right hand rule, wire 2 creates a $\vec{B}$ Field into the page ⊗. This means that because the flux is decreasing and the area stays the same, $\vec{B}$ into the page must be decreasing. In order to oppose this change in flux, Lenz's law states that a magnetic field will be created by an induced current. To compensate for the decrease in $\vec{B}$ into the page, the current must run clockwise to create a $\vec{B}$ into the page.

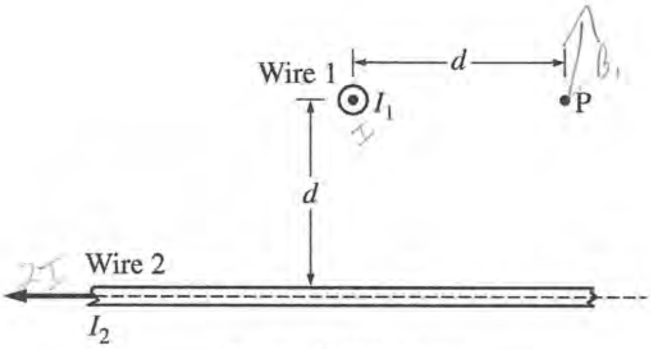


Figure 1. Side view

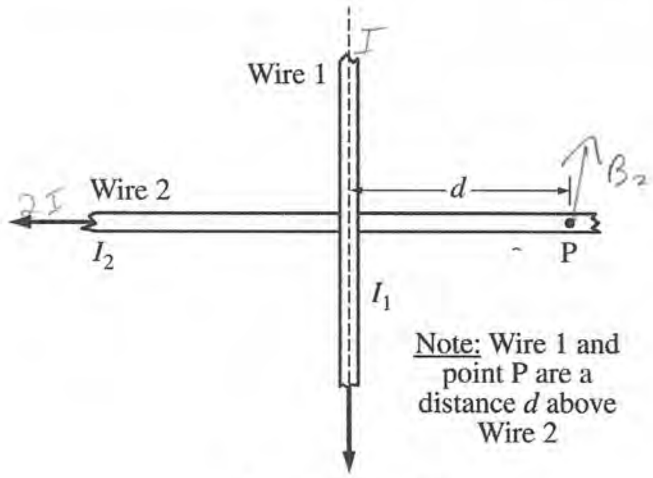


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

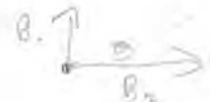
(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

$$\oint B\,dl = \mu_0 I$$
$$B\,2\pi r = \mu_0 I$$
$$B = \frac{\mu_0 I}{2\pi r}$$
$$B_1 = \frac{\mu_0 I}{2\pi d}$$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

$$\oint B\,dl = \mu_0 I$$
$$B = \frac{\mu_0 2I}{2\pi d}$$
$$B_2 = \frac{\mu_0 I}{\pi d}$$
$$\sqrt{\left(\frac{\mu_0 I}{\pi d}\right)^2 + \left(\frac{\mu_0 I}{2\pi d}\right)^2}$$
$$\sqrt{\frac{4\mu_0^2 I^2}{4\pi^2 d^2} + \frac{\mu_0^2 I^2}{4\pi^2 d^2}}$$
$$\sqrt{\frac{5\mu_0^2 I^2}{4\pi^2 d^2}}$$
$$B = \frac{\sqrt{5}\,\mu_0 I}{2\pi d}$$

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.



$$\tan^{-1}\theta = \frac{\frac{\mu_0 I}{2\pi d}}{\frac{\mu_0 I}{\pi d}}$$
$$\tan^{-1}\theta = \frac{1}{2}$$
$$\theta = 26.57^\circ$$

GO ON TO THE NEXT PAGE.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

_✓_ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

Because due to wire 2, the magnetic field at wire 1 is running along wire 1 in the opposite directio. Because they are parallel, there is no magnetic force on wire 1.

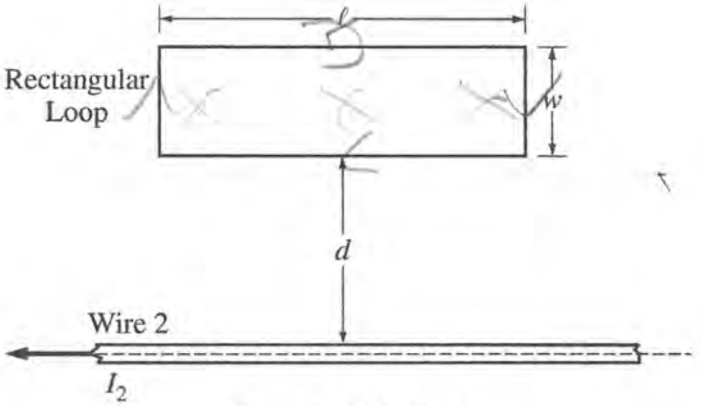


Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1-kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

$\Phi = \int B \cdot dA$

_✓_ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

$\frac{\mu_0 I}{2\pi r}$

$\frac{\mu_0 2 I_0 (1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

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(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

$$\frac{d\Phi}{dt} = \varepsilon = \ln\left(\frac{d+w}{d}\right)\frac{\mu_0 I_0 \ell}{\pi}(-k)$$

$$I = -\ln\left(\frac{d+w}{d}\right)\frac{\mu_0 I_0 \ell k}{\pi}$$

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

_✓_ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

There is a clockwise current because the B flux through the loop is into the page and it's decreasing with time, So there needs to be an induced current that produces a field that is into the page to counteract the decreasing flux. Therefore there needs to be a clockwise loop.

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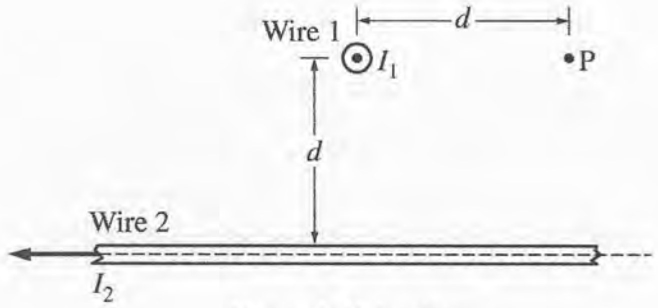

Figure 1. Side view

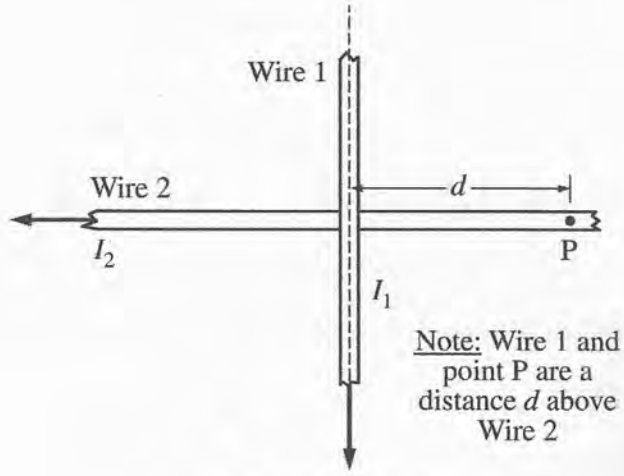

Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance *d* above wire 2. Point P is a horizontal distance *d* from wire 1 and a distance *d* directly above wire 2. Express your answers to parts (a) and (b) in terms of I, *d*, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

$\oint B\,dl = \mu_0 I_{encl}$

$B \oint dl = \mu_0 I_{encl}$ $\quad B = \frac{\mu_0 I}{2\pi d}$

$B(2\pi d) = \mu_0 I$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

$\otimes + \psi$

$B_2 = \frac{\mu_0 2I}{2\pi d}$

$B_{tot} = \sqrt{\left(\frac{\mu_0 2I}{2\pi d}\right)^2 + \left(\frac{\mu_0 I}{2\pi d}\right)^2}$

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

$\tan(\theta) = \frac{B_1}{B_2}$

$\tan^{-1}\left(\sqrt{\left(\frac{\mu_0 2I}{2\pi d}\right)^2 + \left(\frac{\mu_0 I}{2\pi d}\right)^2}\right) = \theta$

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

_X_ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

The B field resulting from the Wire 2 points opposite to the current of wire 1, thus when performing the cross product between current in wire 1 and B field, the direction of the resulting force is to the right, tangent to the bottom of the wire, thus rotating it counter clockwise

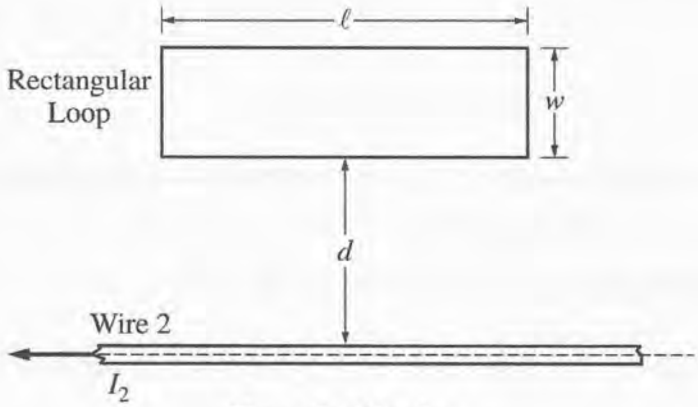

Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1-kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

_X_ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

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(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

$$\frac{d\Phi}{dt} = \mathcal{E} \qquad \left(\frac{\mu_0 I_0 l - \mu_0 I d k t}{\pi}\right) \ln\left(\frac{d+w}{d}\right)$$

$$\mathcal{E} = IR \qquad \frac{-\mu_0 I_0 l k}{\pi}\left(\ln\left(\frac{d+w}{d}\right)\right)$$

$$I = \frac{\mathcal{E}}{R}$$

$$\frac{\frac{d\Phi}{dt}}{R} = I \qquad \frac{\frac{d\Phi}{dt}}{R} = \boxed{\frac{\frac{-\mu_0 I_0 l k}{\pi}\left(\ln\left(\frac{d+w}{d}\right)\right)}{R} = I}$$

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

_X_ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

B field points inwards but due to Lenz's law, the induced field points outword. The resulting current due to the right hand rule, states that current travels clockwise when the field points out.

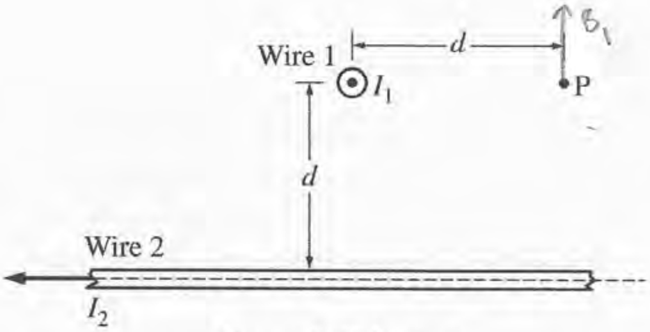

Figure 1. Side view

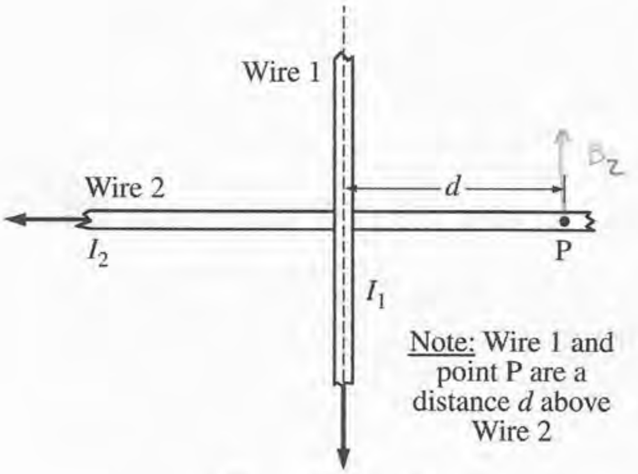

Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

$\oint B_1 \cdot dl = \mu_0 I_1$

$B_1 = \frac{\mu_0 I_1}{2\pi r} = \boxed{\frac{\mu_0 I}{2\pi d}}$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

$\oint B_2 \cdot dl = \mu_0 I_2$

$B_2 = \frac{\mu_0 I}{\pi d}$ $\quad \Sigma B_P = \sqrt{\left(\frac{\mu_0 I}{2\pi d}\right)^2 + \left(\frac{\mu_0 I}{\pi d}\right)^2}$

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

X Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

Wire 1 is parallel to the B field of wire 2, so it will not experience a force due to $F = I\ell \times B$ (must have perpendicular components).

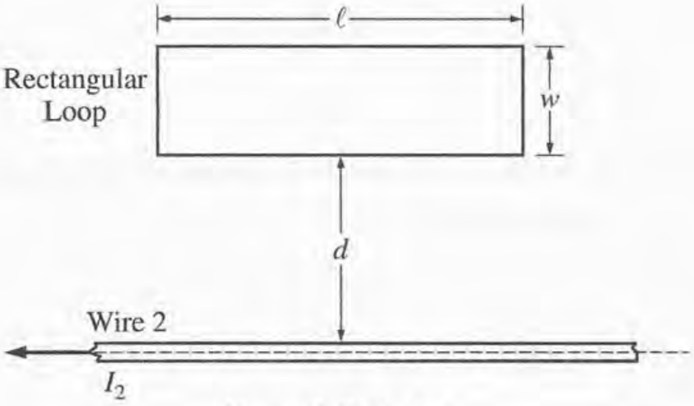

Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

X $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

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(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell(1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

$$\mathcal{E} = \frac{-d\Phi}{dt}$$

$$I = \frac{\frac{-d\phi}{dt}}{R}$$

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

_X_ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

Due to Lenz's Law, the loop will wish to oppose the decreasing B field from the wire by increasing B in the same direction, and Right Hand Rule dictates a clockwise current for a B field into the page.

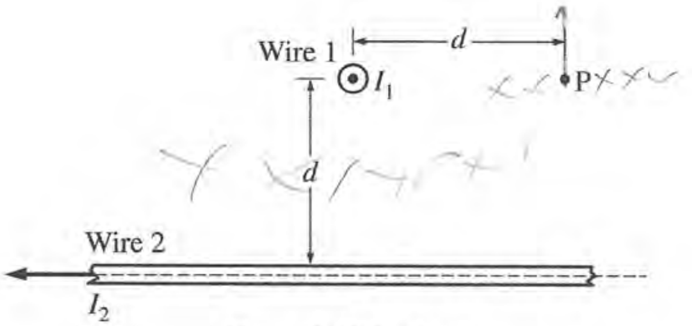

Figure 1. Side view

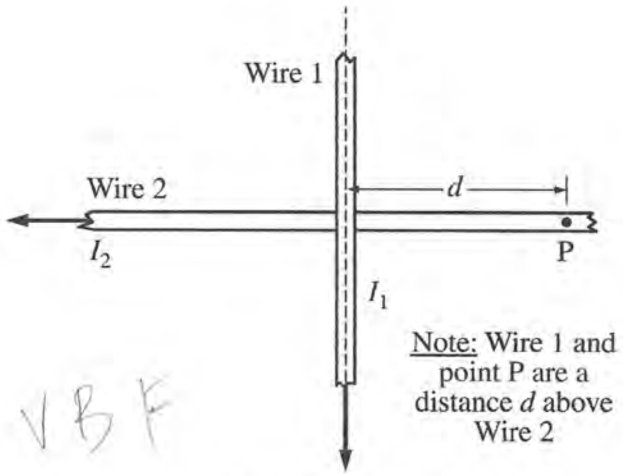

Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B = \frac{\mu_0 I_1}{2\pi d}$$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

$$B_1 = \frac{\mu_0 I_1}{2\pi d} \quad B_2 = \frac{\mu_0 I_2}{2\pi d} \quad B_{total} = \sqrt{\left(\frac{\mu_0 I_1}{2\pi d}\right)^2 + \left(\frac{\mu_0 I_2}{2\pi d}\right)^2}$$

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

$|B_1| = |B_2|$ so angle $= 45°$

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

_✓_ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

$F = BI\ell$

$F = \frac{\mu_0 I_1 I_2}{2\pi d}\ell$

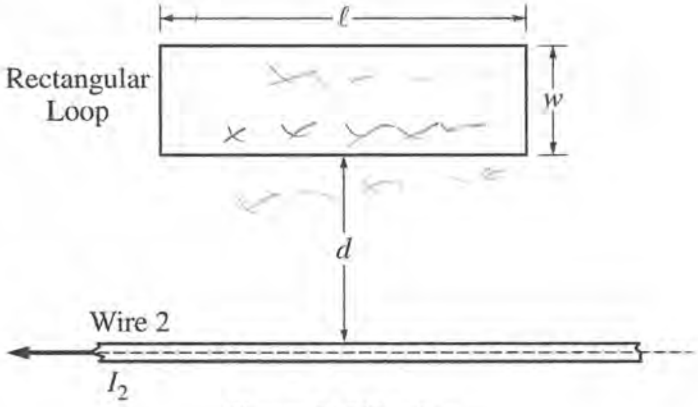

Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

_✓_ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

$\frac{dB}{dt}$ A

Question 3 continues on the next page.

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(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

$$\phi = \frac{\mu_0 I_0 l(1-kt)}{\pi} \ln\left(\frac{d+w}{d}\right)$$

$$\mathcal{E} = \frac{d\Phi}{dt} = IR$$

$$\frac{d}{dt}\left(\frac{\mu_0 I_0 l}{\pi}(1-kt)\ln\left(\frac{d+w}{d}\right)\right) = IR$$

$$I = R\frac{\mu_0 I_0 l}{\pi}\ln\left(\frac{d+w}{d}\right)(-k)$$

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

_✓_ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

Lenz's Law

original strong flux into page

final weak flux into page

response restore flux into page,

So clockwise current

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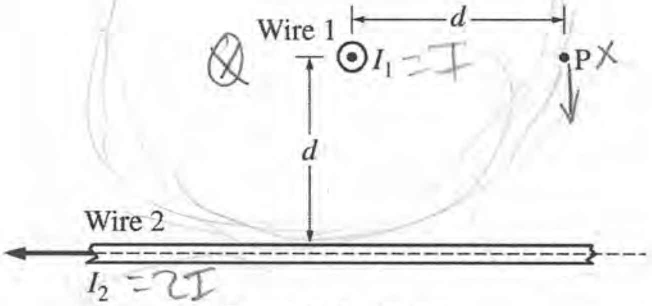
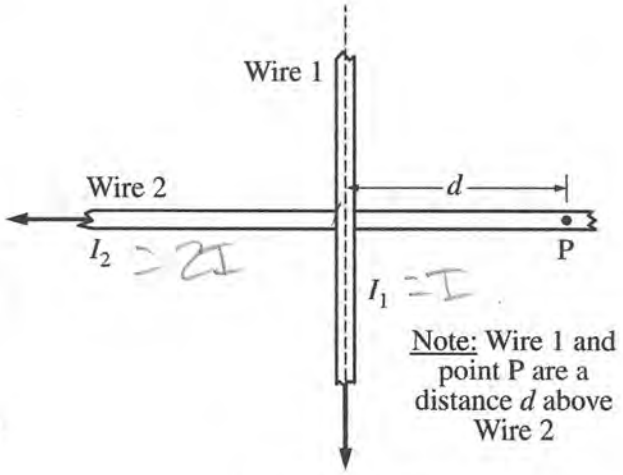


Figure 1. Side view



Note: Wire 1 and point P are a distance d above Wire 2

Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

$\oint B \cdot dl = \mu_0 I$

$B \int_0^{2\pi} r\,d\theta = I\mu_0$

$B = \dfrac{I\mu_0}{2\pi d}$

$dl = r\,d\theta$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

$\frac{1}{4} + 1 = \frac{5}{4}$

$B_1 = \dfrac{I\mu_0}{2\pi d}$

$B_2 = \dfrac{I\mu_0}{\pi d}$

$\sqrt{\left(\dfrac{I\mu_0}{2\pi d}\right)^2 + \left(\dfrac{I\mu_0}{\pi d}\right)^2}$

$\dfrac{\sqrt{5}}{2} \dfrac{I\mu_0}{\pi d}$

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

$\tan\theta = \dfrac{\frac{I\mu_0}{2\pi d}}{\frac{I\mu_0}{\pi d}} = \dfrac{1}{2}$

$\tan\theta = \frac{1}{2}$

$\boxed{26.6°}$

GO ON TO THE NEXT PAGE.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

X Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

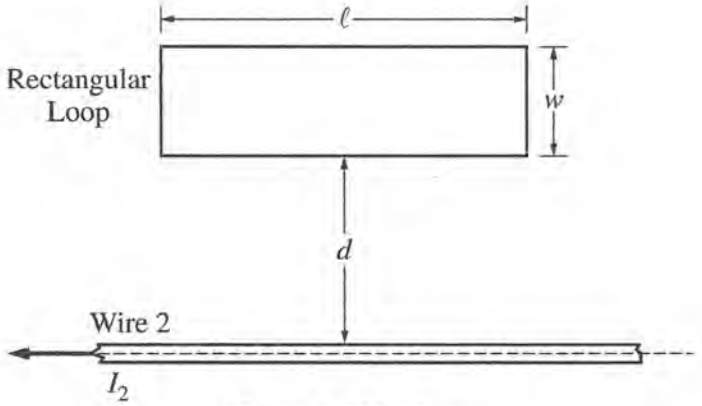


Figure 3. Side view

BA cosθ

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1 - kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

X $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

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(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

$i =$

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise X Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

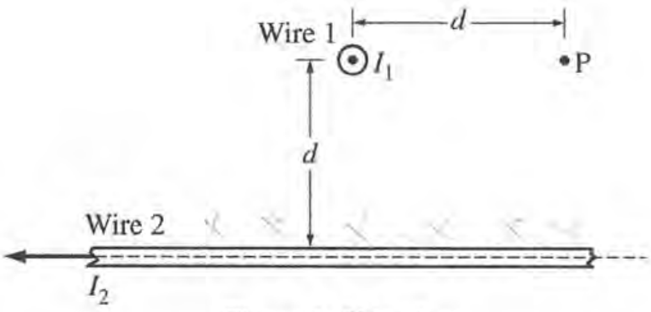

Figure 1. Side view

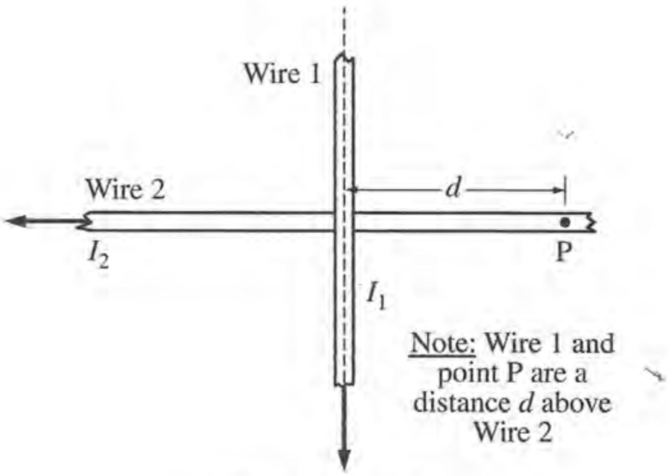

Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$B\,l = \mu_0 I$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

GO ON TO THE NEXT PAGE.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

__✓__ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

The current flowing through wire 1 is parallel to the B field produced by wire 2 at that location. Magnetic force is a cross product that needs perpendicular components.

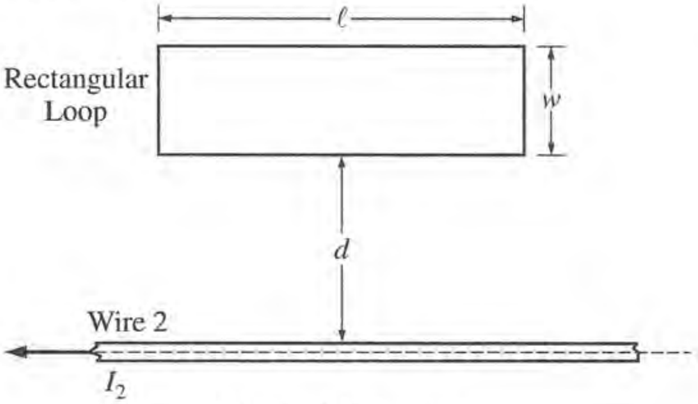

Figure 3. Side view

Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1-kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

GO ON TO THE NEXT PAGE.

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

_✓_ Clockwise ____ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

Current for wire 2 (I_2) is decreasing, which decreases the flux through the loop. The loop opposes change. Flux into the page is decreasing, and the loop wants it to stay the same, so it will induce a current that will create a B field into the page. A clockwise current will increase flux into the page.

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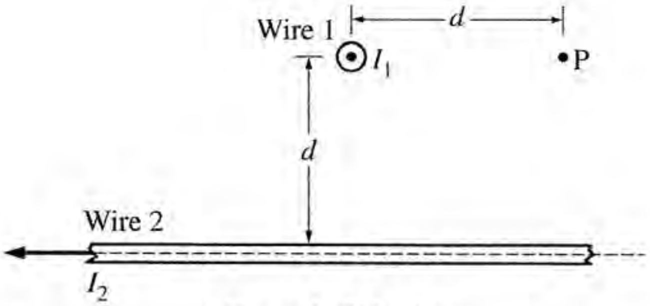


Figure 1. Side view

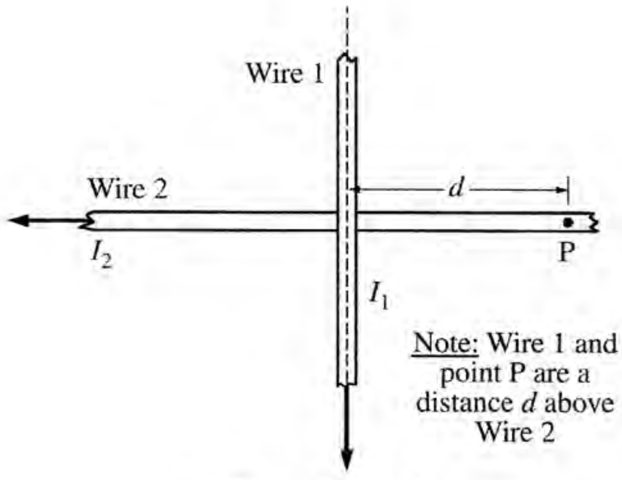


Figure 2. Top view

3. The figures above represent different views of two long, straight, horizontal wires, 1 and 2, carrying currents $I_1 = I$ and $I_2 = 2I$, respectively, in the directions shown. The wires are held in place. In Figure 1, the current in wire 1 is directed out of the page, and wire 1 is a distance d above wire 2. Point P is a horizontal distance d from wire 1 and a distance d directly above wire 2. Express your answers to parts (a) and (b) in terms of I, d, and physical constants, as appropriate.

(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

$$\oint B \cdot dl = \mu_0 I$$

$$Bl = \mu_0 I$$

$$B = \frac{\mu_0 I}{d}$$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

W1 - $B = \frac{\mu_0 I}{d}$ W2 = $B = \frac{2\mu_0 I}{d}$

$$B_{net} = \sqrt{\left(\frac{\mu_0 I}{d}\right)^2 + \left(\frac{2\mu_0 I}{d}\right)^2} = \sqrt{\frac{5\mu_0^2 I^2}{d^2}}$$

$$= \sqrt{\frac{\mu_0^2 I^2}{d^2} + \frac{4\mu_0^2 I^2}{d^2}} = \boxed{\frac{\sqrt{5}\,\mu_0 I}{d}}$$

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

45^0

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

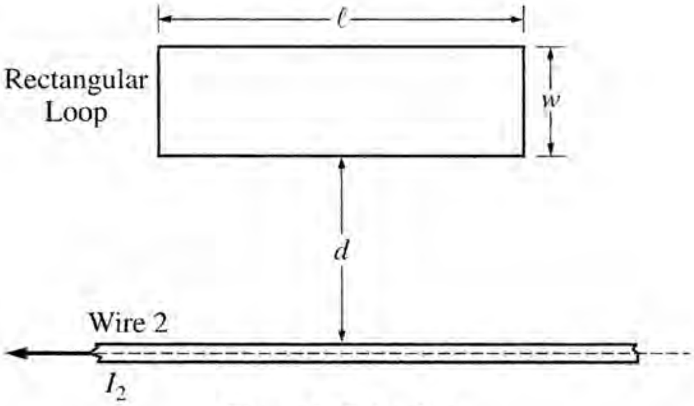
_x_ Wire 1 will move upward as viewed in Figure 1.

____ Wire 1 will move downward as viewed in Figure 1.

____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.



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Wire 1 is now replaced by a conducting rectangular loop of length ℓ, width w, and resistance R. The loop is placed a distance d from wire 2, as shown. The loop, wire, and distance d are all in the plane of the page. The long side of the loop is parallel to the wire. The current I_2 for wire 2 is decreasing linearly as a function of time t according to the equation $I_2 = 2I_0(1-kt)$, where k is a positive constant with units of s^{-1}.

(e) Of the following, select the integration that will give an expression for the flux Φ as a function of time t.

_X_ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)\ell w}{2\pi} dr$

____ $\Phi = \int_{r=d}^{r=d+w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

GO ON TO THE NEXT PAGE.

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \frac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\frac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

$\Phi = \int B \cdot dA$ $\qquad \oint B \, dl = \mu_0 I$

$= BA$ $\qquad B l = \mu_0 I$

$= B l w$ $\qquad B = \frac{\mu_0 I}{l}$

$$\frac{\mu_0 I_0 l (1 - kt)}{\pi} \ln\left(\frac{d+w}{d}\right) = \frac{\mu_0 I}{\cancel{l}} \cancel{l} w$$

$$\frac{1}{\mu_0 w} \left(\frac{\mu_0 I_0 l (1-kt)}{\pi}\right) \ln\left(\frac{d+w}{d}\right) = I$$

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise $\qquad$ __X__ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

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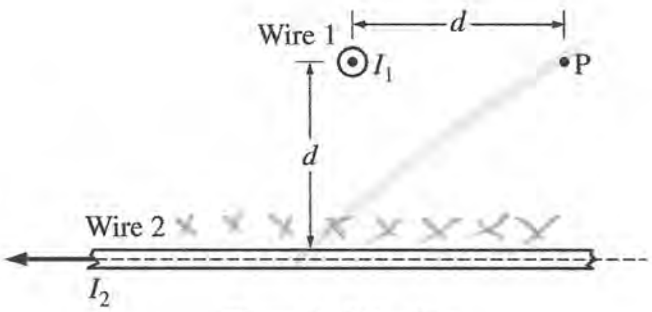

Figure 1. Side view

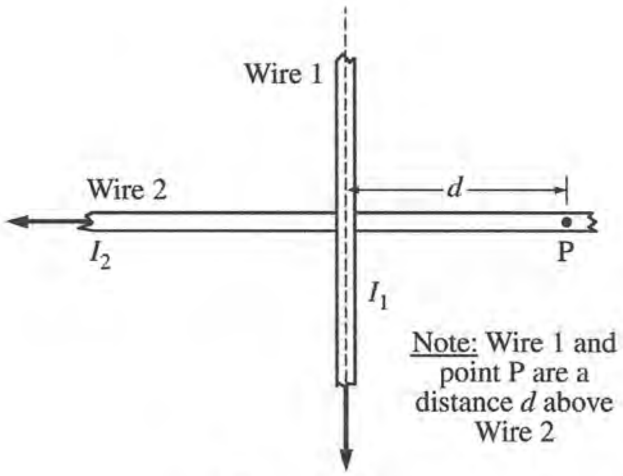

Figure 2. Top view

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(a) Use Ampere's law to derive an expression for the magnitude of the magnetic field at point P due to wire 1.

$B = \frac{\mu_0 I}{2\pi d}$ $\qquad dB = \frac{\mu_0 I \, d\vec{l} \times \hat{r}}{4\pi r^2}$

(b) Derive an expression for the magnitude of the net magnetic field at point P.

$B = \frac{\mu_0 I}{2\pi d} + \frac{\mu_0 (2I)}{2\pi\sqrt{2}\, d}$

$= \frac{\sqrt{2}\mu_0 I + 2\mu_0 I}{2\pi\sqrt{2}\, d}$

(c) Calculate the numerical value of the angle to the horizontal for the direction of the net magnetic field at point P.

(d) Wire 1 is now released. Which of the following best describes the initial motion of wire 1 due to the magnetic field of wire 2 ? Assume gravitational effects are negligible.

____ Wire 1 will not move.

____ Wire 1 will move upward as viewed in Figure 1.

_✓_ Wire 1 will move downward as viewed in Figure 1.

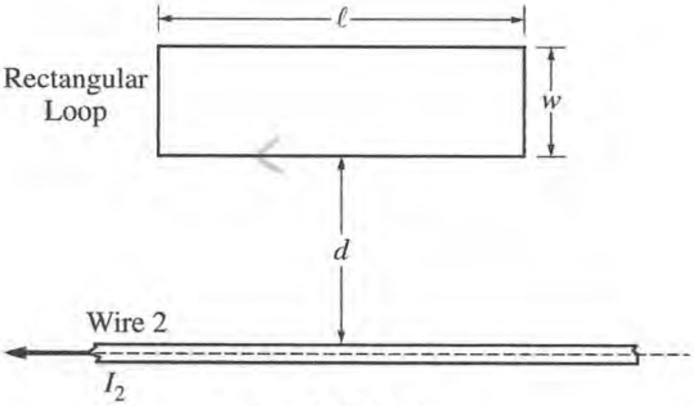
____ Wire 1 will rotate clockwise as viewed in Figure 2.

____ Wire 1 will rotate counterclockwise as viewed in Figure 2.

Justify your answer.

~~Because~~

Because of the right hand rule the magnetic field B will be into the page due to the current in I_2 traveling to the left. Thus when wire 1 is released will travel down because of the downward force from wire 2.



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____ $\Phi = \int_{r=d}^{r=w} \frac{\mu_0(2I_0)(1-kt)}{2\pi r} \ell dr$

Question 3 continues on the next page.

GO ON TO THE NEXT PAGE.

(f) Given that the flux through the rectangular loop as a function of time t is given by the equation $\Phi = \dfrac{\mu_0 I_0 \ell (1 - kt)}{\pi} \ln\left(\dfrac{d + w}{d}\right)$, derive an expression for the magnitude of the current, if any, induced in the loop. Express your answers in terms of I_0, d, r, R, w, k, ℓ, and physical constants, as appropriate.

(g) What is the direction of the current, if any, induced in the loop as seen in Figure 3 ?

____ Clockwise __✓__ Counterclockwise

____ Undefined, because there is no current induced in the loop

Justify your answer.

there is an attrative force between the wires so the currents must travel in the same direction

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Question 3

Sample Identifier: E Q3 A
Score: 14

- a) Full credit ... 2 pts
- b) Full credit ... 2 pts
- c) Full credit (Note that credit is given for substitution, so numerical answer for the angle is not necessary) ... 2 pts
- d) Full credit ... 2 pts
- e) Full credit ... 1 pt
- f) Full credit ... 3 pts
- g) 1 pt for correct selection with attempted justification, 1 pt for showing understanding of Lenz's law, but no metion of the flux

Sample Identifier: E Q3 B
Score: 13

- a) 1 pt for correct answer, but did not show Ampere's law
- b) Full credit ... 2 pts
- c) Full credit ... 2 pts
- d) 1 pt for showing understanding of net force = 0, but no mention of torques
- e) Full credit ... 1 pt
- f) Full credit ... 3 pts
- g) Full credit ... 3 pt

Sample Identifier: E Q3 C
Score: 12

- a) Full credit ... 2 pts
- b) Full credit ... 2 pts
- c) Full credit ... 2 pts
- d) 1 pt for showing understanding of net force = 0, but no mention of torques
- e) Full credit ... 1 pt
- f) 1 pt for taking the derivative of the flux, but no use of Ohm's law and incorrect answer
- g) Full credit ... 3 pt

Sample Identifier: E Q3 D
Score: 10

- a) Full credit … 2 pts
- b) Full credit … 2 pts
- c) 1 pt for relating angle to the fields, but substitution is incorrect
- d) Incorrect analysis of forces and torques … No credit
- e) Full credit … 1 pt
- f) Full credit … 3 pts
- g) 1 pt for correct selection with attempted justification, but no metion of the flux and insufficient explanation of Lenz's law

Sample Identifier: E Q3 E
Score: 9

- a) Full credit … 2 pts
- b) Full credit … 2 pts
- c) Blank … No credit
- d) 1 pt for showing understanding of net force = 0, but no mention of torques
- e) Full credit … 1 pt
- f) 1 pt for proper use of Ohm's law, but did not differentiate the flux and incorrect answer
- g) 1 pt for correct selection with attempted justification, 1 pt for showing understanding of Lenz's law, but no metion of the flux

Sample Identifier: E Q3 F
Score: 8

- a) Full credit … 2 pts
- b) 1 pt for vector sum expression, but did not indicate that $B_2 = 2B_1$
- c) No expression relating the angle to the fields … No credit
- d) No justification … No credit
- e) Incorrect selection … No credit
- f) 1 pt for taking the derivative of the flux, 1 pt for correct use of Ohm's law, but incorrect answer
- g) Full credit … 3 pt

Sample Identifier: E Q3 G
Score: 6

- a) Full credit … 2 pts
- b) Full credit … 2 pts
- c) Full credit … 2 pts
- d) No justification … No credit
- e) Incorrect selection … No credit
- f) Blank … No credit
- g) Blank … No credit

Sample Identifier: E Q3 H
Score: 5

- a) 1 pt for using Ampere's law but incorrect answer
- b) Blank ... No credit
- c) Blank ... No credit
- d) 1 pt for showing understanding of net force = 0, but no mention of torques
- e) No selection made ... No credit
- f) Blank ... No credit
- g) Full credit ... 3 pt

Sample Identifier: E Q3 I
Score: 3

- a) 1 pt for using Ampere's law, but incorrect answer
- b) Full credit ... 2 pts
- c) Did not relate angle to fields and no substitution ... No credit
- d) No justication ... No credit
- e) Incorrect selection ... No credit
- f) Did not differentiate the flux, no use of Ohm's law, and incorrect answer ... No credit
- g) No justication ... No credit

Sample Identifier: E Q3 J
Score: 1

- a) 1 pt for correct answer, but did not show Ampere's law
- b) No vector sum expression and did not indicate that $B_2 = 2B_1$ ... No credit
- c) Blank ... No credit
- d) Incorrect analysis of forces and no mention of torques ... No credit
- e) No selection ... No credit
- f) Blank ... No credit
- g) Incorrect selection and no mention of flux or Lenz's law ... No credit